Play and Learning at Ulster and Sheffield

PLUS

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University of Sheffield
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Play and Learning at Ulster and Sheffield (PLUS)

Executive summary

Children's early mathematics skills are predicted by how much their parents engage in Home Mathematics Activities. Differences in early mathematics skills are closely linked to socioeconomic circumstances (SEC) and predict later participation in advanced maths courses in high school and the likelihood of attending university. This project aimed to explore the relation between home engagement in mathematics and young children's mathematics skills, and how to support families with home mathematics.

This project had three goals: 1) to better understand the relation between Home Mathematics Activities and early maths development by collating and re-analysing existing quantitative datasets; 2) to co-create with parents and early years professionals a set of Home Mathematics Activities that can support 3- to 5-year-olds' early maths skills and put them at the centre of an early intervention; 3) to pilot implementing and evaluating the intervention in Northern Ireland and Northern England, checking how feasible it is for parents to implement and for researchers to evaluate.

<u>**Key findings**</u>

1. Secondary data analysis: Engagement with home mathematics activities and early mathematical skills
We replicated findings of a small positive association between the frequency of engagement with home mathematics activities and children's mathematics skills. Socio-economic circumstances were related to early mathematics skills but not to how often families engaged in home mathematics activities.

2. Co-production: Development of a home mathematics intervention
Through iterative co-production with parents and early years educators, we created a set of home mathematics activities that both groups judged to be **enjoyable** and **appropriate** for 3-5-year-olds and that formed the basis of the early mathematics intervention.

3. Feasibility and pilot study outcomes
After a small-scale promising feasibility study with 11 families, we conducted a

larger-scale pilot study. The pilot involved 84 families who were placed in either the early mathematics intervention group or the matched active control group. The activities were **feasible for both groups to implement**, with **engagement rates above a pre-set threshold** and **positive parent feedback**. Parents adhered to the engagement schedule and instructions, and randomisation, double-blind procedures, and child assessments were all viable.

Recommendations

For researchers: Recruitment: Despite diverse recruitment routes, almost all participating parents had university degrees, raising concerns about whether the intervention – and the study procedures – appeal to families experiencing socio-economic disadvantage. We recommend further qualitative research to understand how to engage with families experiencing socio-economic disadvantage more effectively. We also suggest careful consideration of where any in-person testing takes place. Future recruitment may be better conducted in collaboration with social economy daycares, pre-schools, nurseries or Family Hubs to encourage a more diverse range of families with engagement. On-site assessments during children's sessions would reduce parental burden.

For researchers: Clear communication about activities at different levels: Parents liked our videos which introduced home mathematics activities with suggestions of easier alternatives, yet parents occasionally felt some activities were too advanced. Future work should focus on clear communication, specifically for the easier options. Additional short videos could be offered as parents liked this resource.

For researchers: Understanding how families engage with resources: A great deal of intervention level information was collected about engagement with the study and activities; however, less is known about how families used the materials at home. Collecting data during real interactions, such as observing play and language use, may help refine future guidance.

For funders and researchers: Assessments: A longer-term follow-up, such as at three months post-intervention, is recommended. This would allow time for parents' practices to embed and for learning to consolidate.

For funders and researchers: Building on this project: This study was purposefully not designed to assess the efficacy of our intervention. Given the overwhelmingly positive feasibility and acceptability information was gathered in this project in response to our intervention we suggest that a future, large scale study should be conducted to assess efficacy, when our other recommendations have been actioned.

Take home points for policy makers on working with families: This project highlights the importance of working closely with families when developing interventions to be used at home. Diverse family perspectives should be incorporated into the design from the outset and needs must be accommodated. Feasible and realistic approaches must be used to ensure the best chance for intervention success.

Play and Learning at Ulster and Sheffield (PLUS)

Introduction

A vast body of research suggests that there is wide variation in children's mathematics skills at school entry (e.g. Davis-Kean et al., 2022). This variation is associated with socio-economic circumstances and is predictive of uptake of mathematics courses at high school as well as college attendance. Disparities in mathematical skills widen throughout childhood (Caro et al., 2009), and are negatively associated with health, income, and quality of life, perpetuating cycles of inequality (*Numeracy for Health*, 2015).

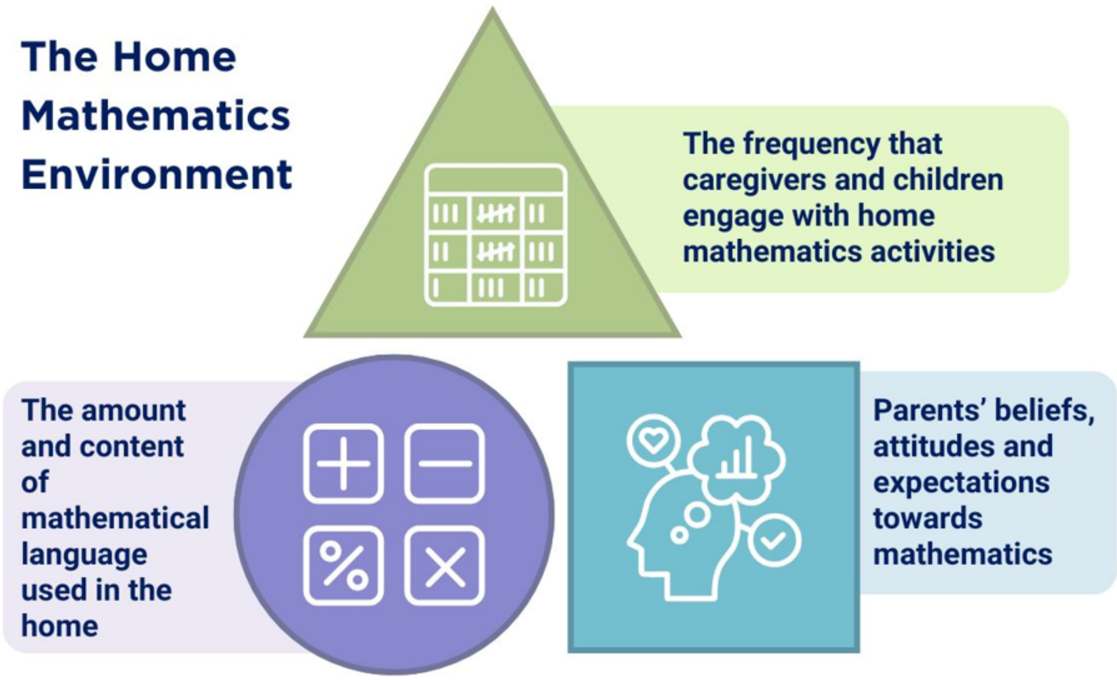
The home mathematics environment

The home mathematics environment (HME) is thought to provide an important basis for children's mathematics skills (Anders et al., 2012; Byrnes & Wasik, 2009; L. Elliott & Bachman, 2018a; J. LeFevre et al., 2010; Muñez et al., 2021), with access to resources (such as books) and parent practices that support learning in the early years being positively related to later academic success (Ribner et al., 2019). Research on early home learning experiences has traditionally focused on literacy development (Lefevre, 2000; Ramani & Siegler, 2015). Given mathematics has been found to be a stronger predictor of overall academic attainment compared to reading skills or attention (Duncan et al., 2007), interest has shifted toward the relationship between families' engagement in home mathematics activities and children's mathematics skills (LeFevre et al., 2009). Recommendations have been made for supporting home mathematics activities in the early years, such as engaging in early quantitative reasoning in the home, with a view to building on these experiences at school entry (Clements, 2010). Overall, findings from numerous studies indicate that home mathematics activities are important for the learning of mathematical concepts (Daucourt et al., 2021; Phillipson et al., 2017) and are associated with children's future mathematics skills (Cahoon et al., 2017). Nevertheless, further research is required to determine if there is a causal relation between doing certain mathematics activities and children's developing mathematics skills, and if there is, what type, format and approach is needed to increase the uptake of home mathematics activities.

What is the home mathematics environment?

The HME encompasses a range of mathematical experiences that may benefit children's mathematical skills and understanding (De Keyser et al., 2020). Typically, researchers commonly measure the HME through the presence of one or more mathematical aspect of the home environment (Figure 1).

Figure 1. The Home Mathematics Environment.



Note: The terms 'beliefs' and 'attitudes' are often used interchangeably in the literature, though they represent distinct constructs within the home mathematics environment.

Caregiver beliefs, attitudes, and expectations

Caregiver beliefs and attitudes are related to children's mathematical skills (LeFevre et al., 2010; Skwarchuk et al., 2014; Susperreguy et al., 2020). Positive beliefs and attitudes towards mathematics may positively impact caregivers' motivation to engage in mathematics activities with their children (De Keyser et al., 2020). Research literature focuses on both parents' beliefs about their own mathematics skills and in relation to their own children's performance (such as what is required for children to be considered 'good' at mathematics; Yılmaz et al., 2025). In addition, positive caregiver attitudes (such as "I enjoy maths) towards mathematics has been observed to be related to the frequency of engagement in home mathematics activities (LeFevre et al., 2010; Missall et al., 2015; Susperreguy et al., 2020), and children's mathematical skills (Kleemans et al., 2012; Susperreguy et al., 2020). Caregivers' expectations, both short and long-term, are also observed to be related to engagement in mathematics activities (Skwarchuk et al., 2014) and children's numerical skills (Segers et al., 2015).

Mathematical language in the home environment

Caregivers use of mathematical language (or maths talk) when interacting with their children predicts children's mathematical skills upon reaching school (Casey et al., 2018; L. Elliott & Bachman, 2018b; Gibson et al., 2020; Purpura et al., 2017; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016) and the effects of increased caregiver maths talk on children's maths skills have even been detected up to one year after experiencing interventions (Susperreguy & Davis-Kean, 2016). Commonly, the occurrence and quality of maths talk are generally collected through structured or unstructured observations (i.e. researcher planned interactions vs natural interactions). Structured cooperative activities have indicated a concurrent relation between maternal maths talk during play and child mathematics skills (Gürgah Oğul & Aktaş Arnas, 2021; Ramani et al., 2015). In a natural observation study of families eating meals together, the frequency of maternal maths talk was related to their child's mathematics skills one year later, after controlling for maternal education, child self-regulation, and the length of time of recordings (Susperreguy & Davis-Kean, 2016).

Frequency of engagement with home mathematics activities

A wealth of literature has investigated the relation between engagement with home mathematics activities and children's mathematical skills (see James-Brabham et al., 2024). Typically, caregivers complete self-report questionnaires documenting their frequency of engagement with a variety of home mathematics activities (e.g. from never to daily) over a prescribed time (e.g. weekly, monthly, etc).

Overall, engagement with home mathematics activities has been found to be related to children's mathematics skills across the globe with increased levels of engagement with mathematics activities in the home typically being positively associated with children's performance on measures of mathematical skills (Daucourt et al., 2021; Ellis et al., 2023; Hunt et al., 2025; James-Brabham et al., 2024; LeFevre et al., 2009; Skwarchuk et al., 2014). However, there are some inconsistencies, with some studies finding both negative (Blevins-Knabe et al., 2000; Ciping et al., 2015), and null associations (Hornburg et al., 2021; James-Brabham et al., 2023; Missall et al., 2015; Zhou et al., 2006) between engagement with home mathematics activities and children's mathematics skills. Some explanations for these inconsistencies lie in how home mathematics activities have been measured, age groups of children in samples, and between country differences in child-care and education systems. In addition, given that when observed, the relation between engagement with home mathematics activities and children's mathematics skills is small (e.g. Cohen's *d*= .1 to .2), mixed results may also be explained by many studies including sample sizes that are too small to detect this effect (De Keyser et al., 2020).

The potential influence of socio-economic circumstances on the HME

Evidence indicates that there is wide variation in children's mathematics skills at school entry, with family socio-economic circumstances (SEC) explaining some of these differences (Caro et al., 2009; Jordan & Levine, 2009). Typically, parental education levels are measured and used as an indicator of SEC, as this factor represents an important aspect of SEC (Hornburg et al., 2021). SEC related disparities in mathematics have been detected as young as age 3 (Blakey et al., 2020; James-Brabham et al., 2023; Sarama & Clements, 2009) and can have lasting effects on children's mathematical performance (Crosnoe et al., 2010; L. Elliott & Bachman, 2018b). Differences in engagement with home mathematics activities may partly explain these gaps, as higher-SEC families often have more resources, time, and confidence to support learning at home, consistent with the family investment model (Conger & Donnellan, 2007).

In contrast to the literature on the association between SEC and children's mathematics skills, the literature on the relation between SEC and the home mathematics environment is mixed and complex. While some studies suggest SEC predicts the frequency, type, and variety of home mathematics activities (Del Río et al., 2017; Galindo & Sonnenschein, 2015), others find minimal (Pan et al., 2018) or no relation at all (James-Brabham et al., 2023). Relatedly, some studies have reported parents from lower SEC backgrounds use mathematical language to a lesser degree than parents from high SEC backgrounds (Levine et al., 2010). However, other research teams have observed no relation between SEC and mathematical language (e.g., Leech et al., 2022; Thippana et al., 2020). The lack of consistency in these research findings highlight the need for clearer understanding before designing interventions to mitigate SEC-related attainment gaps.

Home mathematics environment interventions

Interventions to promote parent support of learning have been observed to increase child developmental outcomes (Jeong et al., 2021), and interventions that include parents are regarded as important for reducing social disadvantage (Clarke & Younas, 2017). A recent systematic review and meta-analyses focusing on home-based interventions for mathematics and literacy outcomes of children aged 3-5 years-old established three main findings (Cahoon et al., 2023). Primarily, there were substantially more interventions focusing on literacy rather than mathematics outcomes (i.e. 28 vs 10 respectively). Secondly, the overall impact of home-based mathematics interventions was consistent but small (d= .18). Finally, many of the included studies reported interventions that required limited levels of parental engagement, that were relatively low intensity (e.g., receiving text messages; (Baroody et al., 2018). The interventions employed various tasks and resources to encourage engagement in the HME, including mathematics games and storybooks, with the two studies that contributed the most weight to the pooled estimate both using games as the primary intervention activity (de Chambrier et al., 2020; Niklas et al., 2015). This suggests that games may play a particularly important role in mathematics learning.

