Nuffield Foundation

Values and variables

Mathematics education in high-performing countries

Mike Askew, Jeremy Hodgen, Sarmin Hossain
and Nicola Bretscher, King's College London

www.nuffieldfoundation.org

Nuffield Foundation
28 Bedford Square
London WC1B 3JS

Telephone 020 7631 0566

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Foreword

The Nuffield Foundation has commissioned several reviews of aspects of mathematics teaching and learning, both to inform its own work and as a contribution to the national debate about future directions for mathematics education.

This review arises from a conviction that we can benefit from looking at educational practices in countries beyond the UK, not necessarily by 'cherry picking' good ideas, but rather through presenting alternative perspectives on familiar assumptions and practices.

Alongside the growth of international comparisons of mathematics attainment in schools in recent decades, there has been a corresponding growth in the volume of research literature exploring and comparing the impact of different national systems of mathematics education. This review is an attempt to identify, explore and explain key messages arising from this extensive body of evidence.

We are grateful to the review team for their diligence in undertaking this review and are confident its publication will make a significant contribution to the continuing debates about policy and practice in the UK and beyond.

Anthony Tomei
Director, Nuffield Foundation

The review team

The review was led by Professor Mike Askew and co-authored by Dr Jeremy Hodgen, Dr Sarmin Hossain and Nicola Bretscher, all of King's College London. Additional support was provided by Professor Margaret Brown and Professor Jill Adler, also of King's College London, and Laurie Jacques.

The Nuffield Foundation

The Nuffield Foundation is an endowed charitable trust established in 1943 by William Morris (Lord Nuffield), the founder of Morris Motors, with the aim of advancing social well being. We fund research and practical experiment and the development of capacity to undertake them; working across education, science, social science and social policy.

Contents

Introduction

This report addresses two questions posed by the Nuffield Foundation:

- **What is the range and type of research evidence from countries with high performance in mathematics that gives insights into the reasons for their relatively high position?**

- **What constitutes high performance in mathematics learning and what factors appear to contribute most to achieving it?**

Our investigation of the first question led us to believe that there was sufficient evidence to suggest that the second question was worth investigating.

The following section discusses how we set about answering the first question. We have identified a wide range of research evidence, and a bibliography arising from this is freely available from the King's College London website at **www.kcl.ac.uk/education/research/mechma**

There then follows a discussion of the assumptions behind international comparisons and the possible limitations to learning from such studies.

From the detailed review of the literature we identified a number of themes each of which is briefly discussed. This is not meant to be an exhaustive list of themes and there are many more we could have included. In the interests of keeping this report relatively brief, we selected themes in areas with strong research evidence, particularly if this evidence could raise questions about practice in teaching mathematics in England. For example, much of the research highlights the importance of culture and learner dispositions, so many of the themes focus on how pupils directly interact with mathematics education – through engaging with their parents, teachers, lessons, textbooks and shadow education. Factors such as school leadership, inspection and money spent on education have less direct impact on the pupil experience and are not considered in detail. This is not to say we consider such aspects as insignificant, but they are beyond the scope of this study.

In discussing these themes we have necessarily identified a relatively small proportion of the papers in our bibliographic database to include in the reference lists in this review. The full set of core references coded against all the themes can be downloaded from the King's College website.

The final section presents a list of conclusions. While some of these relate directly to the themes discussed, this final synthesis draws on our reading of the literature more widely and our professional judgement of the issues that the research raises overall.

Research evidence from countries with high performance in mathematics

Identifying countries with high performance in mathematics

We selected countries to focus on by:

a) identifying mathematics studies conducted on an international scale, for example by the International Association for the Evaluation of Educational Achievement (IEA) and Organisation for Economic Co-Operation and Development (OECD). The studies mainly considered for this identification exercise include:

- the First International Mathematics Study (FIMS) 1963–1967
- the Second International Mathematics Study (SIMS) 1977–1981
- the Third International Mathematics and Science Study (TIMSS) 1995
- trends in Mathematics and Science Study (TIMSS) 1999 and 2003
- programme for International Student Assessment (PISA) on the performance of 15-year-olds in mathematics in 2003 and 2006

b) studying data related to average (median or mean) mathematics performance scores from these studies. For each of these studies, countries whose scores were above the international average were tabularised. We scrutinised the data for those countries that over the years were consistently above the international average and ranked within the top ten countries taking part in these international studies.

From this analysis the following countries were identified as displaying high performance in mathematics: Singapore, Hong Kong, Korea, China, Chinese Taipei (Taiwan), Japan, the Netherlands, Belgium, Hungary, Latvia, Czech Republic, Finland, Australia, Russian Federation, New Zealand, Slovenia and Slovak Republic. Based on the amount of research evidence available, and to make the task manageable, we finally concentrated in the main on the Pacific Rim countries and Finland, although we do make reference to the other countries listed here when appropriate. We note that further comparative research is needed into some of the other high-performing countries in the full list, particularly those whose cultural traditions may be closer to ours than the Pacific Rim countries. Brief vignettes of the education systems in the Pacific Rim countries and Finland follow.

China

We note that China has only taken part in one international survey, conducted at the beginning of the 1990s. Although Chinese students' attainment was high, this survey lacks validity because the sampling of Chinese students was poor. Nevertheless, there is considerable international interest in Chinese mathematics education, which has resulted in a great deal of smaller scale research suggesting that some aspects of Chinese mathematics education perform very well. We have included it on this basis.

Compulsory education in China consists of primary schools (Grades 1 to 6) and junior secondary (Grades 7 to 9). Around 50% of pupils go on to senior secondary (Grades 10 to 12) and just over 20% of all pupils enter higher education. Mathematics education has a long tradition in China, resisting the 'new maths' of the 1960s and 1970s. The Chinese system is nationally governed with, until the mid 1980s, only one allowable series of textbooks. There is a strong emphasis on basic knowledge and skills, with extensive practice bringing these about. Textbooks however, have carefully constructed examples that engage pupils beyond mere 'drill and practice'. Elementary mathematics teachers appear to have comparatively low instructional loads (10–18 40-minute lessons per week), allowing additional time for marking, feedback and lesson preparation (Li, Chen, and Kulm, 2009). Chinese policy-makers are concerned about the extent to which Chinese students can apply mathematics.

Chinese Taipei (Taiwan)

Education in Taiwan consists of six years of elementary school followed by three years of junior high school, then three years of senior high school. Education is compulsory to the end of junior high school. Textbooks are regulated by government but produced by private publishers and schools are free to choose from a list of approved textbooks. The junior high school is differentiated with some vocational schools. Technical training courses are available in the final year of compulsory schooling. Formal symbolic algebra is introduced in elementary school. One feature of mathematics education in Taiwan is that research funds have been used to replicate seminal international research studies (e.g., Lin, 1988).

Finland

Finland's pupils have been considered high performers in mathematics given their success in recent PISA studies. Finland ranked first in 2003 (although the Canadian province of Ontario was the highest scoring) and second after Hong Kong in 2006. This success was a surprise both in Finland and elsewhere (Pehkonen, Ahtee, and Lavonen, 2007). Efforts to understand this achievement have been hampered by a limited research base. The Finnish education system consists of comprehensive school education at both primary and lower secondary levels. Children start school at the age of seven and there are nine years of compulsory schooling. All types of education in Finland are free and well supported. Student groups are heterogeneous: there is officially no tracking or streaming of students, although the flexible school-based curriculum makes it difficult to know how the comprehensive policies play out at the classroom level. Class sizes are small, for example the mean was 18 students in 2003, and 20 students in 2006. Teachers assess students providing descriptive assessments and feedback. All basic (that is primary and secondary) school teachers must hold a masters degree to become permanently employed. A recurrent theme in the research and commentaries on Finland's success is the principle of equity and social justice being central to Finland's education policy. Efforts and resources have been directed so as to provide equal opportunities to all population groups and regions of the country. Early and rapid intervention for pupils falling behind their peers is one of teachers' key priorities, with parental support sought. Teaching is generally acknowledged to be rather traditional.

Hong Kong

The Hong Kong educational system reflects the region's historical links to the United Kingdom. The structure of the system is closely related to the English system in terms of six years of primary school followed by five years of secondary school, then two years of pre-university study, although only the first three years of secondary school are compulsory. Like England, the school curriculum is divided into four key stages and students follow the same curriculum until the end of compulsory education (at around age 15). There is some setting of low-attaining pupils into separate classes. Schools can choose from a list of approved textbooks. Pupils sit national assessments in mathematics at the end of Key Stages 1 to 3. A small proportion of pupils do not progress to the next grade at the end of each year. Some lessons in primary education are taught by specialist mathematics teachers, although not all schools have specialist teachers. Formal algebra is introduced earlier than in England during the equivalent of Year 5. Hong Kong mathematics teaching is largely teacher-directed.

Japan

In TIMSS and PISA studies that look at pupils' mathematics performances across a number of countries, Japan has been consistently ranked within the top ten high-performing countries.

The levels of education in Japan consist of: non-compulsory pre-primary (for ages 3–5); compulsory primary education (ages 6–11); lower secondary (ages 12–14); non-compulsory upper secondary education (ages 15–17 with graduation at generally age 18). There are no entrance or exit criteria at the end of lower secondary and entrance into upper secondary is based on pupils' test scores and report cards from lower secondary. Children with good results stand a better chance of attending the best upper secondary schools in the area. There are night schools or 'cram schools' (Juku), which prepare pupils for upper secondary school entrance exams. Pupils may also choose to attend vocational/technical institutes that combine upper secondary school with vocational higher education. Japan's education system is based on principles that include equal opportunity of access, secular and non-political education. The education system in Japan is highly centralised and relatively homogeneous. Classes are not streamed – there is an emphasis on a uniform educational experience – and much effort is placed in ensuring equality and homogeneity. Differentiation mainly takes place in post-compulsory education, existing between schools instead of within classes and year groups. Following the TIMSS video study, international attention has focused on Japan's apparently more conceptual teaching and on the practices of lesson study (Stigler and Hiebert, 1999). As in China, Japanese educators are concerned about students' problem-solving and application skills (e.g., Hughes, Desforges, Mitchell, and Carre, 2000).

Singapore

Singapore's educational structure comprises six years of primary, four years of secondary and two years pre-university. Only the first four years of primary follow a common curriculum: pupils follow one of two 'orientation' curricula in the last two years of primary, one of these being a reduced curriculum at a slower pace. There is a leaving exam at the end of primary: some pupils take a different exam if they have followed the 'reduced' curriculum. There are three courses at secondary school: around 60% of pupils follow an 'express course' leading to a GCE O-level in four years, 25% a 'Normal' (academic) course leading to O-level in five years (or an N-level in four years) and 15% in a 'normal' (technical) course leading to N-level. Between 20% and 25% of pupils continue to university. While the curriculum is centrally mandated and there is high-stakes assessment, schools have flexibility over the implementation of the curriculum. Since the 1990s there is no longer a single state-mandated textbook, with commercial publishers producing textbooks in an open market. A five-fold curriculum framework emphasises attitudes and meta-cognition as well as skills, concepts and processes. Compared to its near neighbours, Singapore's pupils do report more enjoyment of mathematics.

