A new mathematics GCSE curriculum for post-16 resit students

Draft curriculum

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About MEI

MEI is an independent national charity committed to improving maths education. MEI's support for maths education includes developing curriculum specifications and schemes of assessment, providing professional development opportunities for teachers, and publishing teaching and learning resources. Most of our work is directed towards the maths education of 11–18 year-olds, addressing both academic and vocational pathways, and including maths in other subjects.

MEI's developments of post-16 mathematics curricula and qualifications include the development of the Critical Maths curriculum,[1] which informed the development of the level 3 Core Maths qualifications.

MEI's experience of supporting providers of resit GCSE Mathematics includes working with the Stoke-on-Trent Mathematics Excellence Partnership (MEP) and taking part in the Education Endowment Foundation's research into the contextualisation of resit GCSE Mathematics.[2]

For more information about MEI see https://mei.org.uk/

[1] MEI (2013). Critical Maths: a mathematics-based thinking curriculum for Level 3

[2] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). Embedding contextualisation in English and mathematics GCSE teaching: Pilot report. Education Endowment Foundation.

Introduction

Purpose

This document is part of an exploration of the feasibility of a new mathematics GCSE curriculum which is appropriate to the needs of post-16 students who have already sat GCSE Mathematics and have not achieved at least a grade 4 (a level 2 pass).

Students who have narrowly missed achieving grade 4 and who wish to pursue more mathematical or academic pathways may be well served by resitting GCSE Mathematics. However, many students either resit GCSE Mathematics repeatedly without improving on their KS4 grade, or manage to achieve a pass grade by cramming for the examination, without mastering fundamental mathematical skills and concepts.

It is intended that the new post-16 mathematics GCSE would have a maximum grade of 5, and would be designed so that students achieving grades 4 or 5 (a level 2 pass) would have demonstrated a good understanding of the fundamental mathematics needed to function as effective citizens.

It is not intended to create an 'easier' mathematics GCSE: MEI has a strong reputation for academic rigour in its qualification development. Our intention is to create a mathematics GCSE that is more appropriate for the target students, allowing them to gain valuable skills in applying mathematics and supporting them to develop a positive attitude towards the subject. This would motivate more students to succeed and would help to improve the public perception of mathematics.

Practical considerations are important. GCSE resit students have two terms post 16 before examinations; they are often taught by teachers with limited confidence in mathematics so there will be a need for professional development. The wide range of teachers in FE colleges may well find that it is easier to convince students to learn content which is more directly applicable.

Students may not want to take the course, and there are very large numbers of students in some centres. It is not possible to introduce much new content in the limited time, given that students lack confidence with the content which they have already been taught.

Aims

Students who succeed on the course should:

- develop a positive, confident attitude to mathematics
- be able to use mathematical skills in contexts encountered in work, study and life where specialised mathematical skills are not required
- be prepared for further study that does not require prior learning of abstract mathematics, including Core Maths and vocational courses.

To achieve this, students will need to have fluency with key mathematical skills. Students will have mastered some of these skills before starting on this course but they will have gaps in their knowledge and understanding. GCSE Core Maths has been suggested as a possible name for a qualification based on this curriculum.

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Exemplar contexts

The contexts listed in the following table exemplify what students should be able to do; they are not intended to form an exhaustive list. They are based on the Essentials of Numeracy from National Numeracy,[3] the New Zealand Learning Progressions for Adult Numeracy[4] and the 'General life and personal interest' section of the foundation tier GCSE context grid.[5]

Financial understanding	**Working with measures and shape**
• Understanding discounts in the sales • Understanding household bills • Estimating the cost of weekly food shopping • Splitting a restaurant bill • Shopping around for the best mobile phone deal • Comparing prices for differently sized packages • Budgeting for a holiday or major purchase • Personal budgeting • Managing a budget at work • Understanding interest rates when saving and borrowing	• Being able to read a measuring scale • Knowing your height and weight • Converting between imperial and metric units • Buying enough paint to decorate a room • Using shapes in designing a garden or craft project • Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space • Understanding a map or scale drawing • Understanding measurements relating to personal fitness and health • Giving the right quantity of medicine to children
Planning activities	**Understanding quantitative information**
• Estimating time needed for tasks • Planning a schedule • Understanding staff shifts on a rota • Planning a meal or party for a large number of people • Giving and following directions • Understanding journey times • Understanding a map or scale drawing • Understanding timetables	• Recording numerical information accurately so others can understand • Making sense of statistics in the news • Interpreting the results of an opinion poll and understanding why different polls may produce different results • Understanding results of elections • Understanding food labels • Understanding statistics relating to personal fitness and health • Understanding risk in the news in relation to health

The purpose of this section is to exemplify what students who have successfully completed a course based on this curriculum should be able to do, and so provide a basis for determining what content is appropriate to support competences like these. The content of the curriculum has been organised to show which content supports which types of context. It is intended that this contextualised organisation of content will support teaching and facilitate making links between mathematics teaching and other teaching, including vocational teaching.