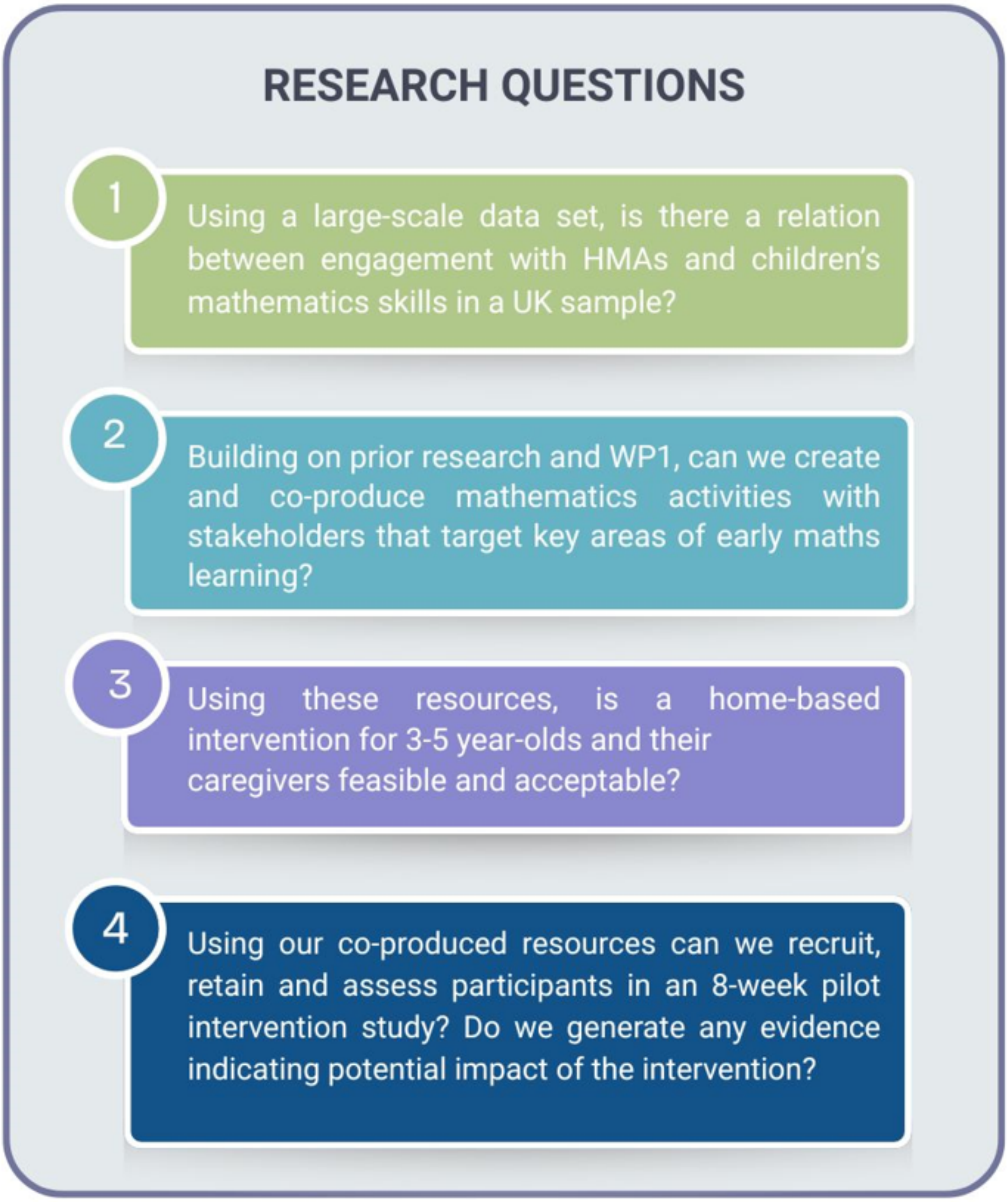
Evidence indicates that preschool-aged children can learn as much, or more, from guided play as from formal instruction (Pesch et al., 2022; Sim & Xu, 2017). Play is thought to facilitate deeper learning by providing abundant opportunity to compare common structures across objects and concepts (Christie, 2022). Moreover, games are enjoyable, which may foster positive emotions during learning (Mayer & Alexander, 2011). The use of games to teach mathematics has long been a research focus (e.g., Bright et al., 1985; Ernest, 1986), and children often show heightened interest in mathematics when numeracy is presented through playful activities (Cheung & McBride, 2017). In addition, games may be particularly beneficial in interventions involving parents who themselves feel negatively about mathematics, as their accessible and playful format can reduce anxiety (Cheung & McBride, 2017).

The feasibility and acceptability of an intervention is predictive of its potential adoption and success (Perepletchikova, 2011; Sanetti & Kratochwill, 2009; Vermilyea et al., 1984). Therefore, the content of home-based interventions must appeal to both parents and young children to sustain engagement and reduce attrition. The use of participatory design and co-production methods has received growing attention in intervention design (Vargas et al., 2022), especially in the context of educational interventions (Honingh et al., 2020). Embedding research users from the beginning of the research process not only ensures that intervention materials are applicable and usable by the target groups, but also the co-production process itself may act as an intervention (Khan & Beltrán-Grimm, 2020).

In the realm of the HME, recent intervention research has aimed to build upon current family routines with the aim of boosting positive mathematical interactions in the home, with intended subsequent benefits for children's mathematics skills. Through co-production processes, researchers have reported that families engage with researchers to understand the value of early mathematics and the potential impact of informal interactions with home mathematics activities for their children's learning (Khan & Beltrán-Grimm, 2020). In addition, Maths Made for Me (MM4M), is a HME intervention which aims to promote positive family mathematics interactions, by building on families' current reported activities, which is tailored to family's needs (Leyva et al., 2023). This intervention used short videos to provide ideas for how families could support mathematics skills and weekly text reminders to engage with mathematics with their children over a 6-week period. Although this study was a pilot study, findings indicated a significant boost in parental mathematical language use (as measured during a video-recorded parent-child play session) and family engagement with home mathematics activities (as measured through a self-report questionnaire). However, no gains in children's mathematics skills were observed.

The current project

Given the previously reviewed literature, the current project aimed to address the following research questions:



Part 1: Using a large-scale data set, is there a relation between engagement with home mathematics activities and children's mathematics skills in a UK sample?[1]

Study Aims and Approach

This study aimed to investigate the types and frequency of home mathematics activities in relation to children's mathematical skills and family SEC using secondary data analysis. Given that little published research has focused on the relation between home mathematics activities and children's mathematics skills in the United Kingdom (Daucourt et al., 2021), this study aimed to combine data from multiple studies to increase understanding of the home mathematics environment and its potential impact in the UK context.

The study took a data harmonisation approach, which combines datasets and allows larger and more diverse datasets to be analysed (Adhikari et al., 2021). Nine UK datasets on 3- to 5-year-olds were combined using data harmonisation, addressing the scarcity of UK studies in this area home mathematics activities measures were grouped into five higher-order categories (for example, operations) based on conceptual similarity. Unlike prior research, which has focused largely on numeracy (Blevins-Knabe et al., 2000), this study included broader domains such as shape and pattern understanding (Ehrman et al., 2023). Confirmatory factor analysis verified these categories. Subsequently, Latent profile analysis (LPA) was adopted, grouping families by patterns of engagement rather than averaging frequency. This can reveal distinct profiles of engagement, such as numeracy-focused or spatial-focused activities, and show how these relate to SEC and children's outcomes (Hickendorff et al., 2018). Ordinal regression was used to assess associations between these subgroups, SEC, and children's mathematical skills. As this was an exploratory analysis, no directional hypotheses were proposed, but both SEC and mathematical skills were expected to be associated with profile membership.

[1] Part 1 is a summary of the following published paper: Hunt, B. W., Cahoon, A., Blakey, E., James-Brabham, E., Matthews, D., & Simms, V. (2025). Is there an association between frequency of home mathematical activities (HMA) and children's mathematical outcomes? Data harmonization and secondary analyses of U.K.-based data sets. *Developmental Psychology*. https://doi.org/10.1037/dev0001987

Methods

Participants

From a potential total of 1,358 participants across nine datasets, only those who provided both mathematical outcome data (child) and home mathematics activities data (parent) were included in the analysis. This resulted in a final sample of 969 parent–child dyads. Of these, 921 provided socioeconomic data in the form of parents' highest educational qualification. Parent participants had a mean age of 35.33 years (SD = 5.51), while children had a mean age of 46.83 months (SD = 5.41, range 35–69 months).

Variables

The key variables from each dataset were extracted (see https://doi.org/10.17605/OSF.IO/7TJCR for details on the variables taken from the nine datasets). These included: (a) home mathematics activities, with corresponding questions, items, and response options; (b) child outcome measures, with standardised and/or unstandardised items; and (c) all available SEC variables. Among the SEC measures, maternal education was the most used across datasets. The extracted variables were then harmonised across datasets (e.g., scaling the responses for equivalence; see below) so that analyses performed on the entire dataset were statistically sound.

Socioeconomic Circumstances

Parents' highest educational qualification was used to index SEC. There were six categories (no qualifications, GCSE, A levels, undergraduate degree, master's degree, PhD). SEC data spanned the range of the socioeconomic spectrum across all nine data sets.

Home Mathematical Activities: Data Harmonisation of Home Mathematics Activities Items

The home mathematics activities questions varied across datasets in how they assessed the frequency of engagement. Some questionnaires used time frames such as *"In the past month…"*, while others referred to *"In an average week…"* (see Supplementary Materials Table 1 for all nine questions). While this inconsistency was noted, the main challenge during data harmonisation was reconciling the different time units used in the Likert scales. To address this, responses capturing weekly frequencies of home mathematics activities engagement (e.g., three times per week) were multiplied by four to approximate a monthly frequency (e.g., 12 times per month). Once the harmonisation process was complete, the original 179 items were mapped onto 36 variables for further analyses.

Mathematics skills

Each dataset included either a standardised measure of mathematics skills (e.g., the British Ability Scales: Early Number Concepts, the Test of Early Mathematics Ability–3, or the Wechsler Preschool and Primary Scale of Intelligence Mathematics subtest) or an unstandardised, domain-specific measure of mathematics skills. For datasets with domain-specific measures, accuracy scores were used to derive an overall mathematics outcome score. Where a dataset contained both standardised and unstandardised mathematics measures, the standardised measure was selected for inclusion in the final dataset. The anonymised dataset and accompanying files (including the key harmonised variables) are available on the Open Science Framework at https://doi.org/10.17605/OSF.IO/7TJCR.

Justification of Variables

By harmonising across the nine data sets, 36 home mathematics activities variables were generated. Following this, only those variables that matched across the majority of data sets (i.e., five or more) were used in the following analyses. Further, variables that contained a large amount of missing data (e.g., >70% missing) were also excluded. This process ultimately led to 14 variables being available for further analyses. After close scrutiny of the literature (e.g., Gilmore, 2023; Milburn et al., 2019; Purpura et al., 2013), the 14 variables were placed into five ‘conceptual categories’ that conformed to early years curriculum guidance (*CEA Curricular Guidance for Pre-school Education*, 2018): Early number: Written number; Early number: Counting and cardinality; Pattern, shape, and space; Operations; Size and quantity. Subsequent analyses were conducted on these five conceptual categories (see Table 1). Finally, to assess goodness of fit, a confirmatory factor analysis (CFA) was conducted on the five conceptual categories. The CFA indicated that the model demonstrated excellent fit for the data ($\chi^2[61] = 123.70$, $p < .001$; CFI = .970; TLI = .955; RMSEA = .033, 90% CI [.024, .041], p-close = 1.00; SRMR = .046).

Table 1. Five conceptual categories with corresponding variables

Conceptual Categories	Variable names	Number of included variables
Early number: Written number	Identifying names of written numbers; Write numbers; Board games; Mathematics activities books	4
Early number: Counting and cardinality	Verbal counting; Counting objects; Using fingers to count; Rhyming related to numbers	4
Pattern, shape, and space	Sorting objects; Building blocks	2
Operations	Scenarios number games; Practice simple sums	2
Size and quantity	Measurements; Discussing quantities with everyday objects	2

Results

Relations Between Home Mathematics Activities, Mathematics Outcomes, and SEC

We explored the correlational relations between the home mathematics activities variables, children’s mathematics skills, and SEC. The five conceptual home mathematics activities categories, mathematical outcome score, and SEC were entered into a correlational analysis. Significant associations were observed between mathematical outcome scores and SEC, Early number: written number, and Operations. SEC was also associated with Size and quantity (Table 2).

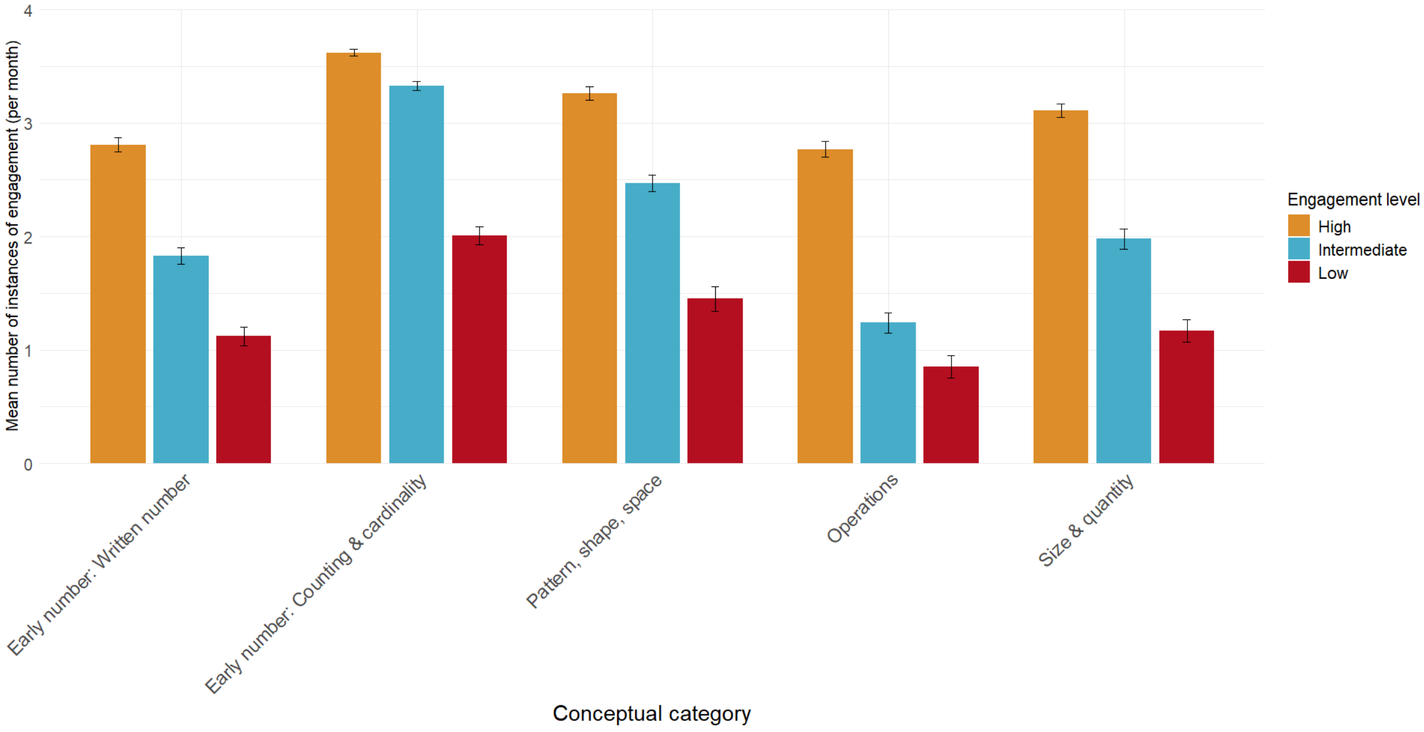
Table 2. Correlational relations between home mathematics activities conceptual categories, child mathematical outcome score, and SEC.

	1	2	3	4	5	6
1. Child maths skills score	-					
2. Socio-economic circumstances	.19***	-				
3. Early number: Written number	.20***	.001	-			
4. Early number: Counting and cardinality	.01	-.01	.41***	-		
5. Pattern, shape and space	-.08	.01	.37***	.47***	-	
6. Operations	.23***	.02	.51***	.28***	.29***	-
7. Size and quantity	.04	.12***	.40***	.37***	.37***	.42***

Latent Profile Analysis

To explore patterns of home mathematics activities engagement, we first calculated average scores for each conceptual category by taking the mean across all variables in that category. We then used Latent Profile Analysis (LPA) to see whether dyads could be grouped into meaningful subgroups based on these scores. Based on the model fit statistics produced by the LPA model, a three-class solution was identified. The model identified 405 dyads in Profile 1, 418 in Profile 2, and 146 in Profile 3. The profiles were designated as being high, intermediate, and low home mathematics activities engagement (Figure 2). Profile 1 was named high home mathematics activities engagement as dyads in this profile had the highest scores in each of the five conceptual categories. The dyads in Profiles 2 and 3 had the second highest and lowest scores in each of the five categories respectively.