South Korea

The Korean educational system comprises six years of elementary, three years of middle school, three years high school and four years of college. Official statistics report 99% of pupils graduating from high school. There is a compulsory core curriculum taught to all pupils to the end of Grade 10. In Grades 11 and 12 there are elective courses. Pupils are assessed at the end of each year and have to pass to proceed to the next grade level. They may remain in a grade level if they fail the end of year assessment, but can only repeat a year once; if they fail a second time, pupils have to move up. Skipping grades is not allowed. Only one textbook series is available at the primary level and all schools use this. Textbooks for secondary mathematics have to follow guidelines provided by the Ministry of Education. Education is hugely important in Korean society, with parents willing to endure personal hardships to support their children's education. But despite the success on international comparisons, Korean policy makers are concerned about pupils' general dislike of mathematics.

England's position

It is worth noting where England sits in all these international studies. England regularly gets positioned towards the top of the group of second ranked countries and its performance is fairly stable over time. It also needs to be noted that the differences between countries' performance are not that large and are usually statistically insignificant. The 'horse race' approach to the rankings produced by international studies – looking to see which position England is placed in and whether or not it has moved up or down the league tables – is not that meaningful, partly because the absolute differences in scores between countries are not that great and partly because the constituent group of comparators changes from study to study and year to year. Overall and over time, England's relative performance is not that worrisome. However, the fact that some countries are consistently ranked highly (Japan, Korea, and Singapore), and some occasionally dramatically improve (Finland), makes a closer study of such countries worthy of attention.

Judging the literature

Having identified the countries to focus on, we selected the research literature using several filters. Using electronic searches in the first instance we tracked down literature from a wide range of studies conducted within various disciplines, including economics, philosophy and psychology, in addition to education. This resulted in an initial database of over 1600 papers. Based on the abstracts of papers (or a quick review of the paper itself where an abstract was not available), each paper was allocated a rating in terms of Priority 1, 2 or 3, with 1 being those papers that looked highly relevant to our study, 2 needing a closer look and 3 not relevant. This initial judgement of relevance was based on the extent to which studies addressed: mathematical attainment, international comparisons, the mathematical system/education in one or more of our identified countries, and factors contributing to mathematical attainment. Priority 2 papers were reviewed and some re-ranked as priority 1, resulting in a core database of around 550 papers.

This core group of papers was recoded for a finer grouping according to the substantive aspect of education addressed: national/societal; schooling system; schools, curriculum; pupils; and teachers. We also coded these according to the type of research: original empirical research; policy research; literature review; and critique of research. The groups of papers that emerged from this coding formed the basis of the findings that are discussed in this report.

What factors appear to contribute most to high performance?

Assumptions underlying international comparisons

Looking to countries that are consistently successful in international comparative studies and expecting to learn directly from studies into these countries embodies certain assumptions.

1. International studies not only measure an appropriate model of mathematical attainment but that countries deemed to be high performing are indeed doing better at teaching mathematics.
2. Mathematics education practices within those countries are stable, coherent, and codifiable.
3. It is differences in these mathematics education practices that mostly account for higher attainment.
4. The practices of higher attaining countries if adopted by other countries would also lead to higher attainment.

Our analysis of the literature indicates fundamental difficulties with all four assumptions.

1. How valid are international studies?

It is not our intention to reiterate the arguments pointing out the difficulties and flaws in studies of international comparisons of mathematics education. For example, there may be considerable differences in the extent to which schools and students feel the tests are important. In PISA 2006 the comparison of first round school participation rates between Finland (100%) and the United Kingdom (76%) is telling. Perhaps more striking is the oft-quoted anecdote from TIMSS 1995 of Korean students marching into the examination hall behind the national flag. Others provide further cogent arguments into the shortcomings of TIMSS and PISA (see for example Brown, 1998; Goldstein, 2004).

Setting aside the problems in large-scale international studies, there are difficulties over what constitutes 'doing well' in teaching mathematics. Our review has revealed that those countries supposedly 'doing well' are themselves not satisfied with simply being ranked highly in international league tables. Korea is concerned that its high ranking is based on too much mechanical test preparation which prevents pupils from developing an 'investigative attitude and other desirable mental habits' (Lew, 2008, p. 45). Singapore feels the need to put in place 'a flexible education structure to cater to the different needs and abilities of the students' (Soh, 2008). Finland is concerned that having achieved high ranking, the changing demography of the country means that current approaches may not continue to be successful (Hargreaves, Halasz, and Pont, 2007). No nation appears to consider itself to be unequivocally successful in their mathematics education.

It is worth noting in the light of this that the 'gaze' across nations for insights into mathematics teaching is not restricted to those countries lower down the international league tables seeking evidence from the higher attaining countries. China is not alone in revising its national curriculum and enshrining many of the qualities that have been encouraged in the United Kingdom, including more emphasis on learner-centred lessons (as opposed to teacher centred) and the need to differentiate provision to meet different learner needs (Xie, 2004).

2. Can stable, coherent, and codifiable practices be identified?

As hinted at above, no education system stands still. The literature indicates that in all high-attaining countries the education system in general and mathematics education in particular is far from stable and is frequently reviewed and revised. Even if it were possible to identify practices that were in place at a particular time and linked to assessment results, if the country in question is revising these practices, what would be the point of adopting them elsewhere? Curriculum reform must be a process of creation not re-creation.

Early international studies examined 'characteristic pedagogical flow' (Schmidt, *et al.*, 1996) or cultural lesson scripts (Stigler and Hiebert, 1999) with the assumption that there are 'typical' national forms of mathematics lessons. Researchers are now questioning the extent to which there are national lesson stereotypes and whether variations within cultures or even within the same classroom might not be more revealing (Laborde, 2006). For example, re-analysis of the TIMSS video data indicates that the much discussed problem-solving approaches of Japanese classrooms may be distinct when teaching geometry but are similar to practices elsewhere when it comes to algebra (Neubrand, 2006).

When codifying practices something must always be left out and we must be cautious of the selective nature of evidence drawn upon. Taking again the example of the Japanese problem-solving approach, rather less discussion is made of: the degree of central control in Japan (over textbooks, curriculum, assessment); the selection procedures in place as pupils move up through school; and the extent of the shadow education system (after-school programmes). Is it possible to codify fully such a complex interplay of elements let alone select which are the most important features?

3. Can differences in mathematics education practices account for differences in attainment?

One of the most striking things the review has shown is that high attainment may be much more closely linked to cultural values than to specific mathematics teaching practices. This may be a bitter pill for those of us in mathematics education who like to think that how the subject is taught is the key to high attainment. But study after study shows that countries ranked highly on international studies – Finland, Flemish Belgium, Singapore, Korea – do not have particularly innovative teaching approaches. Lest this observation be picked up as ammunition for arguing for a 'back-to-basics' style of teaching, we hasten to add that such traditional approaches only appear to succeed because of cultural conditions that support them, particularly through parental expectations and relatively homogeneous populations. And there are many countries that use traditional practices that come low in international rankings. Given these conditions the conundrum is that if, as Hess and Azuma (1991) claim, such traditional approaches are 'not conducive to learning' then why do pupils attain highly? National culture helps explain this – being born into a culture that highly values success in mathematics establishes a 'virtuous cycle' of continuing success.

All too often schools and teachers are written about as if they somehow sit outside the overarching culture, attitudes and policies of their nation states. Obviously this is not the case, education systems are part of the broader cultural milieu. Culture, beliefs and dispositions have all come through strongly as powerful influences in learning mathematics and we explore these in some detail in this report.

4. Will adopting the practices of higher attaining countries lead to higher attainment?

Curriculum reform is littered with attempts to 'cherry pick' particular parts of successful systems to transfer and adopt elsewhere. Piecemeal adoptions are, however, not likely to succeed if features selected are not looked at in terms of their relationship with other aspects of the national culture and school system. For example, the fact that all teachers in Finland are educated to master's degree level should be considered in the context of the high status afforded Finnish teachers, the strong levels of competition to enter teacher education, the elaborate and time consuming screening process of applicants and the relatively 'flat' differentials in salary across the professions (Wilkinson and Pickett, 2009). Or the success of Chinese pupils in the light of the impact of the one child per family policy in raising parental expectations and child obligations, and Chinese teachers' understanding of these expectations and obligations, being themselves in the same position (Tsui, 2005). Success depends on a close coordination and integration of cultural and moral values; economic and policy decisions; the organisation of schools and school systems and leadership within these; the curriculum and how it is taught and assessed.

This complexity can lead to the argument that nothing of relevance can be learned: successful countries by virtue of their uniqueness set themselves apart and make their success difficult to replicate. However, we believe all is not lost and that we can learn from other countries.

Roots of or routes to high attainment?

There is an underlying assumption to many international comparative studies that the answer to successful mathematics teaching is 'out there'; that somewhere, someone has an education system that 'works'. If not in one country, then maybe the answer can be found by looking across several – put together school organisation from one place, with approaches to training teachers from another and pedagogic practices from a third, and a composite solution will emerge.

Our review of the research suggests that finding the 'roots' of success is an unattainable fantasy. The research on individual countries shows that no nation has got it all. There are opportunity costs to high attainment, for example, poor attitudes towards mathematics. And those countries that do perform well on international comparisons have concerns over whether that in itself is the primary goal of mathematics education.

An additional complexity is the fact that for every aspect of mathematics education linked to high performance in one particular country, a contradiction can often be found elsewhere. For example, commentators attribute the lack of a centralised assessment structure in Finland as an important factor in achieving high levels of teacher autonomy and pupil success. Conversely, Singapore also scores highly in international assessments yet has a highly centralised assessment system. Australia has a very mixed population and out-performs many of the more monocultural Pacific Rim countries, challenging the assertion that it is the monocultural natures of Finland and Singapore that account for their success.

One response to such contradictions is to question the evidence and argue that the 'real' underlying cause of differences has yet to be located. Or dismiss certain aspects altogether and look for the solutions elsewhere. But there is another way to view these contradictions and that is to view decisions in education as arising as much from values and moral positions as to rational, evidence-based choices. Instead of trying to reconcile contradictions or dissolve them, we believe the research shows that there are many routes to high attainment. You can have an egalitarian education and high standards (Finland), or you can have a selective one and still have high standards (Singapore). You can have a problem solving pedagogy (Japan) or you can have a teacher-centred one (Korea) and either can lead to high standards. The choice is about the sort of society and culture that education can help support or develop. The choices here go beyond mathematics education to education through the context of mathematics. If the focus remains only on raising standards then is mathematics education being morally responsible?

There is much we can learn from research into high-attaining countries, but it is indirect. It can help us hold a mirror up to our own practice, to raise questions to reflect back on ourselves.


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THEME 1: Impact of teaching

Teaching is a minor factor compared to the match between curriculum and test items in explaining international differences

TIMSS and PISA provide only a broad-brush picture of mathematics education. Findings from TIMSS suggest that teaching is no more than a minor factor compared to the match between curriculum and test items in explaining international differences, but teaching still has a significant effect on mathematical learning. Detailed comparative studies are needed that examine the impact of curriculum and pedagogy.

Over the past 20 years, much policy, research and media attention has been directed at the 'better' teaching methods of countries that perform well on international comparisons. There is little evidence that teaching methods are the major causal factor. For example, in a careful analysis of the SIMS data, Burstein and colleagues concluded that differences in teaching had no discernable effect and the key factors in different performance were the content of the curriculum and pupils prior learning:

> "All we can safely say (we hope) is that students do experience different types of instructional arrangements cross-nationally and the influence of these arrangements generically appears weak relative to such matters as prior learning and the contents of learning opportunities during the course of study. *(Burstein, 1992)*

TIMSS and PISA are different surveys and, whilst complementary, they each assess different aspects of mathematics. PISA, for example, has a specific focus on mathematical literacy and places emphasis on application and realistic contexts. This benefits countries that have a similar emphasis in their curriculum, such as Australia and the Netherlands.