[3] www.nationalnumeracy.org.uk/essentials-numeracy
[4] ako.ac.nz/knowledge-centre/learning-progressions-for-adult-numeracy/
[5] MEI (2017). Contextualisation Toolkit

Contexts from everyday life have been chosen rather than vocational contexts; these will be useful for a wider range of students and fairer as most students will be familiar with such contexts because they do not rely on specialist knowledge. Moreover, the EEF *Embedding contextualisation in English and mathematics GCSE teaching* pilot report[6] found that students tend to respond better to real-life, rather than vocational, contextualisation.

Appendix 1 lists the Foundation tier GCSE Mathematics content and other content which has been included in the draft curriculum, and Appendix 2 lists the GCSE Mathematics Foundation tier content that is not included.

Progression

Students aged over 16 may be taking other courses at the same time as, or soon after, seeking to improve on their GCSE Mathematics grades.

Core Maths

For Core Maths courses, the content of foundation tier GCSE Mathematics is assumed knowledge:

> *It is assumed that students will already have confidence and competence in the content presented in standard and underlined type within the GCSE mathematics subject content. Students will make use of elements of this content when addressing problems within Core Maths but we do not expect these to be explicitly set out in qualification content.*[7]

In practice, most of the content of most Core Maths qualifications builds on GCSE foundation tier in the following subject areas:

- Number
- Ratio, proportion and rates of change
- Probability
- Statistics

Students who have confidence and competence in these areas of foundation tier GCSE Mathematics have the background needed to be able to progress to doing Core Maths.

T levels

At the time of writing, T levels are being developed. The Royal Society's Advisory Committee on Mathematics Education (ACME) has proposed a framework of General Mathematical Competences (GMCs) to inform the development of mathematical content and assessment in T levels:[8]

- Measuring with precision
- Estimating, calculating and error spotting
- Working with proportion
- Using rules and formulae

[6] Runge, J., Munro-Lott, N., & Buzzeo, J. (2019). Embedding contextualisation in English and mathematics GCSE teaching: Pilot report. Education Endowment Foundation.
[7] DfE (2018). Core maths qualifications: technical guidance
[8] The Royal Society (2019). Mathematics for the T level Qualifications: a rationale for General Mathematical Competences (GMCs)

- Processing data
- Understanding data and risk
- Interpreting and representing with mathematical diagrams
- Communicating using mathematics
- Costing a project
- Optimising work processes

The content outline that follows supports the development of these GMCs.

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Subject content: Financial understanding

Exemplar contexts

- Understanding discounts in the sales
- Understanding household bills
- Estimating the cost of weekly food shopping
- Splitting a restaurant bill
- Shopping around for the best mobile phone deal
- Comparing prices for differently sized packages
- Budgeting for a holiday or major purchase
- Personal budgeting
- Managing a budget at work
- Understanding interest rates when saving and borrowing

Example learning outcomes and supporting content from GSCE Mathematics[9]

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order amounts of money in £ and/or pence • Interpret and order negative numbers in the context of a budget; recognise whether a higher value is better or worse	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate the total amount to be paid (without a calculator in suitable context) • Estimate the total to be paid • Divide a bill equally (with or without adding a tip) • Recognise whether a profit or loss has been made • Use a spreadsheet to set up and manage a budget	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Interpret and construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• List possible options when making a financial decision • Decide which option is best, considering total cost and practical considerations such as spreading cost and avoiding waste	5. apply systematic listing strategies

[9] Ofqual (2017). GCSE Subject Level Conditions and Requirements for Mathematics