Figure 2: Latent profiles across five conceptual categories of early mathematics



Predicting profile membership

Upon identification of the three engagement profiles, we then sought to predict category membership from children's mathematics skills scores and family SEC. The model was significant $X^2(6)= 113.59$, $p<.001$, and met the assumption of parallel lines (proportional odds). Further, the model correctly identified 49% of cases and explained 13% of the variance in home mathematics activities engagement profile membership (Nagelkerke pseudo $R^2= .133$). Dyads with higher children's mathematics skills were found to be 29% more likely to be in a higher engagement profile (B= .26; SE= .07; 95% CI= .13 to .39; OR= 1.29; 95% CI= 1.14 to 1.45; *p*<.001). SEC was not found to be related to engagement profile membership (B= .002; SE= .05; 95% CI= -.10 to .10; OR= 1.00; 95% CI= .90 to 1.10; *p*=.961).

Discussion

This study examined how home mathematics activities relate to children's mathematics skills and whether family socioeconomic circumstances (SEC) influence the activities families do at home. Using a large, harmonised dataset of nine UK studies (*n*= 969), we applied Latent Profile Analysis (LPA) to group families by patterns of home mathematics activities engagement.

The analysis showed that families were best grouped by levels of engagement—high, intermediate, and low—rather than by the types of activities they did. Importantly, children's mathematics skills were positively linked to profile membership: children with stronger skills were more likely to belong to higher engagement profiles. This aligns with previous findings that greater home mathematics activities engagement is associated with stronger mathematics outcomes (e.g., LeFevre et al., 2009; Skwarchuk et al., 2014). Our work extends this research by including a broader range of mathematical concepts (e.g., shape, size), which are often overlooked compared to number-based skills.

A key finding was that family SEC predicted children's mathematics skills but not home mathematics activities frequency. This suggests that early attainment gaps in mathematics are not explained simply by how often families engage in home mathematics activities. Instead, other factors, such as the quality of interactions, the use of mathematical language, or access to resources, may be more influential. Children's early mathematics learning is influenced from birth by a variety of contextual and structural factors. These include families' economic and material resources, parenting practices, cultural capital, and access to high quality early years education, as well as individual characteristics pertaining to the child, including genetics, vocabulary, and cognitive abilities (Ribner et al., 2018; Ruthsatz et al., 2014).

These findings have practical implications. Interventions focused only on increasing the quantity of home mathematics activities may not reduce attainment gaps. Efforts may be better directed toward improving the quality of parent–child interactions and supporting families' access to resources, regardless of SEC.

This study had several strengths. We used a novel, person-centred method (LPA) with a large, harmonised sample, covering the full SEC spectrum. This allowed us to test new hypotheses and extend previous research beyond narrow samples and limited measures. However, some limitations remain. Our data only measured frequency of home mathematics activities, not quality, and did not include wider factors such as language or executive function, which are known to mediate SEC effects. The cross-sectional design also means we cannot draw causal conclusions. Longitudinal studies or, preferably, randomised controlled trials are needed to clarify whether home mathematics activities directly influence children's mathematical development.

In sum, our results show that children's mathematics skills are positively associated with home mathematics activities engagement, but SEC is not. This challenges the idea that lower-SEC families engage less in home mathematics activities and suggests that narrowing early attainment gaps will require focusing on interaction quality and broader aspects of the home learning environment, rather than frequency alone.

Parts 2 to 4: Developing a feasible and acceptable evidence-based intervention

Part 2 to 4 of this report summarises work that aimed to develop a feasible and acceptable evidence-based HME intervention. This process followed the Early Intervention Foundation (EIF) guide on developing effective interventions (Asmussen, 2019).

The EIF guide includes a 10-step process for evaluation success. Parts 2 to 4 of this report includes work that frames and lays out the intervention through to piloting the intervention (Steps 1-5). Specifically, Part 2 of this report (Steps 1-3) focused on the co-production of a home mathematics environment intervention, including specific resources. Part 3 (Step 4) consists of a feasibility study to assess the acceptability and practicalities of the intervention. Part 4 (Step 5) includes a pilot study to gain insight into the feasibility and acceptability of the intervention at a larger scale, identify appropriate outcome measures. Overall, this programme of work allows the study team to robustly develop an intervention which may potentially be evaluated at scale (EIF Stages 6-10) and, if proven effective, receive widespread roll-out.

Part 2: Co-production of resources

Aims and Approach

This element of the project aimed to co-produce resources to be used within a home-based intervention. The findings of previous research, the results of Part 1, the study team's knowledge of mathematical development, and the early years framework were used to identify some suggestions of activities. These initial ideas were brought to co-production workshops with early years educators and parents to generate ideas and discussions for new resources. We identified four key areas for the development of the home mathematics activities for the intervention: (1) to develop age-appropriate, educational mathematics activities, (2) to determine the degree of parent-child enjoyment likely to be experienced when engaging in the activities, (3) the likely attrition rate for this study, and finally, to (4) explore the logistical and practical aspects of participating in the study (e.g., appropriate number of home mathematics sessions per week, methods for parents to give feedback during the intervention).

Why co-production?

Co-production approaches – wherein stakeholders with a vested interest in contributing to a knowledge base are actively involved in the design and delivery of services (Clark et al., 2013; Honingh et al., 2020) – are rapidly becoming considered best practice for improving the effectiveness and relevance of research (Albert et al., 2023). With a focus on collaboration, this approach enables integration of the various needs and expectations of participants (Beltrán-Grimm, 2024). Instead of being passive recipients of research and interventions as in traditional research methods (Ishimaru et al., 2018), the power dynamic is shifted, so that participants take an active role in shaping policies that will ultimately affect them (Bell & Pahl, 2018; Ishimaru et al., 2018) which subsequently allows for trust to develop with the community (Fledderus et al., 2014).

With specific regard to education, co-production research with parents of primary school aged children has been shown to have a positive impact on children's academic skills (Honingh et al., 2020). However, in comparison to co-production of childcare services – an area of research that enjoys a substantial existing literature – research user (i.e., parental) involvement in educational research is relatively lacking (Honingh et al., 2020). In addition, most co-production activities that aim to develop interventions targeting child development and learning involves educators and/or parents rather than children themselves. This may be because it is sometimes not feasible, or practical, to include younger children in the co-production of services or curricula. Parents and educators are naturally well placed to offer insights on the types of pedagogical activities that children may enjoy participating in and receive the most educational benefit from.

Method

Participants

Participants involved in the workshops included seven parents of 3–5-year-old children and five senior early years educators (i.e., headteachers of primary schools). Parent participants were recruited through an educational and community orientated organisation based in Belfast (Northern Ireland, NI). Educators were recruited by contacting the schools directly. Parent and educator sessions were conducted separately.

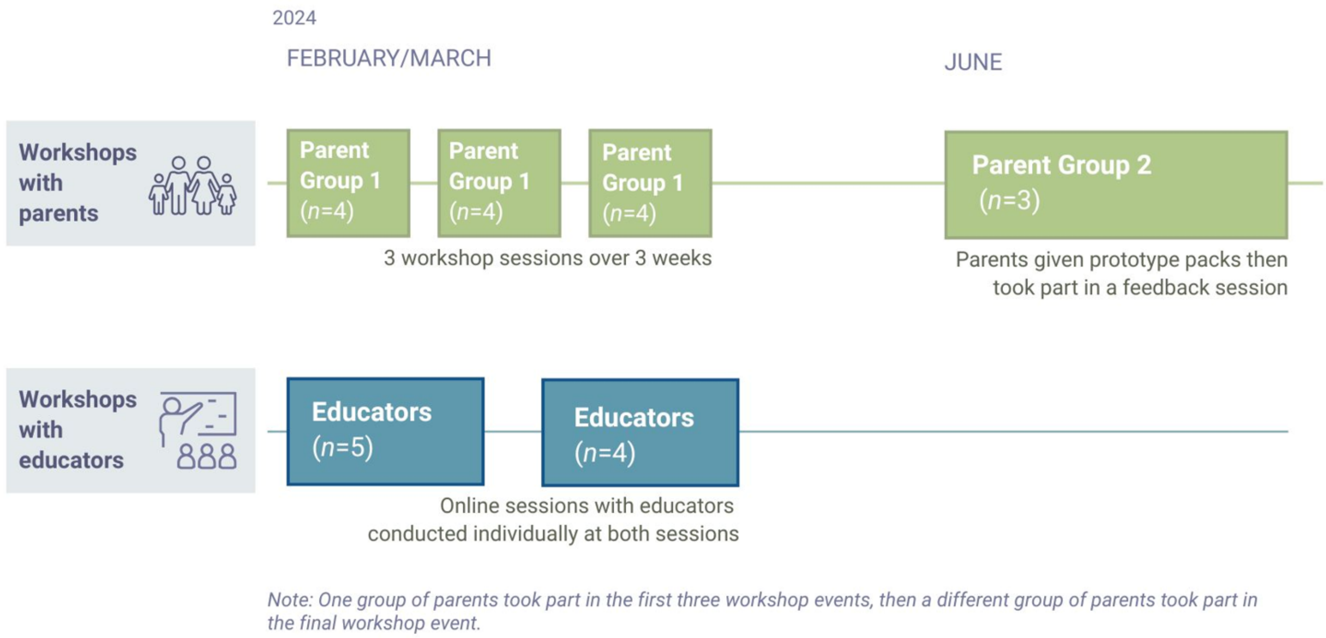
Ethics Statement

This study received ethical approval from the Ulster University Research Ethics Filter Committee (RG3-FCPSY-23-079; December 2023) and both parents and educators provided written, informed consent prior to participation.

Procedure

The co-production timeline is outlined in Figure 3. Early years educators were invited to attend two online workshops. Parents were invited to attend three face-to-face workshops.

Figure 3. Timeline of co-production process



Co-production began with a session with early years educators. The session started by discussing the types of mathematical skills and knowledge that the early years educators felt 3-5-year-olds should display. Then some simple activities aimed at 3-5-year-olds were presented to the group and discussions ensued about the appropriateness of the activities for the target group. Some suggestions of changes to make these activities were made and amendments were enacted before the parent workshops began.

Subsequently, co-production workshops engaged with parents once a fortnight over a period of six weeks. The first workshop included discussions on the types of mathematics activities that the families already did at home (gleaning ideas for new tasks), feedback on the existing activities and then a specific discussion on novel ideas for activities. These ideas were taken and worked into resource ideas for the next session by the research team. During the second session the parent group were shown the updated and new activities and were asked for feedback. In the final session all updated, and some new, materials were presented to the parent group. Parents discussed the resources and provided ideas for final amendments. During this session parents were also asked to provide feedback on the intervention structure, intensity, and delivery mechanisms. After this session the research team completed any requested amendments.

Subsequently, all co-produced resources were presented at a final early year's educator workshop to ensure that the materials were developmentally appropriate for 3–5-year-olds. The resource templates were then sent to a graphic designer to ensure that the resources were presented in an attractive and accessible manner.

Finally, three new parents, who had not been involved in the development process, were recruited and sent all the co-produced resources. These parents were asked to give feedback on the final resources and then any necessary changes could be made before the intervention stage of the project.

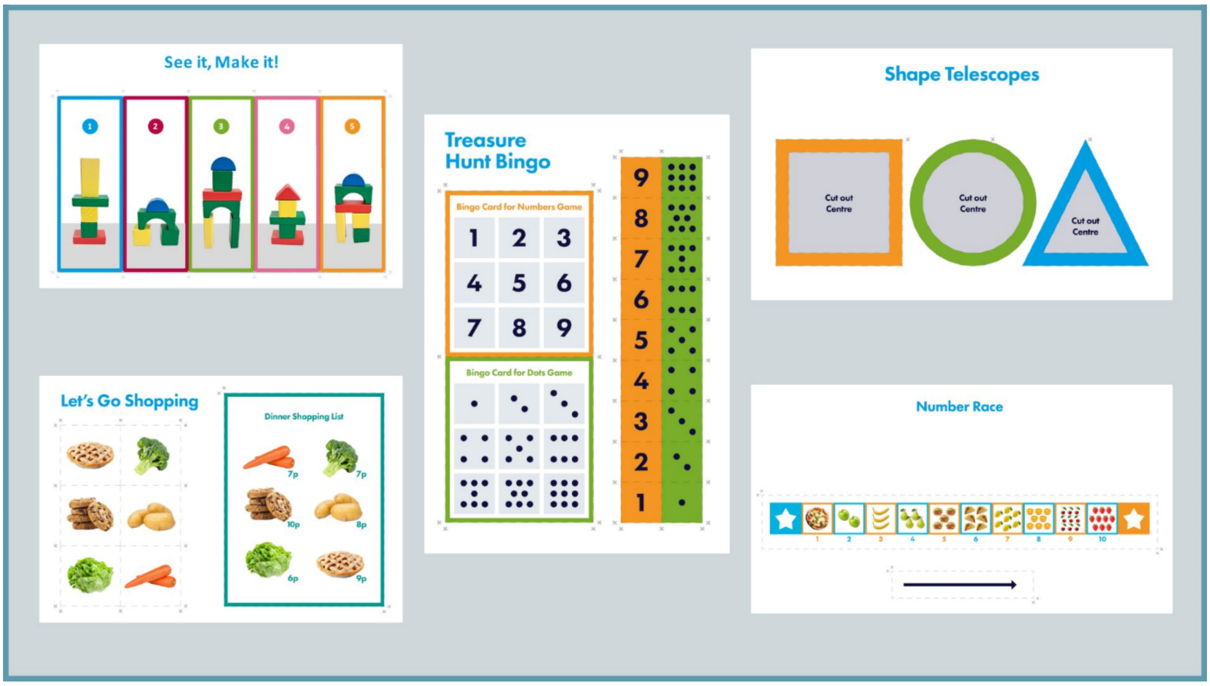
By the end of the process, we had co-produced 16 activities to be used in the intervention (Table 3). The home mathematics activities focused on a wide variety of developmentally appropriate mathematical concepts (e.g. number, patterning, shape) and provided suggestions for parents for related mathematical language that could be used alongside the activities (e.g., bigger, more, less).