There may also be considerable differences in the extent to which schools and students feel that the tests are important. In PISA 2006 the comparison of first round school participation rates between Finland (100%) and the United Kingdom (76%) is telling. Perhaps more striking is the oft-quoted anecdote from TIMSS 1995 of Korean students marching into the examination hall behind the national flag.

Better mathematics performance or better educational performance in general?

A legitimate and interesting question is the extent to which better mathematics performance on TIMSS and PISA is a result of factors specific to mathematics teaching and learning or simply a reflection of more general educational factors. The TIMSS results suggest a fairly strong association between mathematics and science performance. It seems possible, therefore, that there are some common national characteristics across teaching and learning in mathematics and science. Perhaps unsurprisingly, any association between mathematics and reading performance on either PIRLS or PISA is much less clear. In light of this, it may be informative to examine countries where there is a mismatch between performance on mathematics and science. Unfortunately, identifying such countries is not straightforward. For example, in TIMSS 2007, Hong Kong's performance in science is worse than that in mathematics, yet in PISA 2006 Hong Kong was the top performer in science.

The Pacific Rim Countries

There is a great deal of research about mathematics education in Pacific Rim countries that points to two particular factors impacting on attainment: national culture and curriculum content. The research highlighting cultural explanations for high mathematical attainment in these countries converges on, for example; the Confucian Heritage Culture, the stability of mathematics education over the past two millennia (in China), and a national emphasis on effort rather than ability. National cultural effects such as these are doubtless important but are not amenable to change through changes to educational policy (which is likely to reflect national culture anyway). There is also a danger in treating nations as culturally homogenous. We note that, although many claims have been made about China's achievement, there is no robust evidence that the attainment of all Chinese students is high. China has only taken part in one international survey, the IAEP2 in 1990/1, and, whilst Chinese performance was high, the sampling process was not robust (Lapointe, Mead, and Askew, 1992).

Looking at the content of the curricula, all the Pacific Rim countries place a much greater emphasis on algebraic manipulation than in England, although the educational benefits of this emphasis might be disputed. As a result, all outperform England on this aspect of algebra in TIMSS. Similarly, the greater emphasis placed on data handling in the English secondary National Curriculum is reflected in England's strength in data and chance.

There is much to be learnt from curricular comparisons. For example, in Hong Kong, topics in algebra are expressed more coherently and holistically (Sutherland, 2002). Hoyles, Foxman and Küchemann (2002) find less emphasis on real world geometry in Japan and Singapore, and more emphasis on congruence and similarity in Japan. They also found more emphasis on 3D geometry in all countries considered in comparison to England. But such comparisons are best seen as providing guidance.

Understanding Finland's high performance

Finland's performance in PISA is difficult to unpack, largely because there is very little published research on mathematics education in Finland (Pehkonen, Ahtee, and Lavonen, 2007). It is clear that Finland undertook substantial reforms to its education system during the last two decades. A great deal of attention is directed at the quality of teachers, at systemic equity whereby a common curriculum is taught to all students, and at the LUMA[1] curricular reforms of the 1990s. The LUMA reforms were directed at the PISA conception of mathematics and set a target of achieving a top-quartile position amongst OECD countries by 2002 (Kupari, Reinikainen, and Törnroos, 2007). We note, however, that Finland's performance on TIMSS 1999 in Grade 8, whilst above average, is considerably weaker than that of PISA 2000. Hence, it seems likely that the content of the Finnish curriculum is the key factor in Finland's performance in PISA.

Identifying comparable countries

Both TIMSS and PISA necessarily provide a very broad-brush picture. There is much more to be learnt from more detailed comparisons and there is a pressing need to identify international 'near-neighbours' in mathematics education with the potential for illuminating comparative work. We were surprised, for example, to find little comparative work between England and Scotland. Other potentially informative near-neighbours are the Netherlands or the high-performing TIMSS Benchmarking Participant of Massachusetts (for which, unlike Finland, there is a research base and a systemic state reform initiative comparable to those in England, e.g., Riordan and Noyce, 2001).

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1 LUMA originates in the Finnish Luonnonieteet (Natural Sciences) and Mathematiikka (Mathematics).

THEME 2: What rankings tell us

England's performance on international studies needs to be examined beyond looking at rankings

England's success in TIMSS 2007 is significant, but it may overstate the performance. At Grade 8 (Year 9), pupil results in algebra were below the international average and weak in comparison to competitor countries. At Grade 4 (Year 5), performance in number, although above the international average, was relatively weak in comparison to England's overall performance in other areas of mathematics. But these apparent areas of weakness are only important if what is tested in TIMSS is agreed to be important nationally.

England's performance in TIMSS 2007 has risen significantly from TIMSS 2003 at both Grade 4 (Year 5) and Grade 8 (Year 9) (Mullis *et al*., 2008; Sturman *et al*., 2008)[2]. The only countries to perform better than England at both Grade 4 ***and*** Grade 8 were the five Pacific Rim countries: Chinese Taipei, Korea, Singapore, Hong Kong and Japan[3]. However, this broad picture obscures important details, particularly in the attainment of the older age group.

England's general performance

The performance of 14-year-olds in England in mathematics, while 13 points higher than the international average, is still 57 points below that of Japan, the lowest performing of the Pacific Rim countries. Moreover, given that relatively few European countries took part in TIMSS 2007 at Grade 8, England's international ranking (7th) is difficult to judge in terms of our European neighbours, although other results shed some light on this.

For example, England's mathematics performance on the 2006 OECD PISA survey of 15-year-olds is not significantly different to the international average. In total, 18 countries perform significantly better than England, including five Pacific Rim countries (Chinese Taipei, Korea, Macao, Hong Kong and Japan) and seven European countries (Finland, the Netherlands, Belgium, Estonia, Denmark, the Czech Republic and Slovenia) (Bradshaw *et al*., 2007). The highest performing group of countries in PISA 2006 contains just three of the Pacific Rim countries (Chinese Taipei, Korea, and Hong Kong, but not Japan) together with Finland (Programme for International Student Assessment, 2007).

Performance on content

Returning to TIMSS 2007, at Grade 4 (Year 5), England's performance was significantly above the international average in all three content areas: number, geometric shapes and measures, and data display. Performance on number (and computation) was significantly lower than in the other two areas, although still above the international average. This is somewhat surprising given the focus of the National Numeracy Strategy (NNS) over the past decade. However, findings on achievement in number from the Leverhulme Numeracy Research Programme suggest that improvements due to the NNS have been relatively modest (Brown, Askew, Millett, and Rhodes, 2003).

At Grade 8 (Year 9), England's performance differed across the content areas with data and chance strong compared to number and geometry. Algebra was weak in comparison to the other areas with performance just below the international average. There is an emphasis with TIMSS items on traditional algebraic manipulation, which is likely to benefit those countries that emphasise this in their curricula (including the Pacific Rim, but not England). However, recent research comparing English students' understanding of algebra with that of 30 years ago suggests there has been little change in general at Year 9 (Hodgen, Küchemann, Brown, and Coe, 2009). This raises two concerns about participation in mathematics post-16. First, algebra is key to higher levels of study in mathematics and STEM disciplines. Second, algebra is frequently cited by students as a reason for dropping mathematics (Osborne *et al*., 1997).

Performance on knowing, applying and reasoning

There was no significant difference between England's scores for Year 5 pupils across the three cognitive domains of knowing, applying and reasoning. In contrast, England's score in the cognitive domain of applying in Year 9 was only slightly above average and lower than that for the two domains of reasoning and knowing. This lower performance on application may also be reflected in England's performance in PISA 2006 as indicated above. This is a disappointing finding given that in a previous international survey conducted by IAEP two decades ago, England's performance in problem-solving surpassed all others, including the Pacific Rim (Lapointe, Mead, and Philips, 1989). The TIMSS 2007 score may indicate a decline in the key area of application and problem-solving over this period.

General improvement?

At both grades, the increase in overall performance in mathematics was associated with improvements in the middle score range, but there was no significant change to the proportion of students at the highest level of performance, the advanced benchmark. Replication results from the Concepts in Secondary Mathematics and Science (CSMS) study show a similar pattern that students' understanding of some algebraic ideas has declined at the higher levels of attainment (Shayer and Ginsburg, 2009).

In common with both higher and similar attaining countries, the proportion of students with highly positive attitudes to mathematics has fallen at both grades. At Grade 4, 62% of students had highly positive attitudes to mathematics, and at Grade 8, the figure was 40%.

England was relatively unusual in showing no overall gender differences for mathematics performance at either Grade 4 or Grade 8, but, at both grades, significantly more boys showed a high level of confidence in learning mathematics.

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2 To allow for comparison across years, the TIMSS scales are referenced to the 1995 assessments. The scales are set for each grade with a mean of 500 and a standard deviation of 100.

3 At Grade 4: England (541) compared to Hong Kong (607), Singapore (599), Chinese Taipei (576), and Japan (568). At Grade 8: England (513) compared to Chinese Taipei (598), Korea (597), Singapore (593), Hong Kong (572) and Japan (570). In 2007 Korea took part only at Grade 8. England's performance was also below that of three Benchmarking Participants, Massachusetts and Minnesota at Grade 4 and Massachusetts, Minnesota and Quebec at Grade 8.

THEME 3: Attainment and enjoyment

High attainment and pleasure in learning mathematics are difficult goals to reconcile

Education systems, and mathematics education in particular, are determined by values that may not always be explicitly stated. These values relate to 'cultural identities' that may be more or less homogeneous in different contexts. Groups with a cultural identity of being good at mathematics do not necessarily also identify themselves as enjoying mathematics.

“ The new curriculum emphasises: practical mathematics such as problem solving, application and modelling of mathematics, reasoning. More students have to leave school absorbing mathematics and having confidence.

“ Content should be associated with social life and keeping attention on students' interests and practices. Students should be encouraged to take active participation in investigation, field work, communication and cooperation.

“ Find pleasure in mathematical activities; make good use of mathematics in daily life situations.

These quotations might have been taken from any recent English mathematics curriculum policy document. They actually come from presentations by policy makers representing Korea, China and Japan at a conference held in 2005 (Usiskin and Willmore, 2008). As Broadfoot (2000) points out, goals for education are broadly aligned internationally and aspire to high levels of attainment along with learning dispositions and positive attitudes. Our review of research in international mathematics education reveals that high-performing countries are as concerned over pupils' dislike of mathematics as we are in England. As literature from within high attaining nations becomes available (as opposed to findings reported by outsiders gazing towards other cultures), it reveals a dislike of mathematics. (Hirabayashi, 2006; Leung, Graf, and Lopez-Real, 2006).

“ [If in Japan] the aim of mathematics education is to make pupils hate mathematics, then in this point we may have very much succeeded. *(Hirabayashi, 2006)*

Why do East Asian pupils excel?

Leung (2006) raises the question of 'Why do East Asian pupils excel?' in relation to these pupils' high attainment in a context of negative attitudes to mathematics. It has become commonplace knowledge that the Pacific Rim countries place more emphasis on pupils' efforts than on innate ability. Hatano (1990), for example, typically comments that in Asian cultures 'people generally tend to assign much more importance to effort in regard to skills that every member of the society is expected to acquire, ability may be considered more important in the case of optional skills' (p. 112). Two constructs that appear to promote this emphasis on effort over ability are extrinsic motivation and cultural identities.