Example learning outcomes	Supporting content from GCSE Mathematics
• Use multipliers when working with interest rates	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Interpret amounts when given in standard form, including understanding notation on a calculator or spreadsheet (converting a number into standard form is not required nor is calculating with numbers given in standard form)	9. interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a multiplier when working with percentages	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret decimals in the context of money • Round to the nearest pound or penny when appropriate	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate the total spend for a bill • Estimate in UK currency the cost of an item priced in a foreign currency	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round money to the nearest pound or penny, as appropriate • Know the interval that a rounded number of pounds could lie between	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Understand and use the equation of a straight-line graph	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Use formulae for working out costs	2. substitute numerical values into formulae and expressions, including scientific formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Plot points for a time-series graph of cost or expenditure	8. work with coordinates in all four quadrants
• Use a straight-line graph for situations where there is a fixed charge plus a charge per unit • Draw and use a conversion graph • Find an equation for a straight-line graph showing cost	9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
• Interpret gradient and intercept of a straight-line graph in the context of cost or currency conversion	10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
• Interpret graphs of cost against time and recognise the gradient as a rate of increase	14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a graph to find how much you could afford in a situation where there is a fixed charge plus a charge per unit	17. find approximate solutions to linear equations in one unknown using a graph
• Write a formula for a situation where there is a fixed charge plus a charge per unit	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert between standard units of time	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices,) in numerical contexts
• Express one cost as a fraction of another cost	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Express two or three amounts of money as a ratio	4. use ratio notation, including reduction to simplest form
• Divide a sum of money into two or three parts in a given ratio • Solve 'best buy' problems	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

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Example learning outcomes	Supporting content from GCSE Mathematics
• Find a multiplier when working with percentage increase or decrease	6. express a multiplicative relationship between two quantities as a ratio or a fraction
• Be able to use scale factors when working with rates of conversion or rates of pay, e.g. working for half a day gets half as much as working for a full day	7. understand and use proportion as equality of ratios
• Find percentage increase or decrease in financial contexts including profit and loss • Find a new value after a percentage increase or decrease • Find the original value before a percentage change, e.g. calculate the amount without VAT when given the amount including VAT	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Draw and use conversion graphs for currency exchange • Calculate amounts of money in the context of currency exchange	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Use rates of pay to find pay for different lengths of time, e.g. week, year • Use an appropriate inflation rate to estimate prices at a later time	11. use compound units such as speed, rates of pay, unit pricing
• Interpret gradient and intercept of a straight-line graph in the context of cost or currency conversion • Know that a graph for direct proportion is a straight line through the origin	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion
• Use a multiplier to calculate compound interest for loans and saving • Use average inflation rates	16. set up, solve and interpret the answers in growth and decay problems, including compound interest

Statistics

Example learning outcomes	Supporting content from GCSE Mathematics
• Use statistical diagrams in financial contexts	2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

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Additional content

Financial applications
Use mathematics in the context of personal, domestic and simple business finance including loan repayments and choosing the best loan to fit with personal circumstances, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates; be able to calculate income tax – includes understanding terms such as personal allowance and tax rate; be able to interpret a pay slip and use it in the context of budgeting
Be able to use and interpret the output of online calculators in the context of finance, e.g. for loans, savings, taxation, choosing the best deal for electricity or gas
Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, = C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision

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Subject content: Working with measures and shape

Exemplar contexts

- Being able to read a measuring scale
- Knowing your height and weight
- Converting between imperial and metric units
- Buying enough paint to decorate a room
- Using shapes in designing a garden or craft project
- Making and interpreting measurements to decide whether a piece of furniture or household appliance will fit in a given space
- Understanding a map or scale drawing
- Understanding measurements relating to personal fitness and health
- Giving the right quantity of medicine to children

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order amounts (including when given in different units) • Read a thermometer and order temperatures • Use negative numbers in the context of temperature	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate a total distance, knowing that all distances in the calculation must be in the same units • Calculate an area when the sides are given in fractions of a mile (or other unit) • Calculate area of a rectangle when sides are given in decimals • Calculate a missing measurement for a shape with known area or when working out area	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

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Example learning outcomes	Supporting content from GCSE Mathematics
• Use powers in units and formulae when working with area and volume	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Use simple fractions when working with measurements	8. calculate exactly with simple fractions

Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert fractions of a mile, fractions of an inch etc. to decimals and vice versa	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
• Be able to calculate a fraction of a distance	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Use, interpret and convert between standard units	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate lengths, areas and volumes	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round a measure to a given degree of accuracy • Know when it might be necessary to round up, e.g. when estimating how much paint is required • Use inequality notation to specify a simple error interval	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
• When given rounded measurements, calculate the limits of possible area, volume or length • Recognise that rounded measurements may give a misleading impression of whether something will fit into a gap	16. apply and interpret limits of accuracy

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Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use standard algebraic notation in formulae for areas and volumes	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Substitute numerical values into formulae for area, volume and speed • Use a formula to calculate the amount of medication needed for a child	2. substitute numerical values into formulae and expressions, including scientific formulae
• Understand and use standard mathematical formulae in the context of measures	5. understand and use standard mathematical formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Be able to plot points on a graph when solving a problem	8. work with coordinates in all four quadrants
• Draw and use a graph of length against width for a rectangle of given area when solving a problem	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a graph to convert from one unit to another or vice versa	17. find approximate solutions to linear equations in one unknown using a graph
• Write a formula for converting from one unit to another	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Change between different standard units	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) in numerical contexts
• Express a length, area or volume as a fraction of another one • Calculate and interpret waist-to-hip ratio	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