Table 3. Summary of home mathematics activities

Activity (number of versions)	Premise of activity	Mathematical concepts (e.g.)
Treasure Hunt Bingo (2: Dots, Numbers)	Children locate hidden targets marked with dots/numbers from 1-9 and match them with a target board marked with dots/numbers 1-9	Number, ordering, space, mapping.
*Scavenger Hunt (*2: Sorting 2/4 categories)	Children collect items then sort them into 2/4 groups (e.g., large/small leaves).	Patterning, shape, size, equivalence.
Let's Go Shopping (2: Breakfast, Dinner)	Children and parents take it in turns to play shopper and shopkeeper and buy and sell everyday items.	Number, operations, size.
Number Race (2: Forwards, Backwards)	Children and parents take turns to roll a die and advance to the final space on the board.	Number, ordering, space, mapping.
See It, Make It! (1)	Children recreate target structures using building blocks.	Prepositions, shape, space, pattern.
What Comes Next? (2: 2 different sets)	Sequencing activity where children attempt to correctly order events (e.g., daily activities routine, seasons) depicted on cards.	Ordering, sequencing.
Copycat (1)	Parents model a sequence of physical actions (e.g two handclaps) then children repeat this back.	Pattern, counting, sequencing, timing.
Shape Windows (2: Finding shapes, Keeping count)	Children use stencils to find shapes that are present in the environment (e.g., television set; rectangle) and keep count of the number they find.	Shape, space.
Fingers Games (1)	A series of games and children's songs that involve counting on fingers.	Number, operations.
Chalk Rocket (1)	Parents draw a 'rocket', consisting of some squares, on the ground with chalk, and write the numbers 1-10 in each square. Children use a number spinner to determine which square to jump to.	Number, operations, shape, space, symbol-quantity mapping.

Some examples of the finalised home mathematics activities are included in Figure 4.

Figure 4. Examples of finalised home mathematics activities.



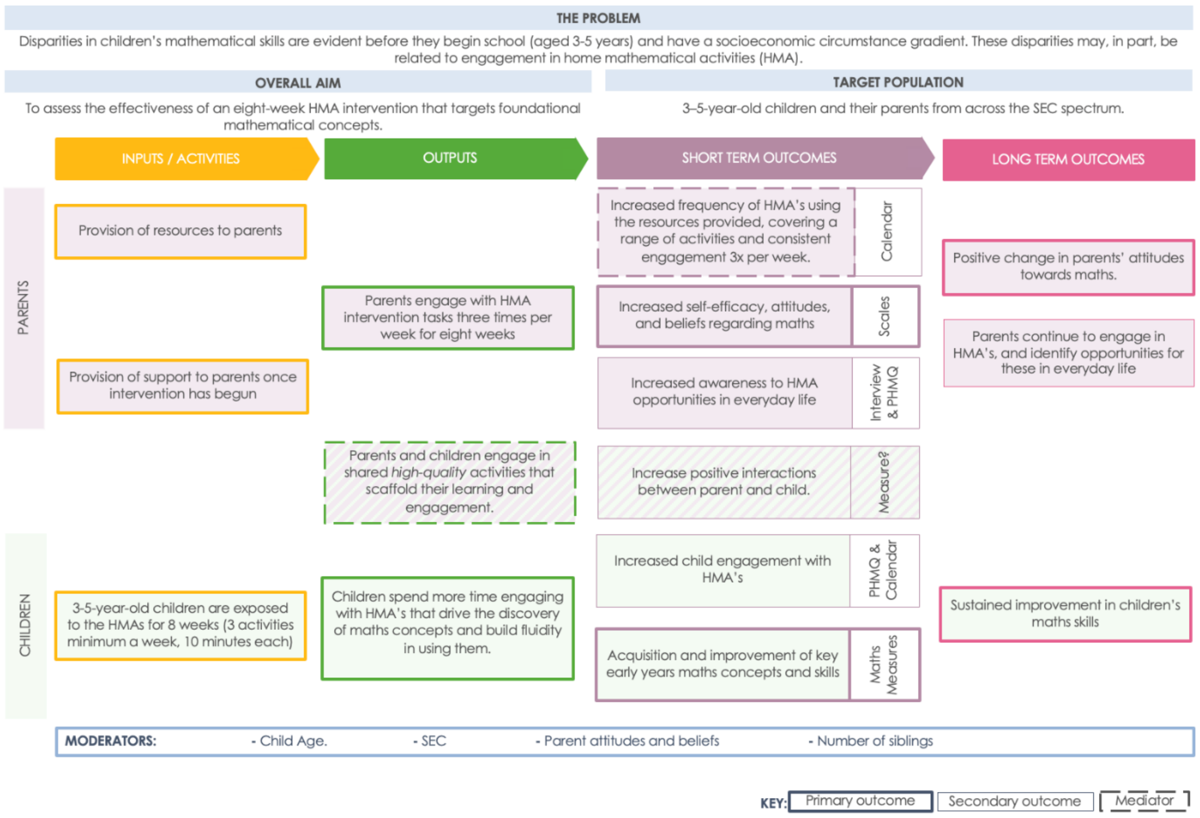
Parent feedback on the proposed intervention

During the workshop sessions, parents were asked about the processes and structure of the intervention. Parents reported that they felt an 8-week intervention period and completing three short activities per week was acceptable. Parents reported that they would like a maximum of two text messages per week as reminders to participate, before communications would become intrusive. This information fed into the building of the intervention and associated logic model.

The intervention

The study team developed a coherent logic model (Figure 5) for the intervention. The logic model specified the inputs, outputs, short-term outcomes and long-term outcomes of an intervention.

Figure 5. Logic model of the intervention



Given prior literature and the information gleaned from our co-production sessions the logic model laid out the components of a study that aimed to ultimately assess the effectiveness of a home mathematics activities intervention. Based on prior literature (e.g. Cahoon et al., 2023) the intervention was determined to be 8 weeks long to allow for sufficient engagement with home mathematics activities resources and the target population was 3–5-year-olds. The intervention provided home mathematics resources and support via text messaging to parents (two times per week), with the aim to engage families in mathematical interactions three times a week over the course of the intervention. Primary outcomes included increased positive mathematical interactions and parent attitudes towards mathematics. Long-term outcomes were identified as positive changes in parents' attitudes towards mathematics and sustained increases in children's mathematics skills.

Control group

The control groups activities focused on the topic of the 'Story of Me' in which parents and children explored topics around their child's interests using crafts and conversations with prompts (such as their favourite things, their dreams, and their favourite places to go). The types of activities included within this resource pack were like those used by local councils (e.g. Bristol City) to promote child wellbeing (UKRI, n.d.) and were designed to be free of maths content/skills directly related to maths. Instead, they focused on developing children's sense of self through autobiographical memories using parent guided art and craft type activities over the same time-period and matched the time and intensity of parent-child interaction in the active intervention group. The control group were provided with a resource box and craft activities. The control group were asked to engage with three activities per week with their child (i.e. two worksheets and one craft activity). Parents were also provided with two messages per week. By the end of programme, children, with the support of their parent had produced a folder containing the products of the 8-weeks' worth of activities.

Measuring fidelity

All participants in both groups received an 8-week calendar, marked with three sessions per week. Parents were asked to indicate on the calendar what activity they completed in each session the date they completed the activity and as to whether the activity went well or not with their child by circling one of the following words: "Great", "OK", or "Not Good" per session. This information was used to measure engagement with both intervention and control activities. In addition, parents were asked to record themselves with their child completing an activity from the resource pack. These videos were used to assess whether parents followed the instructions provided to them.

Part 3: Feasibility study

Aims and Approach

This element of the project focused on assessing the feasibility and acceptability of the 8-week intervention. The aim was to understand if the intervention and co-produced materials were acceptable and feasible to parents and at a broader level to ensure that processes, communication and logistics were feasible for the study to transition to the pilot stage. We had several pre-registered research questions we aimed to address (Table 4). We also pre-registered criteria to judge each research question against.

Table 4. Feasibility study research questions and pre-registered predictions.

RQ1. Do parents consider engaging with the intervention as acceptable?	We expected parents to provide positive acceptability ratings of the intervention and methods of evaluation (>5 on a 7-point scale)
RQ2. Did parents find the mode of evaluation acceptable?	We expected the majority (>60%) to recommend the study to a friend and to continue to use the resources provided to them.
RQ3. Do parents consider engaging with the intervention as feasible?	We expected parents to express positive ratings (average rating >6 on a 10-point scale) on their capabilities, opportunities, and motivations to engage with the intervention. Further, we expected most invited parents (>60%) to agree to book a visit for a researcher to collect post-test measures in person and attend the assessment.
RQ4. Do parents consider the delivery of the intervention as feasible?	We expected most parents (60%) to remain opted into the intervention for the entire eight weeks' duration.
RQ5. How will parents evaluate the delivery of the intervention?	We also addressed the feasibility of the intervention regarding parents' service evaluation. We expected some parents (>30%) to share one home recorded activity video, and for most parents to book an in-person home/lab/family centre visit to engage in the pre- and post-test assessments (>60% both groups, both times).
RQ6. Were there any unintended consequences from taking part in the intervention?	We expected taking part in the intervention to have little to no impact on parents' self-efficacy. In addition, we anticipated that taking part in the study may influence how parents engaged with other activities with their child.
RQ7. Do parents engage with the intervention resources?	We expected parents to engage with the intervention resources three times a week over the eight-week period as prescribed.

Note: RQ= Research question

Method

Ethics statement

The study received ethical approval from Ulster University Research Governance Committee (approval ID: RG3-FCPSY-24-001) and a detailed protocol was preregistered with the Open Science Framework (https://osf.io/enjdc/). The study was conducted in accordance with the ethical standards outlined by the Helsinki Declaration (*Declaration of Helsinki*, 1975), as revised in 2001 ('World Medical Association Declaration of Helsinki', 2001).

Study design

A small-scale study was implemented to ensure feasibility of the processes included in a future pilot study. It was important that the processes for both intervention and control groups were evaluated. Participants were unaware of the broader aims of the study (i.e., single blinding).

Participants

Participants were recruited through the research teams existing database, Eastside Learning (the study community partner), and the research team social media account. Participants were invited to complete an online screening questionnaire through JISC (*JISC Online Surveys*, 2020) to determine eligibility. Participants were then invited to take part, with eleven participating dyads from the Belfast (NI) area took part in the feasibility study. Participants were randomly assigned to either the experimental group (*n*=6) or the control group (*n*=5). Dyads were deemed ineligible if the family lived outside NI, and/or if the participating child was younger than 36 months or older than 60 months (i.e. children who were pre-school age). Families were also excluded if English was not the predominant language used in the home (at least 50% of conversations in the home conducted in English to ensure that the study team could code video interaction data). Eligible dyads were invited to take part in the pre-test assessment, either in participants' homes, in the lab on campus, or in a local community centre.

Table 5 summarises the demographic information for the study participants. All parents participating in the study (in both the experimental and control group) were the primary caregiver of the participating child. Five (83.3%) child participants in the experimental group had at least one sibling, with 1 (16.7%) participant in this group having no siblings. All 5 (100.0%) children in the control group had at least one sibling. None of the 6 (100.0%) experimental group parents indicated their child was diagnosed with a developmental condition (nor were there any parental concerns), whereas, for the control group, 1 (20.0%) child was diagnosed with a developmental condition (specifically expressive speech delay). The remaining 4 (80.0%) child participants in the control group had not been diagnosed with a developmental condition nor were there any concerns on the part of the parent.

Table 5: Demographic data for feasibility study participants

		Group	
		Experimental	**Control**
Mean parent age (years)		35.67 (4.23)	37.60 (4.04)
Mean child age (months)		46.12 (4.83)	53.20 (9.04)
		N **(% group)**	
		6 (100.0%)	5 (100.0%)
Parent gender	Female	6 (100.0%)	5 (100.0%)
	Male	0 (0.0%)	0 (0.0%)
Child gender	Female	2 (33.3%)	3 (60.0%)
	Male	4 (66.7%)	2 (40.0%)
Parent education (SEC)	GCSE/O Level/Level 2 awards	2 (33.3%)	0 (0.0%)
	A Level/BTEC/Level 3 awards	1 (16.7%)	0 (0.0%)
	Level 4 awards	0 (0.0%)	0 (0.0%)
	Foundation degree or Level 5 awards	0 (0.0%)	0 (0.0%)
	Honours degree/Level 6 awards	3 (50.0%)	2 (40.0%)
	Postgraduate degree/certificate/diploma	0 (0.0%)	3 (60.0%)
Birth order	Participating child is first child	1 (16.7%)	2 (40.0%)
	Participating child is second child	3 (50.0%)	2 (40.0%)
	Participating child is third child	2 (33.3%)	1 (20.0%)
Parent ethnicity	White	6 (100.0%)	4 (80.0%)
	Mixed or Multiple ethnic groups	-	1 (20.0%)

Sample size calculations

The objective of this study was to assess the viability of trial processes, such as adherence and acceptance rates and to assess practical considerations around the implementation of the intervention task (Teresi et al., 2022). As such, there was no intention to generate effect sizes from the data with the aim of estimating power for the RCT.

Materials and procedure

Child measures

Cardinal principle knowledge: 'Give-N task', adapted from Wynn (1990) as per Cahoon, Gilmore, et al. (2021).

Participants were required to give a requested number of items (plastic strawberries ~2cm in diameter) to a soft toy animal (“Fluffy”) using a plastic plate, which indicates understanding of the cardinal principle. The numbers 2 through 9 were presented, with the final four trials consisting of two requests for 8 items, and two requests for 9 items. Children were awarded one point for each correct number of items given. The possible range of scores on the task was 0-10. Given the stopping-rules (and therefore missing data) used for the scoring system, internal consistency could not be calculated for the Give-N task (nor any of the child measures).

British Ability Scales (BAS-III) Early Number Concepts assessment (Elliott & Smith, 2011).

The Early Number Concepts subscale of the BAS is a standardised measure designed for children aged 3-6 years and includes questions about number, size, and other numerical concepts (e.g., mathematical language). The subscale contains a maximum of 30 questions, and one point is awarded for each correct answer, except for item 3 (6 points). The possible range of scores was 0-35.

Symbol-quantity mapping: cross-notation comparison task (as per Cahoon, Gilmore, et al. (2021).

This task assesses children's ability to map non-symbolic numerical stimuli onto symbolic numerical stimuli. Children were presented with an array of dots and were asked to quickly (without counting) decide which of two Arabic digits matched the numerosity of the dots. Participants received an accuracy score based on the proportion of presented items answered correctly. The possible range of scores for this task was 0-18.

Digit recognition task (Batchelor et al., 2015).

This task measures knowledge of Arabic digit stimuli. Children were asked to read aloud a series of Arabic digits (ranging from 1 to 20) presented one by one in a non-sequential order. For this task the following digits were used: 3, 2, 5, 8, 7, 9, 12, 14, 11, 16, 20, and 18. One point was awarded for each correctly identified number, with the possible range of scores being 0-12.

Shape task adapted from the Reception Baseline assessment: Shape subtest (Reception Baseline Assessment, 2019).

The Shape sub-test is a standardised measure designed to assess educational abilities of children upon entering formal schooling. Participants were asked to sort shapes (squares, rectangles, circles, triangles) into four groups. Participants were then required to identify the shapes. In the present study, scoring rules deviated from the original sub-task as follows: participants received one point for sorting circles and triangles correctly, but incorrectly sorting the squares and rectangles (e.g., by mixing these shapes), and two points for correctly sorting the four shape types into four separate groups. The task consists of six items, with the range of possible scores being 0-7.

Parent measures

Demographics questionnaire.

This measure captured information regarding parent age, child age, gender, SEC, and broader background questions.

Pre-school Home Maths Questionnaire: Frequency of Numeracy Activities Subscale (PHMQ; Cahoon, Cassidy, et al., 2021).

The PHMQ is a parent-completed questionnaire that assesses different aspects of the home maths environment (e.g., parent expectation, literacy, counting skills, parent-child teaching methods, frequency of numeracy activities etc.). The frequency of numeracy activities component of the PHMQ contains five subscales; parent - child interactions, computer maths games, TV programmes, shape, and counting –

and comprises 22 items in total. The measure is designed to capture the frequency with which mathematics activities are undertaken in the home and is scored on a five-point Likert scale (i.e., “activity did not occur” to “almost daily”). Only the experimental group completed this measure at pre-test, to ensure that participants remained unaware of the broader aims of the study. However, both groups completed the measure at post-test. Internal consistency for the PHMQ was excellent for both pre-test (α= .92) and post-test (α= .90) scores.