Extrinsic versus intrinsic motivation

Various authors argue that in certain cultures exams provide a powerful and accepted source of external motivation. In particular China, Japan and Korea have a long tradition of links between high social and economic status to success in examinations. Pupils in these countries may be less interested in the mathematics itself and more in the status afforded by exam success (Lew, 2008; Soh, 2008; Tsuneyoshi, 2004). In East Asian countries parents and teachers alike accept that working towards success on examinations is a major goal of education and that what is learned is less important than exam success. Policy makers in East Asian countries, however, express concerns over the dominance of this instrumental view of mathematics education (Usiskin and Willmore, 2008).

Cultural identities

The observation that pupils in Asian countries cannot escape mathematics as a 'national… intellectual pursuit' (Hatano, 1990, p. 112) links with studies showing how an individual's cultural identities can impact on test performance. A US study brought together three groups of female Asian high school students that each met with a female Asian researcher. The control group chatted generally before taking a mathematics test. In another group the researcher led the conversation to focus on the fact that they were all women, while the third group were steered to talk about being Asian. This last group performed better than the control group on the mathematics test, while the group tuned into their femaleness performed worse. The argument put forward is that attunement to particular cultural identities – being female or Asian in this case – carries with it particular stereotypes – being worse or better at mathematics than the average – which subsequently get played out (Shih, Pittinsky, and Ambady, 1999). Similar results have been shown for other cultural groups such as black American boys.

Studying hard versus pleasurable learning

Success within such exam-oriented systems requires effort, and any pleasure is a result of the success attained rather than derived through the processes of learning per se. Researchers within the East Asian tradition are split on the value of exam-oriented systems, some arguing for reducing the examination pressure to improve pupils enjoyment, others questioning whether this is a wise move.

> “ If the price to pay for enjoyment is low achievement, should we consider whether the price is too high?
> *(Leung, 2009)*

Implicit goals that arise from nations' different socio-cultural-historical backgrounds have a powerful influence on pupil attainment. These implicit goals make 'borrowing' policies and practices problematic because adopting practices from elsewhere might mean adopting implicit goals that do not fit with England's vision for society or individuals. For example, the Asian emphasis on effort, while a factor in high attainment, is also closely aligned with obedience and authority.

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THEME 4: Parental expectations

Parental expectations are more influential than direct involvement

Parents with accurate or realistic expectations of their children's mathematical attainment have more impact on their children's learning than parents who over-estimate their children's attainment. Parents' realistic expectations have more impact on pupil attainment than direct parental involvement, such as helping with homework.

Parental expectations

The research evidence points to parental expectations being closely related to pupils' achievement. For example, a comparison of US and South Korean data from TIMSS 1995 suggests that one important factor underlying higher attainment in South Korea was a close alignment between the educational objectives and aspirations of home and school (Paik 2001).

Parental expectations play out through how parents judge their particular child against personal and cultural 'benchmarks' of appropriate attainment. A comparative study of 1440 first and fifth grade pupils from Japan, Taiwan and the US found maternal attitudes in the Asian societies to be different from those in the US. American mothers were more interested in their children's general cognitive development than in their specific mathematical achievement. Chinese and Japanese mothers gave more realistic evaluations of their children's mathematical achievements (Stevenson *et al*. (1990). Other studies show that American parents tend to view their children in a favourable light, judging them as doing well against their perceived benchmark standards, whereas Asian parents tend to view their children either modestly or more accurately in relation to their benchmarks (Bell, 1993; Cai and Silver, 1995). The suggestion is that this leads Asian parents to put more pressure on their children to study and work hard in mathematics as they have yet to live up to parental expectations.

Many of these studies have been conducted with middle-class parents, raising the question of consistency of parental expectations across different socio-economic groups. One study comparing parental expectations across different socio-economic status (SES) in both China and the United States found similarities in expectations across SES in China. Questionnaire data (parents and pupils) from Wuhan, a large industrial city, is reported on 990 returns from eighth grade pupils across three secondary schools (one highly selective and two 'regular' schools). The data included information on family income and identified some families as below the official poverty line. Chinese parents in this low-income category did not express lower expectations for their child: 79.7% reported high expectations for their child (exceeding the 74.3% of high-income US parents reporting the same expectations). The Chinese parents, across all SES groups in the survey, reported that, after food, education was the second highest household expenditure with 55% reporting the hiring of tutors and 76% indicating that they would borrow money for their child to go to college (Tsui, 2005).

Tsui suggests that the traditional high status of mathematics in China has been reinforced through market reforms and the one child per family rule. Chinese parents look to their child for security in their old age, so families in urban China have become 'almost obsessed' with academic success and the pressure on children not to let their parents down is great.

Parental involvement

While accurate parental expectations have been shown to be positively associated with pupil attainment, the evidence of the impact of direct parental involvement, for example through teaching children at home or helping with homework, is mixed.

For example, research indicates that Chinese parents are more likely to provide direct instruction in mathematics to their children and help more with homework (Caplan, Choy, and Whitmore, 1992; Chen and Stevenson, 1989). Such direct involvement does appear to have an impact on children's mathematics in the early years (Huntsinger, Jose, Liaw, and Ching, 1997), but once children start school, direct involvement appears to have less impact on the performance than that of indirect involvement through providing tutors or sending children to after school programmes (Cai *et al.*, 1995, Wang, Wildman and Calhoun, 1996).

Some parental involvement comes about through cultural expectations and policy. Chinese parents are required to buy the textbooks prescribed for their children, from primary years onwards for example, and that may influence expectations. Teachers and parents together expect children to work on these with pressure to keep up (Shiqi, 2006). Colleagues in China have expressed surprise that in England textbooks are provided by the state and at the lack of expectation that pupils would do extra work from textbooks at home. While we might expect parents to provide revision books for GCSE and some do buy revision materials for end of KS2 tests, this is different from a culture where providing textbooks is a requirement.

In Finland direct parental involvement appears to be brought about at the instigation of teachers. Finnish schools report a high proportion of pupils with special educational needs, but the high figures appear to be the result of early identification of pupils who are not keeping up with their peers in particular aspects of the curriculum. Early contact is made with parents to discuss what provisions to make (and children themselves are brought into the conversation) (Sahlberg, 2007), thus raising parental awareness of expectations and their child's attainment from an early age.

National cultural factors are difficult to change, but parents and children from all socio-economic groups need to be engaged in conversations about expectations and to receive realistic and specific feedback and expectations about their learning.

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THEME 5: Starting school

Evidence is mixed on the impact of children's mathematics on entry into school

Many researchers argue that parental involvement or pre-school experiences in young children's mathematics directly impacts upon early mathematical attainment. Not all the evidence supports this assertion.

The view that early parental influence is important in mathematical attainment is supported by a study involving interviews with parents of 40 second-generation Euro-American and 40 Chinese-American children from well-educated two-parent families in urban Chicago. The parents of 40 Chinese children were also interviewed from a similar population in Taipei, Taiwan. The 40 children in each of these 3 groups comprised 10 each of preschool boys and girls and 10 each of kindergarten boys and girls (Huntsinger, Jose, Liaw, and Ching, 1997).

The children were given two assessments, a test of early mathematical ability, and an assessment of numeral formation. Parents individually completed a questionnaire, and then mothers and fathers were interviewed together. Neither the questionnaire nor the interview prioritised mathematics – the parents were informed that the research was about supporting their children generally. Huntsinger and colleagues found that the Taiwanese children performed significantly better than the Euro-American children (the Chinese-American children performed best of all). The researchers attribute these differences to two factors.

Firstly, more formal instruction by the Chinese parents, as indicated by their interview responses. In contrast the Euro-American parents spoke more in terms of informal learning and the importance of play.

Secondly, parental attitudes towards mathematics in general and the influence these had on their child-specific beliefs. The more positive the parental attitudes towards mathematics, the higher expectations they had of their children's capability and interest in mathematics. In turn, children highly rated by their parents in terms of mathematical interest and ability scored more highly on the mathematics test. Of course it could be that parents recognised their child's mathematical aptitude and so rated them more highly. Taiwanese parents on the whole expressed more positive attitudes towards mathematics than the Euro-American parents.

While the authors suggest this might mean rethinking what is considered to be appropriate teaching for young American children, they also point out that the Chinese predisposition to calmness in infancy may support children's tolerance of early formal instruction (Kagan, *et al.*, 1994).

Contrasting findings

In contrast, findings about Korean children do not indicate that they start school with any particular mathematical advantage: their performance on an assessment of early years' mathematical skills and understanding was poorer than that of an American sample. This is accounted for by the Korean attitude that it is 'indecent' to engage pre-school children in formal mathematical activities. However, although the American 4- and 5-year-olds out-performed the Korean children, by the age of 7 the situation was reversed and the Korean children performed at a higher level. Reasons suggested for the acceleration of the Korean children include early difficulties they experience in learning two counting systems (an informal one and a formal one) and the greater amount of time devoted to teaching mathematics once they start school (Song and Ginsburg, 1987).

Cross-country comparisons amongst six European countries (Belgium, Germany, Greece, the Netherlands, United Kingdom and Slovenia) of the numeracy development of 5–7 year olds found little difference in attainment across the countries, despite a wide variety in the country's early education policies (Van de Rijt, *et al.*, 2003). In another study, Scottish, English, Australian and New Zealand children's scores on a baseline assessment were examined against the amount of pre-school experience the children had. This experience ranged from zero to six terms of experience: only a 'minimal link was found for mathematics' between experience and attainment, although the data for England alone did show a 'strong and clear' positive relationship (Merrell and Tymms, 2007 p.123). The Effective Provision of Pre-school Education project reports a positive effect of pre-school on mathematics at end of KS2 (although the finding relates to pre-school not formal mathematics teaching in pre-school) (Sylva, Melhuish, Sammons, Siraj-Blatchford, and Taggart, 2008).

We need to ensure that England's early years curriculum maximises children's potential for learning mathematics once they enter school. Research is needed into whether or not our expectations for attainment in early years' mathematics are appropriate.

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THEME 6: The language of number

Asian languages support early adoption of mental strategies

The number names in Asian languages make the base ten structure more transparent. This, and other features of the language, may mean that East Asian children move more quickly in early calculating from counting methods to strategic mental approaches.

Greater transparency of the place value system

Asian number names make the place value structure of grouping in tens more transparent: ten-one, ten-two, ten-three and so on rather than eleven, twelve, thirteen … and two-ten-one, two-ten-two etc. for twenty-one, twenty-two… As well as this more transparent base-ten structure there are fewer number names to commit to memory – one to ten and then combinations of these. There is also less phonetic confusion: whereas English children often encounter difficulties over the similar sounds of 'thirteen' and 'thirty', ten-three and three-ten are clearly differentiated in Chinese, Japanese and Korean. The naming also more closely matches the Arabian numerals: saying 'ten-five' corresponds more to the grapheme '15' than does 'fifteen'. Representation in Asian characters is even more transparent with the numeral 35 written as the equivalent of three-ten(s)-five.