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Example learning outcomes	Supporting content from GCSE Mathematics
• Understand that e.g. a distance twice as long as another has the same relationship in any units	7. understand and use proportion as equality of ratios
• Be able to change between one distance being a fraction of another to the ratio between the distances and vice versa	8. relate ratios to fractions and to linear functions
• Work out a percentage increase or decrease in a measurement	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Use compound units informally (e.g. mg per ml) to compare strengths of similar medicines such as those designed for different ages of children and so understand that they have different concentrations	11. use compound units

Geometry and measures: Properties and constructions

Example learning outcomes	Supporting content from GCSE Mathematics
• Sketch and interpret diagrams for designs for garden or craft projects • Interpret vertical, parallel and perpendicular in 3D in a room or building • Use reflection and rotation symmetry to describe a design	1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
• Know and use the properties of special types of quadrilaterals in the context of a craft or DIY project	4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
• Sketch and interpret plans for simple buildings	13. construct and interpret plans of 3D shapes

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Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert between different units of length and mass • Use the correct units for area and volume • Know that capacity and volume can be measured in the same units • Know that a litre of water weighs 1 kg	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
• Measure lines and angles on a design • Know that a full turn is 360° and that a right angle is 90°	15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
• Estimate the total wall area of a room given length, width and height, and use this to estimate the amount of paint required • Work out the capacity of a fish tank and calculate its total mass when filled with water	16. know and apply formulae to calculate: area of triangles, volume of cuboids and other right prisms (including cylinders)
• Work out the distance round a circular pond or flower bed • Work out the area of a circular table and estimate the amount of varnish needed to paint the top surface	17. know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;
• Use Pythagoras' theorem to check whether a corner is a right angle	20. know and use the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$

Additional content

Financial applications	
Work out or estimate the cost of a craft or DIY project	Use mathematics in the context of personal, domestic and simple business finance including loan repayments, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates
Units of measurement	
Be able to read a measurement from a measuring scale and tell the time using both digital and analogue clocks; understand 24-hour clock and am and pm	
Work with commonly used units of measurement, including imperial units: mm, cm, m, km, litres, ml, cl, g, kg, pounds, stones, inches, miles, pints, gallons. Students should be able to convert between units in the metric system and know the following conversions relating to imperial units: • 14 lb = 1 stone • 8 pints = 1 gallon • 5 miles ≈ 8 km • 1 inch ≈ 2.5 cm • 1 lb ≈ 0.5 kg • 1 pint ≈ 0.5 litre • 1 gallon ≈ 4 litres	
Be able to work with more accurate (given) conversion factors and understand when it is appropriate to estimate	

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Subject content: Planning activities

Exemplar contexts

- Estimating time needed for tasks
- Planning a schedule
- Understanding staff shifts on a rota
- Planning a meal or party for a large number of people
- Giving and following directions
- Understanding journey times
- Understanding a map or scale drawing
- Understanding timetables

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order times (both time of day and intervals); use the symbols =, ≠,<, >, ≤, ≥ • Order amounts given in different units	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate the total time taken for a task • Calculate the length of time a journey takes • Calculate the total cost of running an event • Calculate the cost per mile of a journey • Work out speed, distance or time given the other two quantities	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• List possible options when making a plan • List the possible combinations of meals of 2 or 3 courses from a simple menu	5. apply systematic listing strategies
• Calculate with fractions of hours	8. calculate exactly with simple fractions

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Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert fractions of an hour to decimal parts of an hour and vice versa, including when working out speed	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Interpret measurements on a map or scale drawing • Use standard units for measurements when using an online calculator • Work with speed, distance and time • Work with compound units such as miles per gallon, litres per mile when working out the cost of a journey	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Estimate the length of time needed for a journey • Estimate the cost of a party or other event	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round the amount of food needed for a large party to help with shopping • Express the time needed for a journey as an error interval • Compare an answer from a rule of thumb to an exact answer	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a formula to estimate the length of time needed for a journey	2. substitute numerical values into formulae and expressions, including scientific formulae
• Know that speed = distance/time	5. understand and use standard mathematical formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Use references (e.g. A4) to find a street on a map	8. work with coordinates in all four quadrants
• Sketch a graph of distance against time when travelling at constant speed • Sketch a graph of time against speed for travelling a fixed distance	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