Parents’ beliefs about mathematics (Sonnenschein et al., 2012).

This measure assessed different types of parental beliefs about mathematics learning (e.g., value of academic achievement, relationship with personal growth). For the purposes of this measure, we only administered three quantitative items that measured the importance of children’s maths activities and parents’ beliefs about own maths skills. Participants responded on a five-point Likert scale ranging from “Very unimportant/unconfident” to “Very important/confident”). Similarly to the PHMQ, only the experimental group completed this measure at pre-test. Both groups completed the measure at post-test. Internal consistency at pre-test was good (α= .81), and acceptable at post-test (α= .66).

Child-parent relationship scale (Pianta, 1992).

This measure is a parent-completed questionnaire that assesses the quality of the parent’s relationship with their child, and contains three subscales: dependence, conflicts, and closeness. Example items include “I share an affectionate, warm relationship with my child”, and “My child and I always seem to be struggling with each other.” Items are scored on a scale of 1-5 (“Definitely does not apply” to “Definitely applies”) and comprises 30 items in total. Responses to positively worded items were reverse coded to be consistent with use as an overall measure of child-parent relationship (e.g., instead of using the three subscales individually). Higher scores therefore relate to higher conflict, lower closeness, and higher dependence. Internal consistency was found to be good for both pre- and post-test scores (α= .86).

Me as a Parent Scale (short-form; Matthews et al., 2022)

The Me as a Parent Scale (short-form) is a four-item, parent-completed questionnaire that measures parent self-efficacy (3 items; e.g., “I have confidence in myself as a parent”) and parental self-management (1 item; e.g., “I can stay focused on the things I need to do as a parent even when I’ve had an upsetting experience”). Parents rated their level of agreement with each statement on a five-point scale, ranging from 1-5 (“Strongly disagree” to “Strongly agree”). Internal consistency was good for pre-test scores (α= .85) and excellent for post-test scores (α= .90).

Feedback questionnaire

The post-test survey contained several questions designed to obtain feedback from participants, with the aim of improving several elements of the intervention. Responses were collected using a Likert scale in the first instance, with the opportunity for participants to expand on their response with free text. Questions centred around the difficulty level of the activities, frequency of expected engagement, the number of activities that were provided, whether engaging in the intervention impacted other areas of participants’ lives (positively and negatively), how parent and participants experienced the pre- and post-test assessments, and whether the text messaging reminder system was useful.

Parent-child cooperative block task observation.

Parent-child dyads engaged in a structured cooperative block (Lego™) play task observation (~5 minutes) at the beginning of the assessment session at both pre- and post-test. Participants were instructed to “play like they would at home.” The task was not directly related to mathematical concepts but gave participants the opportunity to refer to them if they wished. The interaction was recorded with a digital camera (Sony DSC-W800 Compact Camera) on a tripod. Verbal interaction (utterances) was coded for mathematical language (e.g., cardinality, dimension, magnitude, spatial language). The coding scheme was a synthesis of several maths language coding schemes used in similar studies (i.e. Daubert et al. (2018), Eason et al. (2021), He et al. (2022), Leech et al. (2022), Melzi et al. (2022), and Schnieders and Schuh (2022)) and was designed to capture concepts presented via the home mathematics activities intervention tasks.

Calendar data – subjective experience of activity sessions

Pilot intervention fidelity and subjective experience of the activity sessions was assessed using an engagement calendar which participants were instructed to complete as part of the intervention. Both control and experimental participants were required to indicate each time they undertook an activity on a calendar, and to subjectively rate whether each activity session was "Great", "OK", or "Not Good" by circling the word(s) that reflected their experience. No predefined threshold was set for this metric, for either the number of sessions engaged in nor parents' subjective ratings of the activity sessions, given the novel nature of the intervention.

Recordings of activities

Parents were asked to record one activity with their child and return this securely to the study team. This video provided additional evidence of the fidelity of how activities were used.

Procedure

An individual not involved in the study based at Ulster University randomly assigned each participant to either the experimental group or a control group. Participants in both groups were then invited to undertake the pre-test assessment at their convenience in which the child mathematics measures and the parent survey were administered. Dyads engaged in a video-recorded cooperative block play task for five minutes whilst the experimenter was out of the room. The experimenters then administered the child maths measures in the following order: British Ability Scales, Give-N task, Symbol-Quantity mapping task, Digit recognition task, NFER shape task. Parents completed the survey on a laptop whilst the child maths measures were administered. Assessments were administered by author BH.

Upon completion of the assessment, participants were provided with a pack of activities to take home, a £10 shopping voucher, and a small soft toy for participation in the pre-test assessment. Experimental and control packs appeared identical so experimenter would remain blind to group. For the experimental group only, mathematics activities were undertaken in the home environment over the course of eight weeks. The control group did not engage in any mathematics activities as part of this intervention, instead they engaged with craft activities with their child. Both groups of participants were instructed to keep a record of the activities that were undertaken each week, and to subjectively rate the quality of each session on a purpose-designed calendar. A text message system was employed to remind parents in both groups to engage in the activities, and to provide support in the form of instructional videos of the activities. The text messages were delivered twice per week (Monday and Friday). Once participants had completed eight weeks of activities, they were invited to attend the post-test assessment which was identical to the pre-test assessment, except for the parent survey which additionally included a feedback questionnaire. Finally, parents were offered a second £10 shopping voucher, and child participants were presented with another small soft toy for participation in the post-test assessment. Families were given a choice of where to complete the session. Most opted to complete the session on the University campus (13 sessions pre- and post) with some at home (8 sessions pre- and post) and one session taking place at a community centre.

Once all post-test assessments had taken place, cooperative block play task footage (pre- and post-test) was coded for mathematical language.

Data analyses

Due to the nature of the study, descriptive statistics for all variables were sufficient to assess feasibility.

Results

Primary findings are presented as mapping onto the preregistered research questions outlined at the beginning of Part 3 (Table 4), with secondary findings presented as tables and figures.

RQ1. Do parents consider engaging with the intervention as acceptable?

Parents reported that they found the intervention acceptable ($M = 5.62$; $SD = .74$), exceeding the pre-registered criterion on the Sekhon et al. (2022) Framework of Acceptability of 5 or above on a 7-point scale. In addition, eight of the 11 (72.3%) of participants' overall scores on this measure were greater than 5, indicating that most of the sample provided favourable acceptability responses.

RQ2. Did parents find the mode of evaluation acceptable?

Eighty-two percent of participants indicated that they would continue to use the resources, exceeding the pre-registered criterion of more than sixty percent of parents indicating that they “probably would or “definitely would” continue to use the resources provided to them.

RQ3. Do parents consider engaging with the intervention as feasible?

Overall, parents reported feeling that they had the capability, opportunity, and motivation to engage with the intervention (M= 8.96; SD= 1.08, with the pre-registered threshold set at an average rating of above 6/10 on a measure adapted from COM-B (Keyworth et al., 2020). Ten of the 11 (90.1%) of participants’ overall scores were greater than 6, indicating that most of the sample provided favourable feasibility responses.

RQ4. Do parents consider the delivery of the intervention as feasible?

Retention across the 8-week intervention was excellent (100.0%) and exceeded the pre-registered retention rate of 60%. Relatedly, all 11 (100.0%) of participants agreed to, and attended, the post-test assessment, which met exceeded the stated minimum of 60%.

RQ5. How did parents evaluate the delivery of the intervention?

All invited parents booked a visit with a researcher to collect pre- and post data and attended these sessions, exceeding our target of sixty percent. Adherence videos were shared by fifty-five percent of parents, surpassing the target of thirty percent.

RQ6. Were there any unintended consequences from taking part in the intervention?

No negative impact on general caregiver self-efficacy was found from participation in the study (Pre-test: M= 4.14; SD= .47; Post-test: M= 4.32; SD= .50).

RQ7. Did with parents engage with the intervention resources three times a week over the eight-week period?

Parents in both groups engaged with around 20 of the 24 total activities that were provided (experimental group = 20.8, control group = 20.3). In addition, 5 out of the 6 parents who returned recordings of engaging with their child with an activity precisely followed the instructions that were provided to them.

Assessing ceiling and floor effects in measures

Figure 6 summarises scores for all child measures at pre- and post-test. Ceiling effects were observed on the NFER Shape task at both pre-test (55%) and post-test (56%). Ceiling effects were also observed for the Give '*N*' task at post-test (45%).

Figure 6. Children's performance on the five maths tasks at (a) pre-test and (b) post-test for both groups overall.

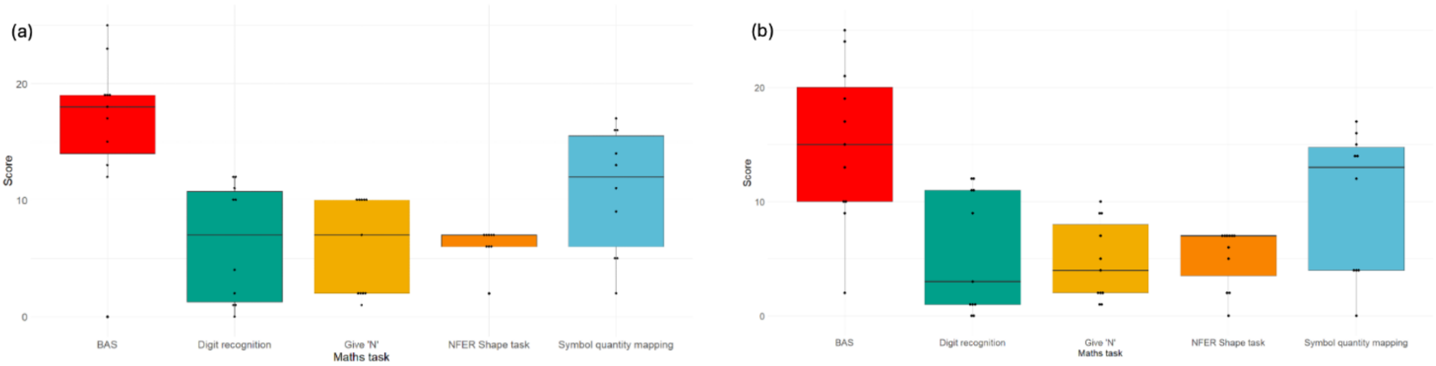
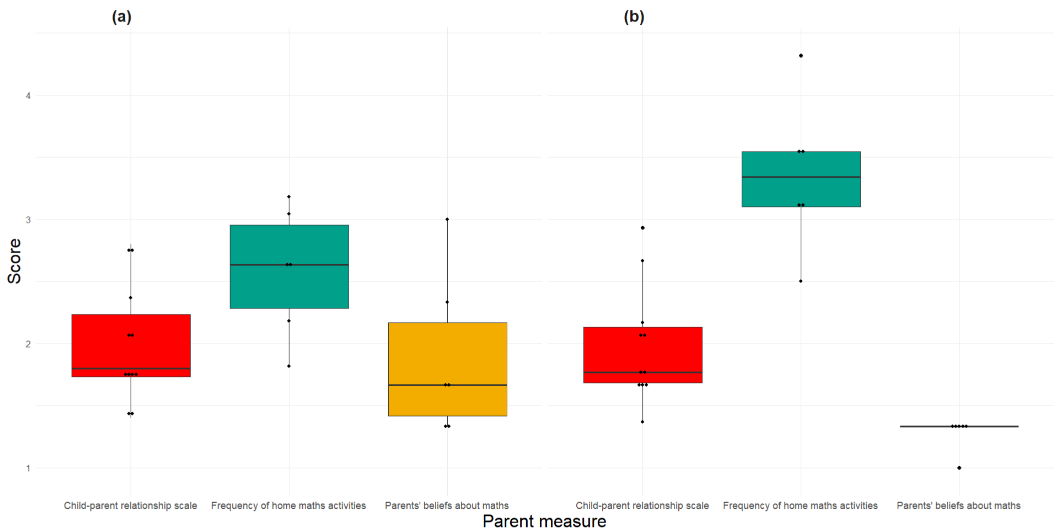


Figure 7 summarises parent responses to the child-parent relationship scale, frequency of home maths activities, and parents' beliefs about maths at (a) pre- and (b) post-test. We observed sufficient variance at pre-test for Parent's Beliefs about Maths, though not at post-test. The decision was taken to continue to use the measure in the pilot study as it is a validated and recognised measure in the literature. Good variation in responses was observed on all other measures.

Figure 7. Performance on parent survey measures for both groups for pre- and post-test.



Interim conclusion

Overall, given the findings of the study, the intervention was deemed feasible to proceed to the pilot study stage. In relation to our pre-registered feasibility thresholds, all criteria were surpassed.

Adaptations from feasibility study to pilot study

Several changes were made to the design and procedure of the feasibility study to progress to a larger-scale pilot study. Firstly, minor changes were made to the parent survey. This included offering a response option for "No formal qualifications". Secondly, as ceiling effects were observed on the NFER Shape and Give *N* tasks, a narrower age range for the pilot was recruited which was expected to reduce mean scores on these tasks. Therefore, only children aged between 42 and 54 months were eligible to participate in the pilot study, this change is inclusion criteria also ensured that children in the study had not yet begun school. Thirdly, the decision was taken to use Duplo™ rather than Lego™ in the cooperative block play task for the pilot for safety reasons, as it was observed in the feasibility study that parent participants often brought younger siblings into the assessment sessions. Fourthly, the pilot study was planned as a collaboration between Ulster University and the University of Sheffield and as such, Sheffield was added as a second testing site for the pilot. To widen the participant pool, participants from across the whole of Northern Ireland and in South Yorkshire were eligible to take part in the pilot study.

Some qualitative comments from parents indicated that they may have found some of the instructions for some of the activities difficult to follow. To aid accessibility brief videos were made to go alongside each activity, and QR codes were provided at the start of the intervention linking to each video. These videos were plain language summaries to help support families (see https://youtu.be/lVS08MC0D5g for an example).

Part 4: Pilot study

Aims and Approach

The pilot study aimed to recruit 80 families split across the control and experimental conditions to assess the feasibility of administering the intervention and assessments. Although the pilot was not designed or powered to detect intervention effects, it aimed to examine descriptive changes in pre- to post-test scores.

Method

Participants

Participants were recruited via a variety of methods, including our existing lab databases of parent volunteers, through contacts with local nurseries, and adverts on social media (Facebook advertising), to maximise inclusivity and reach a diverse sample of families. To be included in the intervention, parents must have had a child who would be between 3 and 4.5 years-old at the start of the study and had not yet started school.

The sample was made up of 84 participants, 42 of whom were recruited through Ulster University, with 42 through the University of Sheffield. With regards to experimental groups there was an equal split, with 21 experimental participants at each site and 21 controls at each site.

The mean age of parents in the experimental group was 36.5 (SD= 4.6) years old. The mean age of parents in the control group was 36.7 (SD= 4.3) years old. The mean age of child participants in the experimental group was 47.8 (SD= 3.1) months, and the mean age of control group child participants was 47.6 (SD= 3.9) months.

Parents reported if their child had (or they suspected they had) any developmental or health conditions (experimental group: 6 children (14.3% for this group); control group: 7 children (16.7%)).