A study investigated whether these differences affected how children might model two-digit numbers. American, Chinese, Japanese and Korean children with an average age of just over six years eight months were shown a variety of two-digit numerals, for example 43 or 62, and asked to represent them using base-ten blocks (which none of the children had met in school). After producing a first representation the children were asked if they could use the blocks to show the same number differently. The vast majority of the US children (91%) initially represented the quantities by counting out that number of single cubes, whereas the Chinese, Japanese and Korean children were much more likely to represent the quantities using tens and ones blocks (81%, 72% and 83% respectively). A group of younger Korean kindergarten children were also included – more of them acted like the US children (59%), but even 34% of these children could make an accurate representation using tens and ones. On being asked to make a second different representation, half the American children could not make a different construction for any of the five numerals that they were presented with – they could only rearrange the single cubes into a different visual arrangement. The majority of the Asian children could create a distinctive second representation (76% Chinese, 79% Japanese, 98% Korean first graders) (Miura, Kim, Chang, and Okamoto, 1988).

Speed of number pronunciation

Digit names in Asian languages (including Chinese, Japanese and Korean) are shorter and more quickly pronounced than their English counterparts (less than a quarter of a second as compared to a third of a second). This means that more digits can be held in working memory (around 2 digits more and 3 in Cantonese (Dehaene, 1997). Citing research comparing American kindergartners with Chinese kindergartners, researchers noted that the former counted more often on their fingers, while the latter counted verbally to find totals (Geary, Bow-Thomas, Fan, and Siegler, 1993). They speculated this may lead to early adoption of strategies other than counting on fingers when calculating with small numbers.

Adopting strategic approaches

There is evidence that Asian children adopt addition strategies based on partitioning (decomposition) of digits, rather than counting in ones, sooner than American peers. The hypothesis is that this is the result of the Asian language number naming system. For example, adding 6 + 8, Korean children might decompose the 8 into 4 + 4 and calculate 6 + 4, then 10 + 4 (Fuson and Kwon, 1992). The suggestion is that the naming of 14 as 'ten-four' allows for the earlier and easier adoption of such strategies by Asian speaking children: since they know 'ten' is an aspect of naming the final total they partition the numbers in order to make ten. Geary's research comparing Chinese and American children's strategies supports this hypothesis: second and third grade children in each nation differed in the 'back-up' strategies used for additions they could not recall: Chinese children used this partitioning strategy earlier and more frequently, while US children reverted to finger counting. These researchers also found there was a greater increase in the Chinese pupils' test scores within the academic year than there was for the American pupils.

The result of language?

Whether these early differences are purely a result of language differences has to be considered in the light of differences in opportunity to learn. The Chinese children in each of the grades 1, 2 and 3 had received between 20 and 25% more mathematics lessons than the American children in the same grades, a difference confirmed by other studies (James W. Stigler, 1988). From a regression analysis controlling for this additional opportunity, Geary and colleagues conclude that the extra teaching time did contribute to differences in performance but did not totally explain them. Differences in the number of mathematics lessons that children experience appears to account for some but not all of the differences in cross-national mathematical performance.

Whether such early differences in achievement have a lasting impact is open to question. Given that (most) English pupils do replace finger counting with strategic methods and recall of number facts, albeit later than Asian pupils do, such early advantages may be thought of as 'washing out'. On the other hand the early use of verbal counting and the partitioning strategy for adding single digits may predispose Asian children to construct mathematical 'mental objects' and to come more quickly to regard mathematics as a mental rather than a physical activity. Other research has shown that relying on counting methods can restrict children's progress (Gray and Pitta, 1996, Askew, Bibby, and Brown, 1997).

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THEME 7: Attainment gaps

High attainment does not necessarily close the attainment gap between pupils from differing socio-economic backgrounds

Teachers' mathematics qualifications do not appear to be strongly linked to average levels of pupil attainment. However, differences in teachers' qualifications are associated with the attainment gap between pupils from high and low socio-economic status backgrounds: in high-attaining countries with a wide attainment gap, pupils from lower socio-economic status (SES) backgrounds are less likely to be taught by well qualified, experienced teachers.

International data shows that countries with high average attainment in mathematics have not all succeeded in narrowing the attainment gap between pupils from different SES backgrounds. An analysis of the TIMSS 2003 mathematics attainment data produced a measure of the gap between pupils from high and low SES backgrounds. Ranking the 41 countries according to this measure of the gap between high and low SES pupil scores, Korea and Taiwan are in the top ten countries with the widest gaps. Korea, with a mean average gap of 132 points, is third after South Africa (140 point gap) and Chile (133 points difference), countries which are ranked bottom and eighth from bottom respectively in terms of mean average pupil scores (Akiba, LeTendre, and Scribner, 2007). Ranked in terms of the average pupil score, Singapore, South Korea, Hong Kong, Taiwan, and Japan are the top five countries: Korea and Taiwan's high average scores mask the attainment gap in these countries.

“ We can see from the figures that both high-achieving countries such as Korea and Taiwan and low-achieving countries such as Chile and South Africa produce large achievement gaps between high-SES and low-SES students …. The data show that high-achieving countries do not necessarily produce a smaller achievement gap between high and low SES students.

(Akiba et al., 2007)

Teacher characteristics and attainment

To explore possible reasons for this finding, the researchers used the TIMSS questionnaire data to draw up five measures of teacher qualities: certification, whether they had studied either mathematics or mathematics education as a 'major' in their training (treated as two separate variables), number of years teaching experience and a composite measure of 'teacher quality'. No data were available for England on certification, so England is neither included on that strand of analysis on certification nor on the composite strand.

Each of these five teacher measures was considered in relation to mean pupil scores and achievement gap scores. Whether or not teachers had studied mathematics (as opposed to mathematics education) produced the most revealing results. In terms of average pupil attainment the researchers found a positive relationship between the teacher having studied mathematics and higher levels of pupil attainment. The teacher 'having studied mathematics' taken together with being 'fully certified' and 'years of teaching experience' resulted in a more positive correlation with pupil attainment. This result may be the consequence of more experienced teachers being assigned to teach higher attaining pupils rather than more experienced teachers being better at raising standards.

Teachers who had studied mathematics were significantly more likely to be teaching groups with a narrower attainment gap between pupils from high or low SES backgrounds. None of the other measures produced a similarly significant effect on the narrowing of the attainment gap.

Opportunity gap

The researchers report an 'opportunity gap' between the percentage of pupils from high SES and low SES backgrounds who were taught by a teacher who reported themselves as having a mathematics major. Chile is reported as having the largest opportunity gap with 24.2 % difference and Taiwan is fifth at 15. 4 %. England was ranked fourth at 17.5% (England does not come in the top ten countries with the largest attainment gap and no figure is given for the size of this gap.)

An anomaly here is South Korea. The reported opportunity gap between pupils of high and low SES being taught by a teacher with a mathematics major is comparatively small at 6.5% (although this still results in Korea being ranked 12 out of 41), and yet there is a wide attainment gap in Korea. Teachers in Korea do not choose which schools to apply to work in. They are hired by the city or province and then assigned to schools. After five years, they are re-assigned to a different school. This regular movement of teachers between schools is designed to even out the quality of teachers across schools (Kang and Hong, 2008). The high uniformity of South Korean schools apparently makes such deployment policies acceptable to teachers, together with incentives for teaching in difficult areas (for example remote or particularly low SES) that include smaller classes, increased salary, better promotion prospects and the chance to choose the next school (which presumably accounts for the figure of 6.5% difference opportunity gap). If, as Akiba and colleagues claim, the attainment gap between high and low SES pupils is partly accounted for by teachers' mathematics qualification, then why does South Korea's policy for equity in access to well-qualified teachers not reduce the country's attainment gap?

Akiba and colleagues argue that the continued attainment gap between high and low SES groups is attributable to shadow education opportunities, citing three contributory factors:

- High-income South Korean families spent almost three times more per child on private instruction than low-income families.
- A positive correlation between parental level of education and access to private tuition.
- Lack of access to shadow education in rural and remote areas.

Other research suggests that shadow education may not have such an impact on pupil attainment. This raises the question of how the Korean policy for teacher deployment actually works in practice: is the deployment of teachers actually as equitable as the policy suggests?

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THEME 8: Shadow education

Shadow education is an important adjunct to state education

Mathematics education outside school – shadow education – is widespread, and, particularly in East Asian countries, parents of all socio-economic status are willing to make sacrifices to pay for extra-curricular mathematics instruction. This extra provision can contribute to maintaining high standards, particularly where the shadow education complements formal education in school. The time spent on shadow education can, however, have adverse effects on pupils' wider social development.

Shadow education refers to a wide variety of out-of-school activities including one-to-one tutoring, exam preparation courses, and private but school-like classes (for example Japanese juku). Systems of tutoring are noted to be widespread in Hong Kong, Singapore, Taiwan, Korea, Greece, and Turkey (Baker, Motoko, LeTendre, and Wiseman, 2001).

In a critical synthesis of studies relating to mathematics education outside school, Schümer (1999) argues for the need to look beyond schools and classrooms to understand Japanese achievement in mathematics. Japanese secondary school pupils invest more time in mathematics outside school and receive more intensive guidance on their learning than pupils in either the United States or Germany (Hiebert, *et al*., 2003). Whilst Japanese secondary pupils get less directed homework from teachers in school, they are more likely to have additional tuition, with two-thirds of eighth graders receiving supplementary mathematics lessons in private tutoring or schooling. Moreover, supplementary tuition in Japan *complements* the more conceptual teaching in school by providing more opportunities for practice and repetition. There are claims that shadow education has reduced the work of Japanese teachers through standardising many pupils' achievement levels in classes (LeTendre, 1999).

Enrichment or remedial?

As Baker and colleagues note, much of the rhetoric around shadow education is couched in terms of 'enrichment'. High stakes testing regimes, however, place a premium on end-of-school exam results and much shadow education is actually remedial in nature (that is, lower attainment pupils engage in shadow education in order to catch up). The analysis by Baker and colleagues of evidence from 1994–95 TIMSS data supports this claim. Baker found only three countries that had a predominantly enrichment focus to shadow education (Korea, Romania and Thailand), although Japan was unusual in having 'rich mixes of remedial and enrichment' shadow education (p. 14). The overall trend is for use of shadow education to be higher in countries where national expenditure on education is lower and without full pupil enrolment (at seventh and eighth grade). However, the analysis did not find evidence to support the hypothesis that nations which are highly motivated to perform well in international tests displayed greater use of shadow education (p. 11). Nor did their findings support the popularly held view that shadow education may account for success in international assessments.

Other studies, however, do suggest that shadow education may be significant in raising standards. A 2001 follow-up study of a 1989 survey of Japanese primary and middle school pupils examined test results and shadow education attendance (juku), SES, and other factors. The results showed that the test scores were generally lower for the 2001 group. The children who did not go to juku experienced the largest decrease in scores (Tsuneyoshi, 2004).

Shadow education and socio-economic status

Kariya and colleagues claim that it was the children of the less-educated parents, who did not have the extra push at home, whose hours of shadow education had decreased the most.

This finding echoes a popular view of shadow education as a class issue: parents with high SES are more likely to want (and be able) to pay for shadow education than parents with low SES. The evidence from the East Asian countries is that SES is not a major factor, with high numbers of parents across the SES spectrum making shadow education provision for their children. For example, high rates of additional tuition are reported for South Korea. Data from 2003 records around 75% of South Korean Grade 7 to 9 pupils getting tutoring or being enrolled in 'cram' schools (Lee, 2005). These figures do not include programmes offered by the state schools themselves, which are reported as making shadow provision for over 22% of pupils in 2008 (Kang and Hong, 2008). Other commentators report figures that 91% of primary, 81.5% of lower secondary and 70% of upper secondary Korean pupils are involved in out-of-school programmes, concluding that 70% to 90% of pupils study mathematics in out-of-school programmes almost every day (Lew, 2008). As this author notes, the most important thing in understanding the Korean education system is how 'Korean parents are willing to submit themselves to their children's education.'