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Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret travel graphs and use them to solve problems	14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Make a spreadsheet to calculate the cost of a large party	21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Find a runner's speed in km per hour or miles per hour	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices) in numerical contexts
• Plan a journey using a map, using the scale to work out distance • Interpret scale drawings • Make a simple scale drawing	2. use scale factors, scale diagrams and maps
• Express a scale on a scale drawing or model as a fraction	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Use and interpret ratio for a map scale	4. use ratio notation, including reduction to simplest form
• Use a scale factor to find a distance either in reality or on the map/diagram	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
• Express and use a scale factor for a map or scale drawing as either a fraction or a ratio	6. express a multiplicative relationship between two quantities as a ratio or a fraction
• Understand that ratios of distances in real life are the same in a scale drawing or map	7. understand and use proportion as equality of ratios
• Be able to change between one distance being a fraction of another to the ratio between the distances and vice versa	8. relate ratios to fractions and to linear functions

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Example learning outcomes	Supporting content from GCSE Mathematics
• Estimate the time taken for a task when different numbers of staff are involved • Find the cost of an event for different numbers of people • Scale up the total time for a task from information from a sample/pilot • Rewrite a recipe to make it for a different number of people	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Work out average speed for a journey using an internet planner which gives distance and time	11. use compound units such as speed, rates of pay, unit pricing
• Interpret the gradient of a distance–time graph as speed	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion

Geometry and measures: Properties and constructions

Example learning outcomes	Supporting content from GCSE Mathematics
• Describe a scale drawing using correct terminology	1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description
• Recognise special quadrilaterals in scale drawings, plans and diagrams and use their properties when interpreting the scale drawing, plan or diagram	4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
• Draw and interpret plans drawn to scale	13. construct and interpret plans of 3D shapes

Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Work out the total time taken for a project	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
• Measure a bearing from a map or scale drawing • Interpret a scale drawing • Measure a route on a map	15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
• Convert measurements from a scale drawing or map to work out areas	16. know and apply formulae to calculate: area of triangles, volume of cuboids and other right prisms (including cylinders)

Example learning outcomes	Supporting content from GCSE Mathematics
• Use formula for area or circumference of circle when making calculations from a scale drawing or map	17. know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;

Additional content

Financial applications	
• Set up and interpret a spreadsheet to plan an event	Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, = C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision
Units of measurement	
• Find the time that a task or journey is expected to end	Be able to read a measurement from a measuring scale and tell the time using both digital and analogue clocks; understand 24-hour clock and am and pm
• Use imperial or metric units of length in scale drawings and maps	Work with commonly used units of measurement, including imperial units: mm, cm, m, km, litres, ml, cl, g, kg, pounds, stones, inches, miles, pints, gallons. Students should be able to convert between units in the metric system and know the following conversions relating to imperial units: • 14 lb = 1 stone • 8 pints = 1 gallon • 5 miles ≈ 8 km • 1 inch ≈ 2.5 cm • 1 lb ≈ 0.5 kg • 1 pint ≈ 0.5 litre • 1 gallon ≈ 4 litres
• Convert between imperial and metric units of length when planning a journey	Be able to work with more accurate (given) conversion factors and understand when it is appropriate to estimate
• Use the output of an online map to work out average speed • Compare the distance given by an online map with the direct distance	Be able to interpret the output of an online map in the context of planning a journey

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Subject content: Understanding quantitative information

Exemplar contexts

- Recording numerical information accurately so others can understand
- Making sense of statistics in the news
- Interpreting the results of an opinion poll and understanding why different polls may produce different results
- Understanding results of elections
- Understanding food labels
- Understanding statistics relating to personal fitness and health
- Understanding risk in the news in relation to health

Example learning outcomes and supporting content from GSCE Mathematics

Number: Structure and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Order risks using either percentages or fractions • Order amounts of nutrients in different units	1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
• Calculate percentages from raw data • Use a probability expressed as a fraction, decimal or percentage to calculate an expected frequency	2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
• Simplify working when calculating without a calculator • Construct a simple spreadsheet formula, knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)	3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
• Understand the use of an exponential model for bacterial growth	6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
• Calculate expected frequency given the risk	8. calculate exactly with simple fractions
• Interpret populations given in standard form	9. interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

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Number: Fractions, decimals and percentages

Example learning outcomes	Supporting content from GCSE Mathematics
• Convert a probability expressed as a fraction to a decimal or percentage and vice versa • Be able to convert between a fraction of the population and the corresponding percentage of the population	10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
• Work out a percentage or fraction of a population	12. interpret fractions and percentages as operators