Thirty-nine (92.3%) participants in the experimental group reported being the mother of the child participant, with one (2.4%) parent indicating they were the child's father, one (2.4%) an adoptive parent, and one (2.4%) not providing a response. Forty (95.2%) participants in the control group reported being the mother of the child participant, and two (4.8%) reported being the father. Thirty-one (73.8%) parent participants in the experimental group indicated being the primary caregiver of the participating child, and one (2.4%) parent indicated not being the child's primary caregiver. Nine (21.4%) parents in experimental group reported having equal care with someone else. Thirty-four (80.9%) parent participants were the child's primary caregiver in the control group, with 8 (19.0%) indicating having equal care with someone else. One (2.4%) participant did not provide a response to this item.

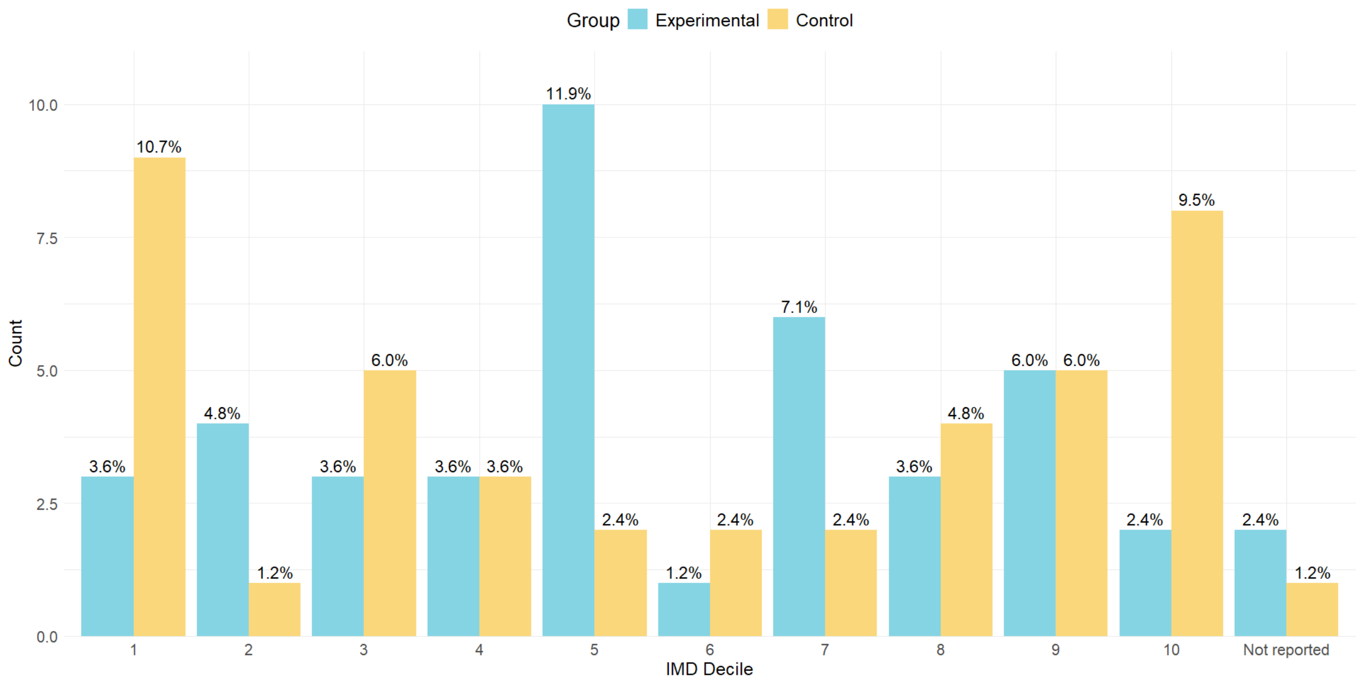
In terms of childcare/early education, 39 (92.3%) participants in the experimental group reported that their child was in receipt of regular childcare, averaging 21.9 (SD= 8.3) hours per week. For the control group, 41 (97.6%) of parents indicated that regular childcare was used, averaging 23.3 (SD= 9.7) hours per week. The most popular form of childcare/early education provision reported by parents was delivered via pre-school settings (64.3% experimental group, 52.4% control group). Table 6 provides further summary demographic details of the participants.

Table 6. Demographic data for pilot study participants according to group.

		Group	
		Experimental	Control
		N (% group)	
		42 (100.0%)	42 (100.0%)
Parent gender	Female	41 (97.6%)	39 (92.8%)
	Male	1 (4.2%)	2 (4.8%)
	Non-Binary	-	1 (2.4%)
Child gender	Female	22 (52.4%)	20 (47.6%)
	Male	20 (47.6%)	22 (52.4%)
Parent ethnicity	White	38 (90.4%)	41 (97.6%)
	Mixed or Multiple ethnic groups	2 (4.8%)	1 (2.4%)
	Asian, Asian British, Asian Irish, Asian Welsh	1 (2.4%)	-
	Not reported	1 (2.4%)	-
Parent education	No formal qualifications	-	1 (2.4%)
	GCSE/O Level/Level 2 awards	1 (2.4%)	1 (2.4%)
	A Level/BTEC/Level 3 awards	1 (2.4%)	3 (7.1%)
	Level 4 awards	1 (2.4%)	-
	Foundation degree or Level 5 awards	1 (2.4%)	-
	Honours degree/Level 6 awards	18 (42.8%)	12 (28.6%)
	Postgraduate degree	17 (40.4%)	24 (57.1%)
	PhD	2 (4.8%)	1 (2.4%)
	Not reported	1 (2.4%)	-
Birth order	Participating child is first child/only child	12 (28.6%)	13 (31.0%)
	First with other siblings	14 (33.3%)	15 (35.7%)
	Second child	10 (23.8%)	9 (21.4%)
	Third child	5 (11.9%)	4 (9.5%)
	Fourth child	-	1 (2.4%)
	Not reported	1 (2.4%)	-

Figure 8 summarises the IMD decile spread of the full participant sample, indicating families from a wide spread of neighbourhoods were recruited into the study. No difference was observed in IMD between recruitment sites (χ^2= 2.7021, df = 9, p-value = 0.975).

Figure 8. IMD decile according to experimental group.



Outcome measures

As per the feasibility study the following outcome measures were administered at pre- and post-test.

Primary outcome measures:

1. BAS-III Early Number Concepts (C. D. Elliott & Smith, 2011)
2. Summed scores from the mathematics tasks, including:

- Cardinal principle knowledge ("Give-N task"; Cahoon, Gilmore, et al., 2021; Wynn, 1990.
- Symbol-quantity mapping (Cahoon, Gilmore, et al., 2021).
- Digit recognition (Batchelor et al., 2015).

- Shape Understanding (Reception Baseline assessment: Shape subtest (Reception Baseline Assessment, 2019).

Descriptive statistics (means, distributions) were examined for these measures. For these primary outcomes, we expected a descriptively higher pre–post change in the experimental group compared to the control group.

Secondary outcome measures:

- Parents' Beliefs about Mathematics Scale (Sonnenschein et al., 2012).
- Frequency of engagement in home mathematics activities (Cahoon, Cassidy, et al., 2021).
- Parent and child mathematical talk during cooperative block play.

For these secondary outcomes, we expected descriptively higher post-test scores for parental beliefs and mathematical talk in the experimental group. Control group scores were expected to remain stable or show modest increases, with medium–large pre–post correlations. No specific prediction was made regarding frequency of engagement in home mathematics activities.

In addition, 'Me as a Parent' (Matthews et al., 2022) and the 'Parent-child relationship' (Pianta, 1992) scales were administered to monitor any potential unintended negative consequences of the study.

Mathematical language

The cooperative block play interactions were video recorded, and recordings were transcribed and coded for frequency in various categories of mathematical language ("maths talk"). See Appendix A for the synthesised coding scheme.

Instances of maths talk were summed to create an overall score separately for each parent and child. In addition, the number of categories (out of 14) of maths talk used in each session were calculated to create a breadth of maths talk score for each parent and children separately.

Sixteen (~10%) of the completed transcripts were double coded for reliability purposes. For these transcripts, the total number of instances of maths language across the individual maths talk categories was calculated. Reliability was found to be "excellent" between coders (Hallgren, 2012) for both parent maths talk (ICC= .78) and child maths talk (ICC= .88). Transcripts from NI participants were coded by author BH and transcripts from Sheffield participants were coded by author ES.

Procedure

Pre-registered outcomes

The procedure for the pilot study followed that of the feasibility study, with a small number of changes. Firstly, we increased the target sample size, with the aim of recruiting 80 dyads evenly split between experimental and control group. The predicted sample size was exceeded, with 82 dyads being included in the study. Blinding was employed in the pilot study, so that data collectors were unaware of which group participants were allocated to for the duration of the study. Lastly, plans were made to conduct follow up interviews to collect further feedback on the study and activities. Qualitative analyses (e.g., Thematic Analysis) were conducted on the transcripts from these interviews.

The following pre-registered (https://osf.io/enjdc) outcomes were the focus of this pilot study:

(a) Recruitment. We aimed to recruit 40 dyads per group (experimental/control) from Northern Ireland and Sheffield to assess feasibility of larger-scale UK recruitment.

(b) Retention. Retention was defined as attendance at the post-test session, with a benchmark of ≥60% across both groups.

(c) Fidelity of engagement. Adequate fidelity was defined as ≥66% engagement with intervention activities (≥16 of 24 sessions), measured via calendar responses.

(d) Data collection feasibility. Success was defined as <20% missing data on the primary outcomes (BAS Early Number Concepts sub-scale scores, individual mathematics domain task scores) across dyads.

(e) Blinding procedures. Feasibility of randomisation and blinding was assessed via number of instances of unblinding across both sites. No threshold for this was specified.

(f) Statistical analysis feasibility. Distributional properties, variance, and correlations of baseline and outcome measures were examined. Primary outcomes were analysed using ANCOVA, with effect sizes reported.

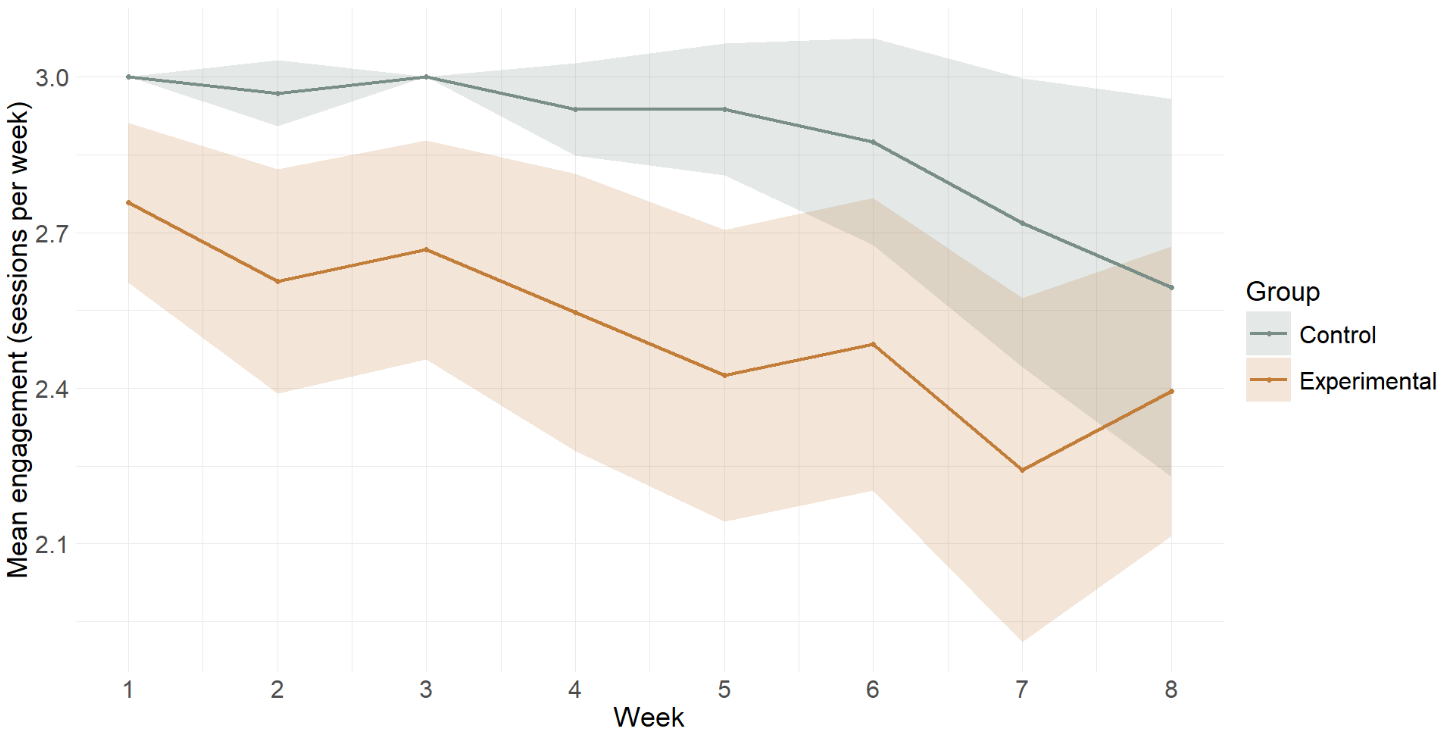
(g) Lack of negative consequences. Parental self-efficacy (Matthews et al., 2022) was monitored to ensure engagement in the study did not induce negative parental consequences. A reduction >4 scale points from pre- to post-test was considered adverse.

Results

Engagement

On average, participants engaged in 21.55 (SD= 3.69) activity sessions over the eight-week intervention period, at a rate of 2.69 (SD= .46) activity sessions per week on average for the two groups (Figure 9). Regarding between group differences in engagement, control group participants were found to engage in a higher number of sessions overall (M= 23.01; SD= 2.28) compared to experimental group participants (M= 20.12; SD= 4.23), $t(50)= 3.47$, $p=.001$.

Figure 9. Time series plot of weekly session engagement according to group. Error ribbons represent 95% confidence intervals.



Out of the 84 participants who began the intervention study, 72 participants (85.7%) attended at post-test, with 94.6% of post-tested participants agreeing to take part in a follow up interview on their experiences of the study. Twenty participants completing follow-up interviews. With regards to testing location, 57 (36.54%) pre- and post-test assessments were conducted in participants' homes, and 99 (63.46%) were conducted in a university laboratory. In addition, an average of 17.4 (SD= 14.0) days elapsed between the end of the intervention and the post-test date.

Change in engagement with home mathematics activities, parents' beliefs about mathematics, and child mathematics skills

Parent-level measures

Table 7 summarises parent reported engagement with home mathematics activities (on an average weekly basis) and their own attitudes towards mathematics.

Table 7. Parental survey responses at pre- and post-test by group. Mean (SD).

	Experimental		Control	
	Pre	Post	Pre	Post
Frequency of mathematics activities	3.08 (.74)	3.24 (.60)	-	2.85 (.69)
Parents' Beliefs about Maths	1.82 (.76)	1.64 (.60)	-	1.73 (.60)

To ensure that we did not encourage parents in the control group to change their behaviour around mathematics, these measures were only administered at post-test for the control group. Frequency of engagement with mathematics slightly increased from pre to post-test for the experimental group. Parents in the experimental group reported more engagement with home mathematics activities than the control group at post-test, $t(71)= -2.67$, $p=.009$.