It is important not to assume that the prevalence of shadow education is simply a consequence of parental demand. It is a huge private enterprise with resources to convince parents that it serves a need. One commentator notes that the industry of shadow education in Japan may be inhibiting educational reform through advertising and distribution of marketing materials that have 'sensationalized the 'failure' of public school education' (Tsuneyoshi, 2004, p. 370).

Shadow education in England

The importance of shadow education in other countries is relevant to England in the light of recent announcements of the introduction of 1:1 tutoring for under-achieving pupils. The notion of shadow education and its acceptability as the norm for many countries suggests that these countries are happy to accept that state education cannot meet the needs of all pupils. Cultural values in England would deem the need to tutor a pupil who was falling behind as an indictment on the quality of a mainstream educational setting. There is a belief in England that the state education package should provide all that is needed during the school day; a belief that may not be shared by high-attaining countries.

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THEME 9: Problem solving

High attainment in international comparisons does not imply high attainment in problem solving

TIMSS results may be largely attributable to proficiency in computation and solving relatively predictable and routine problems. When solving non-routine problems, differences between nations appear to be less pronounced. Examination of pupils' problem-solving strategies reveals that pupils using abstract and symbolic approaches to problem solving generally perform more successfully: use of such approaches may be a result of pedagogical emphases rather than being a developmental issue.

Problem solving: learning outcomes

In a series of studies spanning some 15 years, Jinfa Cai has explored differences between Chinese and American pupils' problem solving strategies (as opposed to simply looking at differences in correct answers). For example, a major study compared US and Chinese pupils on three types of items:

- computation
- routine problems
- non-routine problems

While the Chinese pupils scored more highly than the US pupils on the computation and routine problems, the two populations were much more closely matched on the non-routine problems, with the US pupils marginally out-performing the Chinese pupils (Cai and Hwang, 2002).

In another study Cai explored performance of Chinese fifth and sixth graders on long division calculations and division set in a problem context that required the answer to be rounded up (people travelling on buses). Over 80% of the Chinese pupils could correctly calculate 11.28 ÷ 3.6 and over 90% correctly answer 3480 ÷ 60. However, only around 25% of these same pupils gave an acceptable answer to the division with remainder problem, which Cai points out is about the same level of success that US pupils attained on a very similar task reported in NAEP (Cai, 1995; National Assessment of Educational Progress, 1985).

Problem solving strategies

One major finding from this research is the evidence of more use of formal and abstract strategies by Chinese pupils than by American counterparts. For example, when solving non-routine problems, sixth grade American pupils were more likely to use drawing or diagrammatic representations to find solutions than Chinese peers who preferred numerical or symbolic representations. Interview data indicated that American teachers did not expect Grade 6 pupils to use algebraic methods whereas Chinese teachers of Grade 6 did.

The Chinese pupils' preference for the symbolic may be a result of differences in curriculum ordering: Chinese pupils tend to be introduced to formal recording, and algebra, earlier than American pupils. To examine this hypothesis Cai carried out a study in the United States and China where the year groups were matched on whether or not they had been taught formal algebra rather than matched for age. The results showed that US pupils who had been taught algebra (eighth grade) were more likely to use symbolic representations than US pupils in Grades 6 or 7, but such use was still much less than that demonstrated by the Chinese pupils who were more likely to use symbolic representations at all grades, irrespective of whether or not they had been taught formal algebra. These differences positively correlated with levels of performance: in both China and the United

States pupils who used symbolic (arithmetical or algebraic) representations scored more highly than those using pictures or words. Analysis across both populations of pupil methods based on concrete strategies showed similar success rates; pupils using abstract representations were more successful and more Chinese pupils worked in this way (Cai, 2002).

Concrete or abstract?

It might be expected that the Chinese preference for abstract representations is due to less exposure to concrete materials and pictorial representations. However, Stigler and Perry report that in classes observed in China and Japan, teachers did make extensive use of manipulatives and real-world materials or scenarios, more so in fact than the American teachers in the same study (Stigler and Perry, 1988). Another study describes videos of two lessons, one Japanese, one American. Both lessons introduced the formula for finding the area of triangles to upper primary school pupils, but differed in the approach to establishing this. The Japanese lesson was structured around children manipulating images to find informal methods for calculating the areas of triangles and with the teacher subsequently drawing these together into the accepted formula at the end of the lesson. The American teacher introduced the formula early in the lesson: the pupils then applied this to several problems (Zhou, Peverly, Boehm, and Chongde, 2000).

There are, however, differences reported in the way that concrete materials are used across nations. Observers report that Japanese lessons often focus on one or two problems, with a variety of ways of modelling and solving them being explored in a single lesson, with concrete representations being explored alongside and in parallel to more abstract representations. This parallel use of concrete representations is different from an approach that starts with the concrete, with the expectation of this providing a foundation for the abstract, and follows on with the concrete gradually being withdrawn. The Effective Teachers of Numeracy study at Kings' noted that the most effective teachers in that study worked with a connected range of representations in parallel (Askew, Brown, Rhodes, Wiliam, and Johnson, 1997).

Research is needed into whether encouraging more abstract representations will raise attainment or whether it is the case that higher attaining pupils are more likely to use abstract representations.

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THEME 10: Textbooks

English textbooks are more routine and involve less variation than those of many other countries

In England, procedural fluency and conceptual understanding are largely seen as mutually exclusive aims. This polarising of procedural and conceptual is not helpful. Pacific Rim teaching is largely dominated by procedures and hence supportive of procedural fluency, but the procedures used tend to be explicitly grounded in mathematical principles and consequently more mathematically coherent and meaningful than those most commonly used in the United Kingdom. In the Pacific Rim, mathematically informed procedural teaching is introduced and promoted through carefully constructed textbooks.

Textbooks are the major resource for teaching mathematics. In TIMSS 2007, 65% of Grade 4 teachers and 60% of Grade 8 teachers internationally use textbooks as the main basis for mathematics lessons. However, research on textbooks and textbook use is limited and, aside from TIMSS, there is no large-scale study of textbook use in England or internationally.

Textbook use in England

Textbook use in England is lower than that in the highest attaining countries. At Grade 4, textbook use in all the Pacific Rim countries is higher than the international average and the proportions of teachers using textbooks as the primary basis for lessons is as follows: Chinese Taipei (94%), Hong Kong (84%), Japan (83%) and Singapore (75%). In contrast, textbook use in English primary classrooms has fallen to only 15% with Mullis *et al.* (2008) commenting that England 'appear[s] to be working towards only supplemental use or no use at all for almost all students' (p. 289). Evidence from the Leverhulme Numeracy Research Programme supports this finding and suggests that, in primary classrooms, there is widespread use of worksheets with at times dubious quality control.

At Grade 8, the comparison is less stark, although textbook use in England is still comparatively low. Teachers in all the Pacific Rim countries use textbooks more than England and for all except Singapore the use of textbooks as the primary basis for lessons is greater than the international average. In contrast, textbook use in Scotland is relatively high. Anecdotal evidence suggests that, whilst secondary teachers often use alternative resources at the beginning of lessons, textbooks provide the principal source of the problems set for pupils in class.

Given the paucity of comparative research in this area, the reasons for these differences in textbook use are not clear, although the lower English figure may reflect a greater use of internet resources, initiatives by the National Strategies promoting the use of alternative resources alongside published textbooks, and a view amongst the educational establishment that schools over-rely on textbooks rather than undertaking their own detailed planning.

Content of textbooks

Over the past 20 years, the educational system in England has been subject to frequent reform and review. One consequence of this has been to limit the time available for the development and trialling of textbooks, with publishers competing to produce textbooks quickly. This has led to a reduction in the quality of textbooks and, in their study comparing textbooks in three European systems, Haggarty and Pepin (2001) find English textbooks to be less coherent and more routine than French and German textbooks:

> English textbooks appeared simple, both in terms of complexity and coherence. Questions were mostly straightforward applications of the worked examples provided. They were the routine-type where a 'taught' method was applied in relatively impoverished and non-real contexts and they only rarely required deeper levels of thinking from pupils. Pupil access to textbooks was limited, with many having no access beyond exercises set by teachers in class. Consequently, it is speculated that pupils could not have a sense of 'ownership' of the textbooks, nor of the textbook as a source and support of their independent learning. This, in turn, resulted in a complete reliance on the teacher as the only source of mathematics.
>
> *(Haggarty and Pepin, 2001)*

In contrast, Gu, Huang and Marton (2004) demonstrate how Chinese textbooks use systematic variation in the choice and structure of problems. By considering both non-routine examples and 'non-concepts', teachers extend students' understanding of the concepts. For example, they show how the concept of 'opposite angles' is illuminated by the consideration of near opposite angles (see Figure 1). The discussion is better developed in geometry and some research is needed to extend these ideas to number, algebra and data handling.

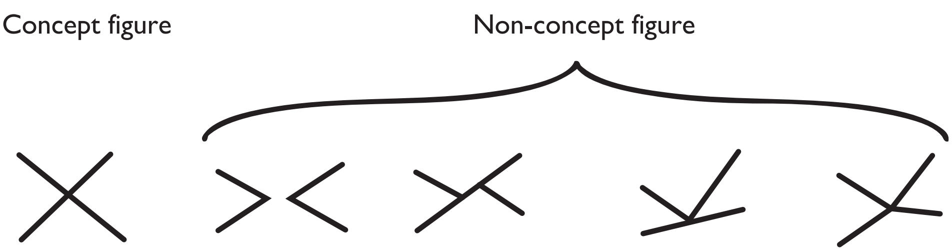

Figure 1: Concept and non-concept examples relating to opposite angles (Gu *et al.*, 2004)

Teachers' use of textbooks

In an examination of Hong Kong and Korean teaching, Leung and Park (2002) found that teachers use teacher's manuals in planning their teaching. They also found Hong Kong and Korean teaching to be largely procedural, but a striking contrast is that these procedures are more coherent, more developed and more mathematically based than those commonly used in the United Kingdom. For example, whilst the common procedure for the division of fractions in England is 'turn the divisor over and multiply', the teachers in Leung and Park's study refer to the use of the associative and commutative laws in the explanation of the algorithm given in the teacher's manual.

In contrast to the situation in England, in Japan, Korea and Singapore, textbooks are subject to a process of official textbook approval (International Review of Curriculum and Assessment Frameworks Internet Archive (INCA), 2009). It may be that this process of approval has the effect of improving the quality of textbooks in these countries, although there is no strong evidence of a causal link. Textbook development is also aligned with curriculum changes: policy makers, curriculum developers and textbook writers work in parallel so that curriculum reforms are coherent and supported by manuals and textbooks.

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THEME 11: Teacher subject knowledge

Teaching emphases within lessons may depend on cultural factors as much as teacher subject knowledge

Teachers' mathematical subject knowledge is widely regarded as an important basis for effective teaching of mathematics. However the relationship between teacher subject knowledge and pedagogy is not simple, and pedagogy depends upon tacit values and expectations as well as knowledge. What a teacher emphasises within a lesson may depend on cultural factors such as beliefs about learners as much as on the teacher's subject knowledge.