Number: Measures and accuracy

Example learning outcomes	Supporting content from GCSE Mathematics
• Interpret units on a statistical graph • Give the appropriate units when constructing a graph • Work with compound measures such as people per km^2 or per 1000 people	13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
• Compare different people's estimates of the number of people in a crowd from a photograph • Estimate a value from a statistical diagram when the scale will not allow accurate reading off	14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology
• Round a mean calculated from rounded values	15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
• Express an error interval for a mean calculated from rounded values	16. apply and interpret limits of accuracy

Algebra: Notation, vocabulary and manipulation

Example learning outcomes	Supporting content from GCSE Mathematics
• Understand and use a trendline formula for a scatter diagram produced by software	1. use and interpret algebraic notation, including: • ab in place of $a \times b$ • $3y$ in place of $y + y + y$ and $3 \times y$ • a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$ • $\frac{a}{b}$ in place of $a \div b$ • coefficients written as fractions rather than as decimals • brackets
• Substitute values into a formula for a line of best fit	2. substitute numerical values into formulae and expressions, including scientific formulae

Algebra: Graphs

Example learning outcomes	Supporting content from GCSE Mathematics
• Plot points for a scatter diagram	8. work with coordinates in all four quadrants
• Find the equation of a line of best fit for a scatter diagram	9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient
• Interpret the gradient and intercept for the line of best fit on a scatter diagram	10. identify and interpret gradients and intercepts of linear functions graphically and algebraically
• Recognise when a straight-line model is appropriate for a scatter diagram, and when it is not	12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

Algebra: Solving equations and inequalities

Example learning outcomes	Supporting content from GCSE Mathematics
• Use a line of best fit to estimate an unknown value	17. find approximate solutions to linear equations in one unknown using a graph

Ratio, proportion and rates of change

Example learning outcomes	Supporting content from GCSE Mathematics
• Use and interpret units on a statistical diagram	1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices,) in numerical contexts
• Find the fraction of the sample or population in a particular category • Find the fraction or percentage of daily recommended amount in a particular food	3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
• Use ratio to compare risks • Use ratio to compare the size of different groups in a sample or population	4. use ratio notation, including reduction to simplest form
• Use a ratio from a sample to estimate the number of people	5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
• Understand risk given as either a fraction or a ratio	6. express a multiplicative relationship between two quantities as a ratio or a fraction

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Example learning outcomes	Supporting content from GCSE Mathematics
• Relate a fraction of a population or sample to a ratio between two parts of a population or sample	8. relate ratios to fractions and to linear functions
• Find the percentage of the sample or population in a particular category • Interpret probability as a percentage of the population	9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics
• Work out the number of calories or grams of a nutrient in a portion of food when given the amount in 100 g	10. solve problems involving direct and inverse proportion, including graphical and algebraic representations
• Use and interpret compound units in statistical diagrams	11. use compound units such as speed, rates of pay, unit pricing
• Interpret the gradient of a line of best fit in context including knowing that it gives the increase in the dependent variable for an increase of 1 unit in the independent variable	14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion

Geometry and measures: Mensuration and calculation

Example learning outcomes	Supporting content from GCSE Mathematics
• Use standard units in statistical diagrams • Interpret different units on food labels	14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

Probability

Example learning outcomes	Supporting content from GCSE Mathematics
• Record and analyse the outcomes of a game involving probability • Analyse the outcomes of a simple clinical trial	1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
• Calculate the expected number of people with a particular condition in a randomly selected group of given size	2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
• Use expected frequencies to find a probability	3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

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Example learning outcomes	Supporting content from GCSE Mathematics
• Given the lifetime probability of suffering from a particular illness, find the probability of not suffering from that illness • use probabilities summing to one when working with tree diagrams or two-way tables	4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
• Use spreadsheet simulations to estimate probabilities	5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
• Use frequencies to show the results of a clinical trial in a two-way table, Venn diagram or tree diagram • Use a systematic strategy to work out the number of ways to do something	6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
• Find the probability that all three children in a family are boys • For a genetic illness with a given probability of transmission, find the probability that two children are free of the disease	7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
• Use expected frequencies to work out the probability of a false positive in a medical test	9. **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables**

Statistics

Example learning outcomes	Supporting content from GCSE Mathematics
• Given the results of an opinion poll or survey, estimate the number of people with each view in the population	1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
• Show the nutrients in a group of foods using an appropriate statistical diagram • Show results from a survey or a series of surveys using an appropriate statistical diagram or table • Interpret statistical diagrams on websites or in political leaflets	2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
• Use calculator or spreadsheet statistical functions to calculate quartiles and know that they split the data into four groups of equal size • Compare earnings for male and female workers or house prices in different parts of the country • Have an informal understanding of outliers as unusual values which may have a large effect on the mean and range and understand that the median is not affected by outliers	4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data, • appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers,)