Parents' beliefs about mathematics declined slightly from pre to post-test for the experimental group. At post-test, parents in the control group had slightly more positive attitudes towards mathematics than the experimental group, but this difference was not significant ($p>.05$).

Child-level measures

Table 8 summarises the measures of child mathematics skills from a standardised assessment and bespoke tasks.

Table 8: Pre- and post-test performance on child maths measures (raw score means and standard deviations) for participants who were assessed at both pre- and post-test, according to group.

	Experimental		Control	
	Pre	Post	Pre	Post
BAS: Early number concepts	16.00 (5.32)	17.49 (5.02)	16.00 (6.15)	16.74 (6.08)
Maths domain tasks				
Give *N*	6.35 (3.17)	6.27 (3.07)	4.97 (2.80)	5.27 (3.14)
Symbol-quantity mapping	10.63 (4.17)	11.14 (4.58)	9.94 (3.81)	10.57 (4.17)
Digit recognition	6.17 (3.83)	7.03 (3.36)	4.50 (3.48)	5.35 (3.67)
NFER Shape task	5.62 (1.38)	5.92 (1.48)	5.38 (1.58)	5.97 (1.58)
Overall domain tasks (%)	59.6 (22.0)	63.9 (20.6)	50.9 (19.5)	57.1 (20.2)

Note: BAS, British Ability Scales; NFER, National Foundation for Educational Research

British Ability Scales (BAS) III: Early Number Concepts

An ANCOVA was conducted to examine whether group predicted BAS scores at post-test while controlling for pre-test BAS scores. The covariate (pre-test score) was significantly related to post-test scores (p<.001, $\eta^2 g$ = .51), indicating that higher pre-test scores were associated with higher post-test scores. However, there was no significant effect of group on post-test BAS scores (p=.425, $\eta^2 g$ = .01).

Pre- and post- test scores were correlated for both the experimental group, $r(35)$= .68, p<.001, and the control group, $r(33)$= .73, p<.001.

Mathematics Overall Domain Task Scores

A second ANCOVA was performed to assess whether group predicted overall domain task (%) scores at post-test whilst controlling for overall domain task scores at pre-test. While pre-test scores were significantly related to post-test scores (p<.001, $\eta^2 g$= .75), there was no effect of group on post-test scores (p= .806, $\eta^2 g$= .001).

Pre- and post-test scores were correlated for both the experimental group, r(35)= .91, p<.001, and the control group, r(33)= .82, p<.001.

Mathematical Language

Child maths talk

An ANCOVA was used to assess whether group predicted post-test total child maths talk whilst controlling for pre-test total child maths talk. Pre-test scores were significantly related to post-test scores (p<.001, $\eta^2 g$= .17). However, there was no effect of group on post-test scores (p=.075, $\eta^2 g$= .05). Pre- and post-test scores were correlated for the experimental group, r(35)= .53, p<.001, but not the control group, r(33)= -.04, p=.800.

An ANCOVA was used to analyse whether group predicted post-test breadth of children's maths talk, whilst controlling for pre-test breadth of maths talk score. Pre-test scores were not significantly related to post-test scores (p=.191, $\eta^2 g$= .02), and there was no significant effect of group on post-test scores (p=.073, $\eta^2 g$= .05). Pre- and post-test scores were not found to be correlated for either the experimental group r(35)= .18, p=.300, or the control group, r(33)= .01, p=.900.

Parent maths talk

As for previous analyses, ANCOVAs were performed to assess the effect of group on post-test total parent maths talk, controlling for pre-test parent total maths talk. While pre-test scores were significantly associated with post-test scores (p<.001, $\eta^2 g$= .23), there was no significant effect of group on post-test scores (p= .350, $\eta^2 g$= .01). Pre- and post-test scores were correlated for both the experimental group, r(35)= .45, p=.005, and the control group, r(33)= .50, p=.002.

To investigate whether Group predicted the breadth of parent maths talk scores at post-test whilst controlling for breadth of parent maths talk scores at pre-test, another ANCOVA was conducted. Pre-test scores were not significantly related to post-test scores, (p=.225, $\eta^2 g$= .02). However, there was a significant effect of group on post-test scores (p=.019, $\eta^2 g$= .08), with the experimental group scores (M= 4.85; SD= 1.53) being identified as significantly higher than control group scores (M= 4.20; SD= 1.16). Pre- and post-test scores were not found to be correlated for either the experimental group $r(35)$= .17, p=.300, or the control group, $r(33)$= .04, p=.800.

Potential unintended negative consequences

Table 9 summarises parents' reports of their self-efficacy as a parent ('Me as a Parent', Matthews et al., 2022) and their relationship with their child (Parent-child relationship', Pianta, 1992).

Table 9. Parental survey responses at pre- and post-test by group. Mean (SD).

	Experimental		Control	
	Pre	Post	Pre	Post
Me as a Parent Scale	4.00 (.45)	4.09 (.39)	3.84 (.80)	3.76 (.97)
Child-parent Relationship Scale	3.16 (.26)	3.13 (.24)	3.07 (.34)	3.17 (.35)

No group differences were found for the 'Child-parent relationship' scale or the 'Me as a parent' scale at either pre or post-test (all *p*'s >.05).

No pre- to post differences were found for the 'Child-parent relationship scale' or the 'Me as a parent' scale for either the experimental group or the control group (all *p*'s >.05).

Process analyses

Feasibility of blinding procedure

To assess the feasibility of the blinding procedure, experimenters kept a note of the number of times they became unblinded at post-test – the point in the study in which unblinding was most likely to have occurred. Experimenters were unblinded during 16.7% of post-test assessments (Ulster= 7; Sheffield= 5). However, no unblinding occurred during the 8-week intervention (e.g. through using the text messaging support system etc).

Missing data

Missing data was checked to assess feasibility of data collection for home and university laboratory testing locations on the child-level measures. Data was typically missing due to a child's refusal to engage with a task. At pre-test, for home-tested participants, there was 3.5% missing data for each of the four summed maths domain tasks separately (give-N, symbol-quantity mapping, digit recognition, shape understanding). There was no missing data for the BAS for home tested participants. Overall, for home-tested participants, 2.8% of data was missing.

For laboratory-tested participants, there was 3.6% missing data for the BAS and the Give-N task, 7.3% missing for the shape understanding task, 9.1% missing data for the digit recognition task, and 14.6% missing data for the symbol-quantity mapping task. Overall, 7.6% of data was missing for lab-tested participants.

At post-test, for home-tested participants, there was no missing data for the BAS, Digit recognition, or Shape understanding tasks. For the Give-N" and Symbol-quantity mapping tasks, 3.6% of data was found to be missing. A total of 1.4% of missing data was found for home-tested participants overall.

For lab-tested participants, there was no missing data for the BAS, Give-N, Symbol-quantity mapping, or the shape understanding tasks. For the digit recognition task, 2.3% of data was missing. Overall, a total of 0.5% of data was missing for lab-tested participants at post-test.

Participant feedback

Parents were asked a range of questions to explore participants' affective attitudes (Sekhon et al., 2022) towards the intervention and activity tasks. When asked about the developmental appropriateness of the activities, 92% of experimental participants and 78 % of control participants felt that the activities were "very much" or "mostly" right for their child's developmental stage. Participants also reported the number of activities provided was gauged well, as 70% of experimental group participants, and 62.2% of control group participants felt that the number of activities provided in the intervention were "just right." In response to a question on the burden of participating in the study, 84% of experimental group participants, and 92% of control group participants reported not having to give anything up to take part in the study. The length of the intervention was found to be highly acceptable among participants, as 89% of experimental group participants, and 86% of control group participants felt the length of the eight-week intervention was “just right.”

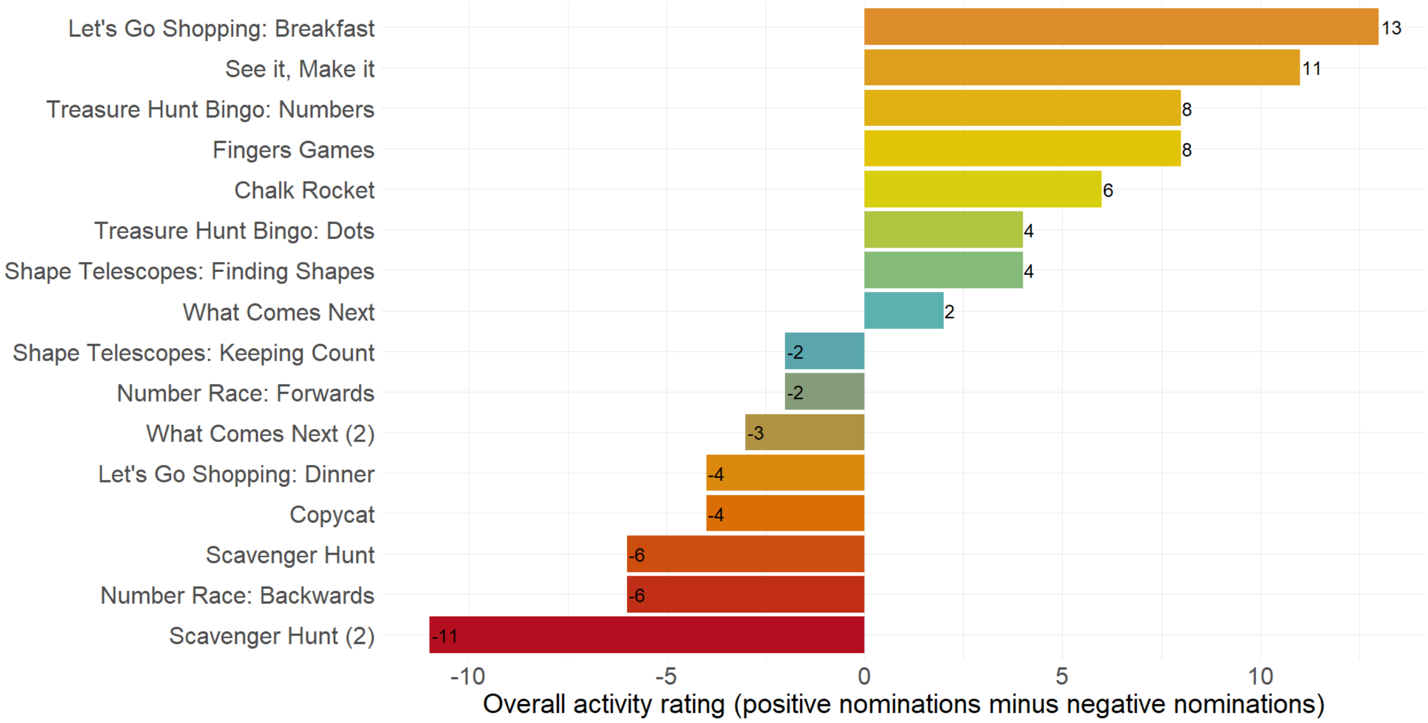
Finally, 64% of experimental participants and 70% of control participants felt their involvement in the intervention influenced how they engaged in other activities with their child, and 94.6% of experimental participants and 100% of control participants felt that the activities provided a positive opportunity, other than for learning, to interact with their child.

Parents ratings of individual activities

Participants in the experimental were required to nominate their three favourite and three least favourite activities from the intervention at post-test. For each activity, the total number of like least (LL) nominations was subtracted from the total number of like most (LM) nominations to give an overall net “liking” value for each activity. The activities were then ranked in order of their overall net value to determine the most and least popular activities (Figure 10). A similar process was engaged with for the control group, but instead of rating activities, parents rated their most and least liked weekly topic, due to the structure of this arm of the intervention (Figure 11).

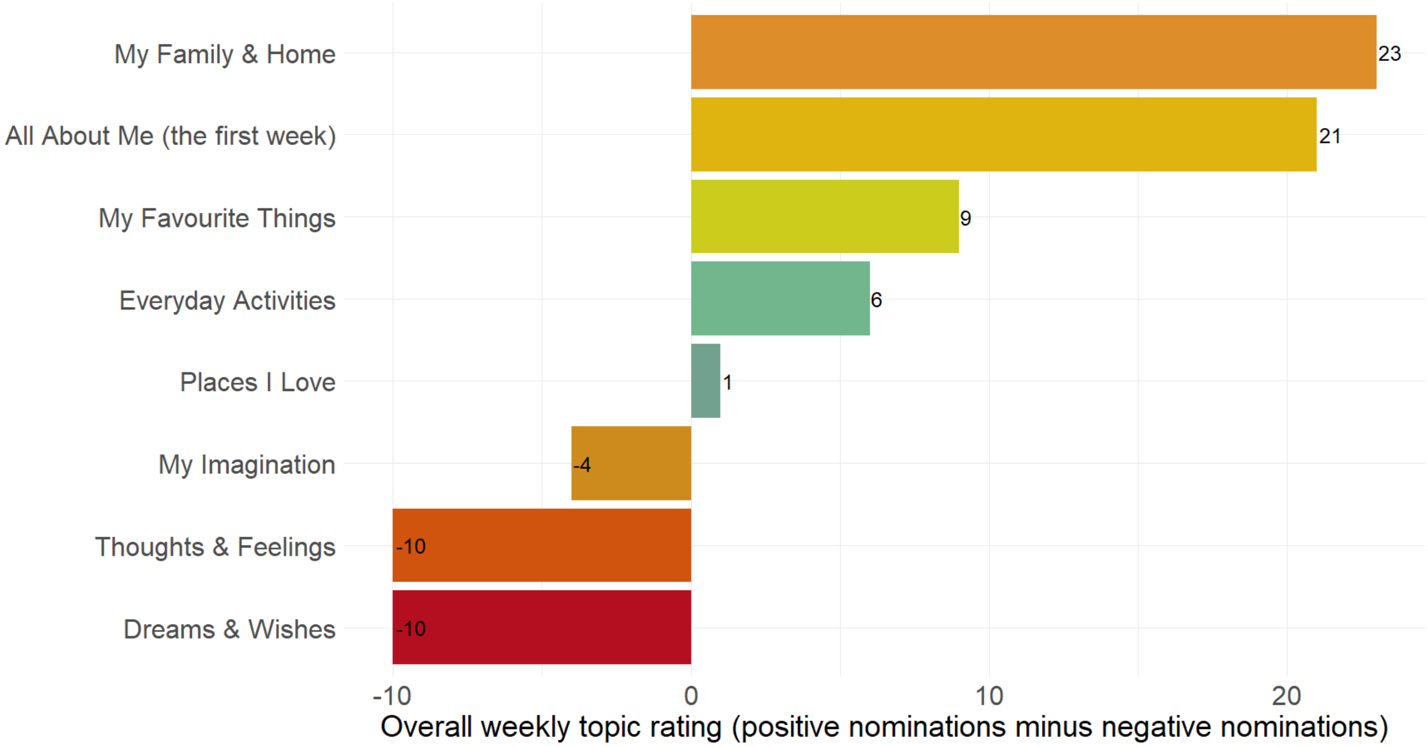
For the experimental group, ‘Let’s Go Shopping: Breakfast’ was the most popular home mathematics activities with a net positive score of 13, followed by ‘See it, Make it’ with a net negative score of 11. ‘Scavenger Hunt’ (2) was the least popular activity with a net negative score of 11 (Figure 10).

Figure 10. Summary of ratings for individual home mathematics activities for the experimental group



For the control group, ‘My Family & Home’ was the most popular activity, with a net positive score of 23, followed by ‘All about Me’ with a net positive rating of 21. ‘Dreams & Wishes’ and ‘Thoughts & Feelings’ received the lowest net negative rating out of the control activities with -10 given for both activities (Figure 11).