Liping Ma investigated US teachers' conceptual understanding in many areas of elementary mathematics and found it to be noticeably weaker than that of Chinese teachers. Ma's research has been widely cited internationally as demonstrating the need for teachers to have a "profound understanding of fundamental mathematics". Ma's study involved a small group of teachers, and researchers are interested in whether or not her findings apply more generally. One such study exploring this examined 160 American and Chinese teachers' knowledge of teaching fractions. To assess subject matter knowledge the teachers had to complete a number of problems about fractions. The Chinese teachers significantly outperformed the Americans, on all three strands of concepts, calculations and word problems. In interview, the American and Chinese teachers displayed similar levels of pedagogic content knowledge about the difficulties that children might have, but the Chinese teachers were reported as more knowledgeable about which were the most important fraction concepts to teach, the relevant knowledge that pupils might already have and teaching approaches that might lead to understanding (Zhou, Peverly, and Xin, 2006, p. 441). The authors argue that the Chinese teachers demonstrated greater 'longitudinal coherence' (Ma, 1999) in knowing when different aspects of knowledge of fractions were introduced into the curriculum and how these related to what had been previously taught and what was still to come in later years.

Subject knowledge and teacher practices

Leung questions the relationship between Ma's construct of 'Profound Understanding of Fundamental Mathematics' (PUFM) and actual teaching practices through findings from a small-scale replication study using some of Ma's items. His Chinese teachers, when probed, demonstrated elements of PUFM but expressed beliefs in the importance of teaching procedures to pupils. These beliefs were grounded in their pragmatic knowledge of what pupils could be expected to understand: they saw understanding as following on from procedural competence and stressed the importance of memorisation. Leung suggests that the role of memory in East Asian approaches to teaching should not be equated with the Western idea of 'rote' understanding (memorisation without understanding) (Leung, 2001; Leung and Park, 2002). Emphasising the idea of repetition rather than rote, researchers argue that the emphasis is on the 'intention to both memorise and to understand' (Dahlin and Watkins, 2000).

Procedural or conceptual?

On a larger scale, researchers took responses to 20 items from the (third) TIMSS eighth grade (13-year-olds) teacher questionnaire they considered elicited views on: the nature of mathematics; mathematical pedagogy; and learning mathematics. The items were grouped into two categories according to whether or not they indicated a perception of mathematics as procedural and algorithmic, or as conceptual and forming a connected whole (Philippou and Christou, 1999). The analysis of teacher responses on these items broadly indicated that teachers from the four Eastern Asian countries surveyed (Japan, South Korea, Hong Kong and Singapore) had responded in ways consistent with an algorithmic (procedural) view of mathematics and its teaching. Teachers from the four European countries (French Belgium, Sweden, Germany, and England) gave responses classified as towards the conceptual end of the classification.

Further differences in teaching emphases beyond conceptual or procedural are revealed in a qualitative study in which four teachers from Japan and four from the United States viewed a video of a lesson either from their home country or from the 'foreign' country. The teachers were invited to stop the tape at any point to comment, and transcripts of their comments were subsequently coded and classified.

> "The Japanese teachers in our study stressed that the following components are necessary for a well-taught math lesson: students' understanding of the material, not too much talking by the teacher, a slow enough pace to give students at all levels time to understand, and proper use of materials and the blackboard-depicting a variety of solution strategies and neatness. … The American teachers in the study had a different take on what qualifies as an effective math lesson. They wanted to see: behavioral indications of student engagement, clear language and explanations by the teacher, not too little talk by the teacher, a fast enough pace to ensure that the bright students won't become bored, and the use of materials and the blackboard-especially by the teacher and knowledgeable students.
>
> *(Zhou, et al., 2006)*

The Japanese teachers' comments were coded as being much more critical of both lessons than those of the American teachers. For example, the US teachers commented on whether pupils were on task while the Japanese teachers focused on whether pupils appeared to understand and, if not, commented that the teacher should accept this as the result of inadequate instructional approaches. Both groups commented on the quality of teacher talk but the US teachers focused more on when they thought teachers gave good explanations, while the Japanese teachers picked out utterances of which they were critical. Only around half the US teachers' comments about pace were about it being too quick, whereas almost all of the comments on pace from the Japanese teachers indicated that they thought it was too quick, with concern noted for not all the learners being engaged, particularly the 'slow' learners. The American teachers expressed more concerns for the 'quicker' pupils getting bored.

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THEME 12: Pedagogy

Within-country differences in pedagogy need to be examined

Within-country differences in pedagogy are more than simple distinctions between national teaching 'scripts' or lesson content. Looking at tasks set does not reveal how teachers develop these in practice. What may look like a procedural task can give rise to cognitive activity that addresses pupils' understanding and, equally, a potentially 'open' task can be made procedural by the teachers' directions and interventions.

Lesson scripts

The first TIMSS video study (1995) concluded there were differences in the 'lesson scripts' of different countries (United States, Germany and Japan), that were greater than within country differences. This conclusion led to the view that teaching was a 'cultural activity' (Stigler and Hiebert, 1999) and although teachers within a country would clearly vary in their behaviours, there were broad patterns of pedagogic styles and approaches within a country. Major differences between countries appeared to be in the use of problems in lessons and whether or not the emphasis was on conceptual understanding or procedural fluency.

The later TIMSS 1999 Video Study involved seven countries: Australia, Czech Republic, Hong Kong, Japan, the Netherlands, Switzerland and the United States. This study looked further into the problems pupils worked on in the video lessons and classified tasks and problems as one of three types: using procedures (typically arithmetical calculations) or routine problems; stating concepts (use of conventions or concepts, for example plotting points or constructing an isosceles right triangle) and making connections (constructing relationships and having to reason and generalise) (adapted from Hiebert *et al*., 2003). On the basis of this classification, Hiebert and colleagues found a predominance in five of the countries for 'using procedures problems', the exceptions being Japan and then the Netherlands.

Secondary analyses of the video data from these studies shows that differences both between and within countries are more nuanced than these classifications suggest.

Within country variation

A re-analysis of lessons from the 1995 video study looked more closely at the cognitive activity that might have arisen from the different tasks set in the lessons and concluded that we need to look beyond the surface differences of the form and structure of the lessons. In particular, that cross-national differences are 'deeply rooted in different mathematical demands on the posed problems' (Neubrand, 2006, p. 303). But the analysis raises the question of whether or not 'mathematical demands' are similar across different content. More of the Japanese lessons were focused on geometry than algebra than was the case in Germany or the United States. Japanese geometry lessons were more focused on conceptual problems, but in algebra they were more procedural.

Neubrand's analysis of the TIMMS 1999 video study data supported differences of emphases between countries but further argues that these are largely a result of the content of problems: geometry or statistical problems were much more likely to be conceptual 'making connections' problems, while algebraic problems tended to be procedural. The majority of Japanese problems in this study were geometrical (84%), as was also the case, to a lesser extent, in the Netherlands.

Looking beyond the types of problems posed to examine how pupils were actually expected to solve them shows that looking at a problem and judging the type of mathematical demand it might provoke cannot be done in isolation from how the problem played out in practice. Neubrand's analysis indicates that problems coded as 'making connections' problems tended to retain this focus in the course of the teaching. However, in course of the lessons based around problems initially classified as 'using procedures', between 20 and 55% of these problems developed, through teaching emphases, into 'making connections' or 'stating concepts' problems. The author suggests that using 'making connections' could be a central distinctive issue of teaching in high achieving countries' (p. 315), and that simply looking at the nature of problems posed cannot reveal this. How the teacher develops the problem is the key to the type of mathematical demand pupils ultimately become engaged in.

Another study re-analysing TIMMS 1999 video data compared the contexts and approaches to problem solving in Dutch and Japanese classrooms. Although the Dutch lessons were originally coded as displaying more real-life connections, in examining the way teachers developed the problems within the lesson, the author concludes the Japanese lessons were actually closer to the spirit of 'Realistic Mathematics Education' in that the pupils were involved in 'mathematising' situations. In the Dutch lessons, despite the contexts of the problems, problem solving was much more routine and procedural (Mosvold, 2008).

More mathematical teacher talk?

Drawing on the work of the Mathematics Quality Analysis Group within the TIMSS Video Study, Leung (2005) finds that, although Hong Kong classrooms were dominated by teacher talk, the quality of learning and instruction was nevertheless high. Video data were studied from all countries participating in the TIMSS Video Study except Japan. In Hong Kong, students were found to have experienced more opportunities to learn new content (around 75% of lesson time) and more advanced content than all others (except the Czech Republic). In Hong Kong, 90% of lessons were found to be thematically coherent compared to between 30% and 65% for the other countries. In addition 75% of the lessons from Hong Kong were found to be either fully or substantially developed, a figure more than three times higher than in the Netherlands.

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THEME 13: Confidence and attainment

There is a surprising association of low confidence in mathematics with high attainment

Pupils' self-perceptions of whether or not they are good at mathematics, and whether or not they like mathematics, may impact on their attainment. To raise attainment, however, it may be that changing the curriculum and teaching would be more effective than trying to change pupils' attitudes directly.

Given the widely held goal of wanting pupils to find mathematics enjoyable, on the assumption that this will raise standards, there is a paradox in the research findings that suggest these are inversely related.

Within and between country comparisons

A study looking for relationships between pupils' self-perception and their achievement in mathematics examined pupil responses to three questions from each of TIMSS 1995, 1999 and 2003: I enjoy learning maths; I usually do well in maths; maths is an easy subject. Correlation analyses were performed using data at the individual pupil level for within-country analysis and aggregated data for between-countries analysis. The within-country findings support the conventional wisdom that pupils tend to achieve more highly if their self-perception is good: pupils' achievement was higher for those who rated themselves highly on all three measures of enjoyment, doing well and finding maths easy (Shen and Tam, 2008).

Between-countries findings, however, revealed negative correlations between achievement and all three measures of self-perception that were consistent across all three waves of TIMSS. The authors note that pupils from the high-achieving East Asian countries tend to downgrade themselves. However, removing East Asian countries from the analysis did not change these negative correlations.

In line with Shen and Tam, Leung (2002) reports that pupils in the top-scoring East Asian countries (Hong Kong, Singapore, Japan and South Korea) believe they do not achieve well in mathematics. In addition, excepting Singapore, they do not enjoy mathematics relative to other countries. Leung suggests that low perceptions of confidence in mathematics ability may be necessary for high attainment.

> "Over-confidence may lower students' incentive to learn further and cause them to put very little effort into their studying, and hence result in low achievement. This is exactly the kind of justification for the stress on humility or modesty in the East Asian culture. The Chinese saying 'contentedness leads to loss, humility leads to gain' illustrates the point well.
>
> *(Leung, 2002)*

Findings comparing mathematics with other subjects present a slightly different picture. A survey of eighth grade students in Wuhan found that Chinese pupils rated mathematics their joint favourite subject alongside English (Tsui, 2005). Hong Kong pupils compare mathematics favourably to Chinese and Chinese History: they report finding mathematics mentally engaging whereas they associated the latter subjects with memorising facts (Jablonka, 2005). Pupils' perceptions of mathematics do not 'stand alone' but are related to their experience of other subjects: perceptions are not absolute but 'benchmarked'.

Living down to low expectations?