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Example learning outcomes	Supporting content from GCSE Mathematics
• Compare age distributions in different countries or towns	5. apply statistics to describe a population
• Plot a scatter diagram of match ticket prices v league table position for football teams and understand that neither causes the other even if there is correlation • Recognise that less scatter means a stronger relationship • Use a scatter diagram of a time series to make a prediction and understand that a prediction close to the times where data are known is more likely to be accurate	6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Additional content

Financial applications	
• Interpret a time series of exchange rates over time	Use mathematics in the context of personal, domestic and simple business finance, including loan repayments, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates
• Interpret statistical output from a spreadsheet • Choose a suitable statistical graph from a spreadsheet menu	Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills: • use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers e.g. = C2 + D2, =C2*1.05^D2 knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these) • select, draw and format suitable graphs and charts using a spreadsheet • interpret output from a spreadsheet in the context of solving a problem or making a decision
Risk	
• Be able to interpret a statement about risk in the form 1 in *n* and convert this form to and from a probability given as a fraction or decimal	
• Be able to use a frequency tree in the context of risk to find a probability	
• Be able to use information about conditional increase in risk to form a frequency tree using natural frequencies and calculate associated probabilities in simple cases	
• Be able to interpret relative and absolute risk	

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Appendix 1 GCSE Mathematics Foundation tier content and other content which is included in the draft curriculum

The content below is based on the foundation tier GCSE Mathematics content for teaching from September 2015 which supports working in realistic contexts such as those listed above. There is some content from the higher tier and also some additional content which is not part of GCSE Mathematics. The foundation tier GCSE Mathematics content which is not included in this draft curriculum is shown in Appendix 2.

Where content is taken from foundation tier or higher tier Mathematics, the references, underlining and **bold** text are as for the GCSE content. Content shown in **bold** is higher tier only in GCSE Mathematics.

Number

Structure and calculation

1. order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥
2. apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
3. recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions; use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
5. apply systematic listing strategies
6. use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
8. calculate exactly with simple fractions
9. interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Fractions, decimals and percentages

10. work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)
11. identify and work with fractions in ratio problems
12. interpret fractions and percentages as operators

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Measures and accuracy

13. use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

14. estimate answers; check calculations using approximation and estimation, including answers obtained using technology

15. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

16. apply and interpret limits of accuracy

Algebra

Notation, vocabulary and manipulation

1. use and interpret algebraic notation, including:
    - ab in place of $a \times b$
    - $3y$ in place of $y + y + y$ and $3 \times y$
    - a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
    - $\frac{a}{b}$ in place of $a \div b$
    - coefficients written as fractions rather than as decimals
    - brackets

2. substitute numerical values into formulae and expressions, including scientific formulae

5. understand and use standard mathematical formulae

Graphs

8. work with coordinates in all four quadrants

9. plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient

10. identify and interpret gradients and intercepts of linear functions graphically and algebraically

12. recognise, sketch and interpret graphs of linear functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

14. plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

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Solving equations and inequalities

17. find approximate solutions to linear equations in one unknown using a graph

21. translate simple situations or procedures into algebraic expressions or formulae

Ratio, proportion and rates of change

1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices) in numerical contexts

2. use scale factors, scale diagrams and maps

3. express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

4. use ratio notation, including reduction to simplest form

5. divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)

6. express a multiplicative relationship between two quantities as a ratio or a fraction

7. understand and use proportion as equality of ratios

8. relate ratios to fractions and to linear functions

9. define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease and original value problems, and simple interest including in financial mathematics

10. solve problems involving direct and inverse proportion, including graphical and algebraic representations

11. use compound units such as speed, rates of pay, unit pricing,

14. interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct proportion

16. set up, solve and interpret the answers in growth and decay problems, including compound interest

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Geometry and measures

Properties and constructions

1. use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

4. derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

9. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference

13. construct and interpret plans of 3D shapes

Mensuration and calculation

14. use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

15. measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

16. know and apply formulae to calculate: area of triangles, volume of cuboids and other right prisms (including cylinders)

17. know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes;

20. know and use the formulae for: Pythagoras' theorem, $a^2 + b^2 = c^2$

Probability

1. record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

4. apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

5. understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

6. enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

7. construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

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8. calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

9. **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables**

Statistics

1. infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling

2. interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

4. interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

    - appropriate graphical representation involving discrete, continuous and grouped data,

    - appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

5. apply statistics to describe a population

6. use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

Additional content

Financial applications

Use mathematics in the context of personal, domestic and simple business finance including loan repayments, budgeting, VAT, understanding the idea of a measure of inflation; be able to calculate using exchange rates

Be able to use and interpret the output of online calculators in the context of finance, e.g. for loans, savings, taxation

Use spreadsheets to model financial, statistical and other numerical situations, including using mathematical content from the rest of the specification and the following additional content and skills:

- use and interpret simple spreadsheet formulae including sums, differences, percentage change, using cell references or absolute cell references, four functions and powers
  e.g. = C2 + D2, = C2*1.05^D2
  knowing that A2 refers to the number in cell A2 and how this will change when the formula is copied down or across the spreadsheet (students are not expected to know that a spreadsheet formula should start with = or that * is used for × in a spreadsheet formula but they should be able to interpret formulae which include these)
- select, draw and format suitable graphs and charts using a spreadsheet
- interpret output from a spreadsheet in the context of solving a problem or making a decision

Units of measurement

Be able to read a measurement from a measuring scale and tell the time using both digital and analogue clocks; understand 24-hour clock and am and pm

Work with commonly used units of measurement, including imperial units: mm, cm, m, km, litres, ml, cl, g, kg, pounds, stones, inches, miles, pints, gallons. Students should be able to convert between units in the metric system and know the following conversions relating to imperial units:

- 14 lb = 1 stone
- 8 pints = 1 gallon
- 5 miles ≈ 8 km
- 1 inch ≈ 2.5 cm
- 1 lb ≈ 0.5 kg
- 1 pint ≈ 0.5 litre
- 1 gallon ≈ 4 litres

Be able to work with more accurate (given) conversion factors and understand when it is appropriate to estimate.

Be able to interpret the output of an online map in the context of planning a journey

Risk

Be able to interpret a statement about risk in the form 1 in *n* and convert this form to and from a probability given as a fraction or decimal

Be able to use a frequency tree in the context of risk to find a probability

Be able to use information about conditional increase in risk to form a frequency tree using natural frequencies and calculate associated probabilities in simple cases

Be able to interpret relative and absolute risk

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Appendix 2 GCSE Mathematics Foundation tier content which is not included in the draft curriculum

The following content has not been included because it does not support working in the kinds of contexts which this curriculum aims to support. In order to ensure that students taking a resit programme have sufficient time to succeed with the mathematics which is most useful to them, it is necessary to reduce the content of Mathematics GCSE.

Number

Structure and calculation

4. use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

7. calculate with roots, and with integer indices

8. calculate exactly with multiples of π

9. calculate with standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer

Algebra

Notation, vocabulary and manipulation

3. understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

4. simplify and manipulate algebraic expressions (including those involving surds) by:
   - collecting like terms
   - multiplying a single term over a bracket
   - taking out common factors
   - expanding products of two binomials
   - factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares
   - simplifying expressions involving sums, products and powers, including the laws of indices

5. rearrange formulae to change the subject

6. know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments

7. where appropriate, interpret simple expressions as functions with inputs and outputs

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Graphs

11. identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically

12. recognise, sketch and interpret graphs of quadratic functions, simple cubic functions

Solving equations and inequalities

17. solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation);

18. solve quadratic equations algebraically by factorising; find approximate solutions using a graph

19. solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph

21. derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

22. solve linear inequalities in one variable; represent the solution set on a number line

Sequences

23. generate terms of a sequence from either a term-to-term or a position-to-term rule

24. recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)

25. deduce expressions to calculate the nth term of linear sequences.

Ratio, proportion and rates of change

1. change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in algebraic contexts

11. density, pressure

12. compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

13. understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; interpret equations that describe direct and inverse proportion

Geometry and measures

Properties and constructions

2. use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

3. apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel

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lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

5. use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

6. apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

7. identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)

9. tangent, arc, sector and segment

11. solve geometrical problems on coordinate axes

12. identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

13. construct and interpret elevations of 3D shapes

Mensuration and calculation

16. know and apply formulae to calculate: areas of parallelograms, trapezia

17. surface area and volume of spheres, pyramids, cones and composite solids

18. calculate arc lengths, angles and areas of sectors of circles

19. apply the concepts of congruence and similarity, including the relationships between lengths, in similar figures

20. know the formulae for the trigonometric ratios $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ and $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$; apply them to find angles and lengths in right-angled in two dimensional figures

21. know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

Vectors

24. describe translations as 2D vectors

25. apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors

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