Figure 11. Summary of ratings for weekly topics for the control group



Qualitative findings

Participants were invited to take part in an online semi-structured feedback interview after completion of the programme. In total, 20 participants scheduled an interview with us (n=10 in each arm). Thematic analysis was conducted using a mostly deductive approach, centred around developing a richer understanding of strengths of the programme and areas for improvement. Five themes were identified. Firstly, parents felt the programme benefited them, their child, and their relationship with their child. They valued the practical and helpful activities, easy to understand and organised materials, and the opportunity to provide some structured learning and have valuable one-to-one time with their child. Secondly, parents had thoughts on areas where the programme worked less well. Some parents perceived some activities as too challenging; with some activities also considered too easy. A small group of parents reported challenges around their child's dispositions (e.g., confidence) or how accessible the materials were. Thirdly, some found fitting in the activities challenging, but emphasised they made it work and tried their best. Parents mentioned quicker activities were easier, but many found a routine, and some were delighted to see children sometimes initiating activities. Fourthly, parents had suggestions for areas to consider if the study was to be conducted again. Key suggestions included further video guidance, fewer activities, and control craft activity materials to be set aside for each week. Finally, parents noticed changes after taking part in the programme. Primarily, it helped them to have more ideas for incorporating maths into their daily lives. Parents also reported noticing that as the programme progressed their child not only improved in mathematics but also observed growth in language skills and confidence. Overall, parents reported feeling the study had a positive impact on themselves and their child. Importantly, when asked, all participants said they would take part in a similar study again in the future, noting they “really gained from it” and it “benefitted us.”

Interim discussion

Through the evidence gathered from the pilot study it was observed that it was feasible to recruit and retain a larger sample of families to engage in an 8-week long home-based intervention. Parents providing overwhelmingly positive feedback on their experiences engaging with the materials with their child for both the intervention and control group). Through follow-up interviews parents gave some suggestions for improvements for a future study, such as additional videos that would explain more activities, or decreasing the number of activities within the intervention. These qualitative suggestions must be held in tension with the overwhelmingly positive quantitative feedback.

Although feedback was positive, overall, we did not observe any changes in child attainment or parent self-reported behaviours. This is perhaps not surprising given the brevity of the intervention and the length of time after the intervention when children’s mathematics skills were measured (i.e. mean= 6.7 days; range= 1-4 weeks). There is a clear theoretical and empirical argument that in order to observe boosts in children’s mathematical skills long-term follow up may be required as children may need compounded exposure to mathematics activities that their caregivers may adopt into their everyday routines and/or new skills may take a protracted period of time to emerge through conceptual development (Leyva et al., 2023).

General discussion

This project aimed to further explore the relation between home engagement in mathematics and young children's mathematics skills, and to develop a feasible and acceptable programme to support families with home mathematics. There are several key findings from this project.

First, through our **secondary data analyses** which harmonised datasets from nine UK studies, we replicated previous findings that indicated a small, positive relation between the frequency with which families engage with mathematics at home and their child's mathematics skills (e.g., Ellis et al., 2023). Of course, these data are correlational and therefore emphasise the need for intervention studies to be able to understand the causal link (if any) between family engagement with home mathematics activities and children's learning. In addition, we observed the regularly reported relation between SEC and children's mathematics skills, with children experiencing lower SEC having lower mathematical skills (James-Brabham et al., 2023). However, importantly, we found that SEC was not associated with frequency of engagement with mathematics at home. We noted that instead of simply focusing on encouraging families to 'do more', we may need to focus on the quality of the mathematics interactions - and the range of the mathematical content that families engage with. This study emphasises the power of collaborative working across research teams, and the opportunity that data sharing can have to produce meaningful research contributions.

Secondly, the research team believes that co-production with research users is essential for effective and engaging intervention development. Through a rigorous, multi-pronged **co-production process** with early years educators and parents of 3–5-year-olds, and by building on prior evidence and data and the early years framework, we developed a set of activities and a programme for the intervention that was deemed as feasible and acceptable by parents. Although co-production is time consuming and requires substantive resources (Oliver et al., 2019), the feedback from parents in the **feasibility study** suggests it paid off. Parents deemed that the activities were suitable for their children to engage with, were enjoyable to use and their rates of engagement – reported through completed calendars – showed that general engagement instructions were followed. Some feedback from parents did suggest that some activities were more advanced than others, and this led the research team to emphasise in communication with parents in the pilot study that children are varied in their skills and development.

Third, in our **pilot study**, which included 42 dyads in an arts-and-crafts based control group and 42 dyads in our home mathematics activities intervention group, parents indicated positive feedback around the activities, with some specific mathematics activities and craft themes being particularly popular. Parents in the pilot study showed that they followed instructions on the rate of engagement with activities over the eight-week period through calendar completion. The purpose of this study was to ensure that we could randomise dyads to one of two groups, conduct the study with a blinded procedure (i.e., experimenters being naïve to group membership), complete assessments with children, and gain substantive information from parents. These factors were generally all completed successfully.

This component of the project was not sufficiently powered to detect effects of the intervention, as this was not the aim of the study. However, looking just at descriptive data, we saw no changes from pre-to-post-test in child mathematics skills or parent and child mathematical language use. Why might this be? The period over which the intervention took place (i.e., 8 weeks) may have not been substantive enough to induce change in children's skills. However, most parents indicated that the intervention period, and the required frequency of engagement was about right. In addition, as already mentioned, we did not include a long-term follow-up assessment, and it can be argued that consolidation of learning may take some time to embed (e.g., Leyva et al., 2023). Finally, although we have evidence on the frequency with which families engaged with our materials, we have little insight into *how* families engaged with them. Some families in the feasibility study sent adherence videos when completing a small number of activities- indicating that they were able to successfully follow the activity instructions, but we cannot extrapolate that this was the case for all participants.

A striking aspect of the pilot study was the sample characteristics of the participants recruited into the study. Our aim of the project was to attempt to target families whose children may have weaker mathematics skills at school-entry, specifically families experiencing low SEC. Participants were recruited through a variety of methods, including existing research team databases, community partners and social media. Throughout the recruitment process, IMD scores were monitored to ensure the demographic spread of participants which was achieved based on this measure. However, once all families were recruited, we were able to look at the educational background of participants, which revealed that the majority had at least a university undergraduate degree. Therefore, an important next step will be to adopt different recruitment strategies and ensure the intervention is appealing for families experiencing low SEC, especially when the intervention requires adoption of activities in real life. We would suggest that more qualitative research should be undertaken with families experiencing low SEC to understand barriers and facilitators to participation in this type of research.

Recommendations

Several recommendations for future research have been identified through this project:

Recruitment: We attempted recruitment for the study using diverse means, including through existing lab databases, local nurseries and community learning partnerships. However, given the high education levels of the families in the pilot study, we recommend that future recruitment should take place predominantly through social economy daycares, pre-school, nursery settings or Family Hubs, involving trusted existing contacts to reach out to families. This approach may help encourage a more diverse range of families to engage with the intervention, especially given the evidence that suggests that families experiencing SEC may respond more favourably to key gatekeepers in their communities (e.g. early years educators, Hoskins et al., 2024).

Assessments: Assessments should take place during children's on-site sessions at (e.g.) nurseries or daycare, reducing the burden on parents to attend pre-and post-test appointments. Importantly, this would not impact randomisation of dyads to groups (at a class level) or the blinding of experimenters to group membership. Most children within the pilot study attended pre-school or childcare settings, so this is deemed a feasible suggestion. Given that we observed no descriptive changes in children's mathematics skills or parent or child mathematical language, we would propose a longer-term follow-up assessment, perhaps at three months post intervention. This follow-up would provide time for embedding of parent practices and consolidation of learning.

Communication vs information sharing: In response to the feasibility study feedback, we secured additional funds to develop brief video descriptions of the activities. Parents provided favourable responses to these in the pilot study. However, there were still issues with parents reporting that some activities were too advanced for their child, even though we had provided written information to reassure them to be responsive to their child's skills. Therefore, in the future we would spend additional time co-producing this communication with parents, perhaps developing more short video resources, to ensure that this message is clear and relevant.

Understanding how families engage with resources: We received excellent information from families at the end of the study, including calendar records that showed rates of engagement over the intervention period. However, we know little about how families engaged with the materials. Perhaps by spending time collecting data whilst families are interacting with the materials and observing the language that they use whilst playing, would help tailor further guidance.

Link between early years educators and parents: This project solely focused on children and their parents in the home environment. The study team has previously worked within childcare and early years settings and, using co-produced materials and training, have observed a boost in children's mathematics skills, specifically for children experiencing low SEC (Scerif et al., 2025). Given the strong evidence base on the importance of parent and educator connection and communication (Desforges & Abouchaar, 2003) - an effective approach may be working through setting-parent partnerships to deliver this type of home-based intervention.

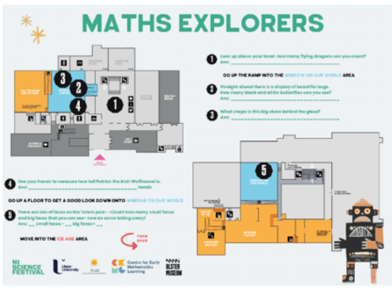
Building on this project: This study was purposefully not designed to assess the efficacy of our intervention but instead to examine the feasibility and acceptability of a new programme to support maths learning at home. The intervention produced excellent feasibility and acceptability evidence. Therefore, we would suggest that a future, large scale study should be conducted to assess efficacy of the intervention, after actioning our other recommendations.

Public engagement and dissemination

A key component of this project was a broad approach to public engagement. We aimed to share our findings and promote the importance of early maths. In addition, in some cases, we aimed to use the opportunities as a chance to gather further feedback on our programme. We detail these approaches below.

Maths trail maps

During the Northern Ireland Science Festival 2024 and 2025 we distributed ~3,000 Maths Mile and Maths Explorers maps (see below).



Parents and their children were encouraged to physically move around Belfast city centre, Derry city centre or the interior of the Ulster Museum to complete some maths challenges with their child. Engagement was excellent - families emailed completed copies of the maps to the team and they were entered into ballots for prizes.

Maths Explorers Events

Two family engagement events were organised in collaboration with a play company “Rabbit and Lark”. The company used our co-produced materials as inspiration to generate interactive activities for families at the events. The first event was held in October 2025, and as such, was themed around autumn (see below).


Seventy families attended and feedback was excellent, for example *“Absolutely amazing event for children and parents to see how everyday maths can be integrated into play. My 3 year old and 5 year old thoroughly enjoyed it!”* and *“Lovely ideas that I can use myself at home!”.* The second event was hosted in February 2026 as part of the Northern Ireland Science Festival programme, with a valentines day theme.

Educator workshop

In November 2025, we ran an event for the Sheffield Hallam Maths Association. A mix of trainee teachers and experienced teachers working in EYFS attended (n=15, 20% trainees, 80% experienced teachers). In the workshop, we discussed current evidence on how early maths skills develop and then we showcased several of the activities we co-produced as part of PLUS.

We were keen to hear feedback from educators on how they might work in their classroom. In the feedback form gathered at the end, 100% of attendees said it was helpful in informing their practice and 100% said they’d use the activities. They especially loved treasure hunt bingo, and one teacher said they wanted to make this a permanent install in their classroom. Comments included: "*Great to see that simple resources can have so much to adapt and use in a lesson", "Great activities and clear explanation of how to use them in classes", "Lovely practical games with resources we can find at school", "They look like lots of thought has gone into them*."

Early Childhood Maths Group

In February 2025, we gave a talk to the Early Childhood Maths Group (ECMG) who are a mix of practitioners, teachers, researchers and teacher educators in maths education from birth to seven. They are keen to promote early maths and produce guidance and resources for adults working with children. A blog post about the PLUS project with example resources is hosted on their website.

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Appendix A. Synthesised mathematical language coding scheme

Category Definition	Examples	Tally	Category score
Cardinal values mentioning or asking for the number of objects immediately, without counting	"Can you move six now?" "Here's your 3p" "I need 10p"		
Counting counting objects or asking for the number of objects immediately, without counting	"Mummy is going to move 5, 1,2,3,4,5" "One, two we have to move to five 3,4,5." "Do you see on the dice 12345"		
Naming digits referring to numbers, reading of numbers on paper and in other forms of media or recognising names of written numbers when seen, or writing numbers	"That looks like a number 7" "I think that says 5p" "Isn't that a 10p not a 2p?" "That is not number 10" "You need all your five moves"		
Measuring Using numbers to refer to units of measure	"You just turned 4." [after the child said he turned 5] "Count on 6 spaces" "So, you move your whale four circles"		
Conventional nominatives using numbers as labels for things or dates	"No, only high fives for me. Punch five hurt me; could you give me a high five?"		
Number comparisons Comparing numbers in a sequence Comparing numbers to see which is smaller or bigger (Schnieders & Schuh, 2022)	"What comes after 5?" "So, the highest number is six and what's the smallest number?		
Ordinal numbers Using ordinal numbers, such as first, second, third, etc Describing or asking about the numerical series (Eason et al., 2021)	"The first thing on my list is an apple." "You can go first, and I will go second," "What comes after twelve?		
Adding/subtracting Performing or asking to perform adding and subtracting calculations	"Here is a 1p and a 5p to make 6p." "Three, that's two and a one." "Did you ask 3 divided by 7?"		

Naming shapes Identifying and describing geometric shapes (Schnieders & Schuh, 2022)	"What shape is this here?" "Rectangle at the top but what do you call this here?" "So, the die only has six squares on it"		
Magnitude comparing two quantities or talking about relative quantities (Eason et al., 2021)	"Look, you're nearly at the end? "Oh, they are the same are they?" "I've got three more things on my list"		
Using spatial words Describe the location or direction of an object within the task (Schnieders & Schuh, 2022)	"Does it go on top of these or down beside it?" "So, this one's down at the bottom" "What's in the middle?" "Go back to the start". "That's on this one"		
Estimating Guessing a quantity without counting (Schnieders & Schuh, 2022)	"My piece is about 20 places away from yours" "How many goes do you think we could do to get to Mr Seahorse there?" "Do you think we could get him there in three goes?"		
Mathematical prompt Adult or child instigates a mathematical interaction.	"How many blocks do you have there?" "Who has the biggest tower?"		
			Total Score

Supplementary materials

Table 1: Harmonised frequency of HMA engagement

Cahoon	James-Brabham 1	James-Brabham 2	Van Herwegen	Bennett	Trickett	Duncan	Simmons	Chang	Final harmonised categories
Activity did not occur	Activity did not occur	Activity did not occur	Not at all	Rarely/never	Rarely/never	Activity did not occur	Never	Rarely or Never	Never/ rarely occurred
A few times a month	A few times a month	Activity occurred one-three times per month	Once a week	Monthly	Monthly	A few times a month	Occasionally	1 to 3 times a month	1-3 times a month
About once a week	About once a week	Activity occurred about once per week	A few times a week	Weekly	Weekly	About once a week	About once a week	About once a week	4 times a month
Few times a week	Few times a week	Activity occurred a few times a week (2-4 times)	Every day	Several days per week	Several days per week	Few times a week	Several times a week	About 3 to 5 times a week	5-16 times a month
Almost daily	Almost daily	Activity occurred almost daily	Several times per day	Most days per week	Most days per week	Almost daily	About once a day/ several times a day	Almost every day	17+ times a month