Shen and Tam's hypothesis for their paradoxical findings is that low standards and expectations beget low performance. In countries with low expectations, pupils perceive mathematics to be easy, contributing to an inflated self-perception of their attainment in mathematics, resulting in complacency and a lack of motivation to improve their performance.

A study comparing attitudes, expectations and achievement in mathematics of 9 to 10 year-old children in economically deprived areas of Russia, England and the United States supports the argument that low expectations go hand in hand with low achievement. Russian children scored more highly overall and had a far smaller tail of low achievers than either England or the United States. The Western children tended to overestimate teachers' perceptions of them; children in Russia underestimated them. Western pupils were found to have higher self-perceptions and were more satisfied with their performance and work-rate, perceiving less room for improvement than the Russian pupils (Elliott, Hufton, Illushin, and Lauchlan, 2001). The explanation put forward for this finding is that Western teachers believe that positive messages motivate pupils; hence there is a gulf between their overt messages to children and teachers' beliefs about pupils' actual attainment. Elliott and colleagues report similar findings for adolescent children in a parallel study. Shen and Tam warn, however, against the assumption that raising standards and expectations will automatically raise performance. Instead they suggest a gradual rise in standards may be necessary, because new values need time to become embedded.

The pupils' fault?

Hess and Azuma (1991) argue that almost universally, the teaching of mathematics is based around out-of-date curricula and traditional teaching approaches that are 'not conducive to learning', but that rather than change the curriculum or essence of teaching, teachers work either on pupil dispositions (making them adaptive to the classroom culture) or to make lessons more appealing (making them 'fun' rather than adopting new pedagogies).

> “Cultures differ in the emphasis they place on these two strategies. Japanese tend to stress developing adaptive dispositions; Americans try to make the learning context more attractive. National difference in educational achievement may be more completely understood by analysis of cultural differences in student dispositions. The interaction of student characteristics and teacher strategies creates very different climates in the two countries. *(Hess and Azuma, 1991)*

There is danger in reading into such studies an underlying assumption that the key to raising standards may lie in changing pupils' perceptions. Promoting adaptive dispositions – accept the need to work harder – may indeed have an impact, but a more constructive approach may rest in overhauling the curriculum and teaching approaches.

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THEME 14: Wider goals

Mathematics education or education through the context of mathematics?

Other countries address learning goals as well as mathematical goals in mathematics classes. For example, Japanese mathematics teachers have the goal of developing personal qualities, such as working cooperatively or persevering, alongside the mathematical outcomes. Attending to developing personal qualities may be as important as attending to content goals.

The Japanese approach to professional development through 'lesson study' has become a popular recommendation for professional development activity in England. While the concept is bound to be adapted to the different context, one thing that appears to have been 'lost in translation' from Japan to England is the nature of the goals set for the lessons being studied. For example, the NCETM website suggests teacher groups select a topic – something particularly difficult, new, important, or coming later – and agree a 'joint statement of the learning goals for that topic'.

This places the emphasis firmly on mathematical content: no mention is made of having any goals based around developing personal or collective qualities. Yet the Japanese place equal emphasis on selecting a second goal, such as working cooperatively or persevering, and the design of the lesson is built just as much around this goal as the content one (Isoda, 2007).

The distinction here can be characterised as a focus on mathematics education or on education through the context of mathematics. The mathematics education literature contains much advice on the advantages of group work and collaborative activity, but this is primarily presented as a means to an end: such pedagogic approaches are enlisted in the service of improving mathematical understanding. The primary goal is learning mathematics, mathematics education.

Whilst not denying the importance of learning mathematics, the Japanese attention to the development of personal goals alongside the mathematical content suggests more of a culture of 'education through the context of mathematics'. Mathematics lessons go beyond simply teaching mathematics. They acknowledge that education is also concerned with identity formation.

Goals for teaching mathematics

This distinction requires a re-imagining of the goals of teaching mathematics. All too often it seems that mathematics is bracketed off from the rest of the curriculum, afforded a distinct status that allows particular practices that would not be countenanced in other subjects. For example, it is common practice in UK primary schools to play 'maths champion' games – head to head challenges – that establish who is 'best' in class at mathematics, while the same teachers would not publicly establish the 'pecking order' in literacy.

The development of qualities such as perseverance may not simply be in addition to mathematical understanding but contributing to it. Despite the wealth of literature citing the East Asian emphasis on effort over innate ability, none of the large-scale international studies measure this directly (indeed they probably could not). One group of researchers created a proxy measure of effort, the Student Task Persistence (STP) variable. This is based on the students' completion of the (Third) TIMSS student background questionnaire and the percentage of questions answered out of the total of items asked. This provides a proxy for motivation to stay on task, as well as ability to complete the task (read the questions, mark the answers etc.) and willingness to have a go at questions that pupils might not be sure of the answer to. The hypothesis was that all three of these factors would have a bearing on performance on the mathematics test. The researchers found statistically significant correlations between mean STP scores and mean mathematics scores at all five grade levels with particularly strong correlations in grades 7 and 8 (Boe, May, and Boruch, 2002).

Because students completed the questionnaire after taking the mathematics test it is possible that student's awareness that they had not done well on the test could have discouraged them from completing the questionnaire: poor performance on the test may have accounted for a low STP score. In looking at a sub-sample of students in the 15th, 50th and 80th percentile the researchers found evidence that higher scoring pupils did indeed answer more of the questionnaire (82% for 80th percentile, 65% for 15th). However, high levels of correlation between the variability of scores in each of these attainment bands and the variability of the mean scores for the nation led the researchers to conclude there was no support for the hypothesis that national-level STP scores might be partly explained by individual test scores.

Overall this research supports the hypothesis that student effort is a key determinant of mathematical attainment.

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Conclusions

These conclusions are a summary of our key findings and recommendations. They are based on our reading and analysis of the research evidence and our professional judgements of the implications of these findings. We have chosen to concentrate on findings and recommendations that largely apply across all phases of schooling.

Behind the headline findings

England's success in TIMSS 2007 is significant, but rankings alone mask important differences in aspects of performance. For example, at Grade 4 (Year 5), performance in number, although above the international average, was relatively weak in comparison to England's overall performance on other areas of mathematics. At Grade 8 (Year 9) pupils' results in algebra were below the international average and weak in comparison to competitor countries. But these apparent areas of weakness are only important if what is tested in TIMSS is agreed to be of national priority and fits with our curriculum emphases *(see Theme 2: What rankings tell us, page 18)*.

Findings from repeated TIMSS and PISA studies add to our knowledge of changes over time, but these international studies are limited by their lack of longitudinal data that examines learning through tracking the same pupils over several years of schooling *(see Theme 2: What rankings tell us, page 18)*.

Not all high attaining countries have closed the attainment gap between pupils from differing socio-economic backgrounds *(see Theme 7: Attainment gaps, page 28)*.

Findings from TIMSS suggest the match between curriculum content and the TIMSS test items matters more than teaching in explaining international differences, although the quality of teaching still has a significant effect on mathematical learning *(see Theme 1: Impact of teaching, page 16)*.

Cultural influences

Internationally, explicit goals for mathematics education are similar in valuing high standards in knowledge and skills, learning dispositions and positive attitudes towards mathematics. No country, including those with high attainment in international studies, appears to have succeeded in achieving all three of these goals. Groups with a cultural identity of being good at mathematics do not necessarily also identify themselves as enjoying mathematics *(see Theme 3: Attainment and enjoyment, page 20)*.

High attaining countries address wider learning goals within the context of mathematics education. For example, Japanese teachers have the goal of developing personal qualities in mathematics lessons, such as working cooperatively or persevering, alongside the mathematical outcomes. Attending to developing personal qualities may be as important as attending to content goals in securing high attainment *(see Theme 14: Wider goals, page 42)*.

Implicit goals arising from nations' different socio-cultural-historical backgrounds have a powerful influence on pupil attainment. These implicit goals make 'borrowing' policies and practices problematic because adopting practices from elsewhere might mean adopting implicit goals that do not fit with England's vision for society or individuals. For example, the Asian emphasis on effort, while a factor in high attainment, is also closely aligned with obedience and authority *(see Theme 3: Attainment and enjoyment, page 20)*.

Contextual influences

Parents who have accurate and realistic expectations of their children's mathematical attainment have more impact on their children's learning than parents who over-estimate their children's attainment. Realistic parental expectations have more impact on pupil attainment than direct parental involvement, such as helping with homework *(see Theme 4: Parental expectations, page 22)*.

Mathematics education outside school – shadow education – is widespread, and parents from all socio-economic backgrounds are willing to make sacrifices to pay for extra-curricular mathematics instruction, particularly in East Asian countries. This extra provision can contribute to maintaining high standards, particularly where the shadow education complements formal education in school. The time spent on shadow education can, however, have adverse effects on students' wider social development *(see Theme 8: Shadow education, page 30)*.

Pupils' self-perceptions of whether or not they are good at mathematics, and whether or not they like mathematics, impact on their attainment. To raise attainment, however, it may be that changing the curriculum and teaching would be more effective than trying to change pupils' attitudes directly *(see Theme 13: Confidence and attainment, page 40)*.

Teachers' mathematics qualifications do not appear to be strongly linked to **average** levels of pupil attainment. However, in high attaining countries where there is a wide attainment gap, pupils from lower socio-economic status backgrounds tend to be lower attainers and are less likely to be taught by well qualified, experienced teachers. Hence differences in teachers' qualifications are associated with the attainment gap between pupils from high and low socio-economic status backgrounds *(see Theme 7: Attainment gaps, page 28)*.

Pedagogical influences

High attainment in international comparisons does not imply high attainment in problem solving. TIMSS results may be largely attributable to proficiency in computation and solving relatively predictable and routine problems. When solving non-routine problems, differences between nations appear to be less pronounced. Examination of pupils' problem-solving strategies reveals that pupils using abstract and symbolic approaches to problem solving generally perform more successfully: use of such approaches may be a result of pedagogical emphases rather than being a developmental issue *(see Theme 9: Problem solving, page 32)*.

In England, procedural fluency and conceptual understanding in mathematics are largely seen as mutually exclusive aims. This is not helpful. Teaching in Pacific Rim countries is largely dominated by procedures and hence supportive of procedural fluency, but the procedures used tend to be explicitly grounded in mathematical principles, and hence more mathematically coherent and meaningful than those most commonly used in the United Kingdom *(see Theme 10: Textbooks, page 34 and Theme 11: Teacher subject knowledge, page 36)*.

In Pacific Rim countries, mathematically informed procedural teaching is introduced and promoted through carefully constructed textbooks. England's textbooks are more routine and involve less variation in the construction of examples than those of many high attaining countries. Well-constructed textbooks not only support teachers but also provide support for pupils to work independently out of class *(see Theme 10: Textbooks, page 34)*.

Pupils from high attaining countries have, through textbooks, access at home to the materials they and their teachers have used and will use in class. This enables them, and their parents and siblings, to work on catching up or preparing for new ideas *(see Theme 10: Textbooks, page 34)*.

Teachers' mathematical subject knowledge is widely regarded as an important basis for effective teaching of mathematics. However the relationship between teacher subject knowledge and pedagogy is not simple and pedagogy depends upon tacit values and expectations as well as knowledge. What a teacher emphasises within a lesson may depend on cultural factors such as beliefs about learners as much as on the teacher's subject knowledge *(see Theme 11: Teacher subject knowledge, page 36)*.

Descriptions of national teaching 'scripts' or lesson content may not reveal important characteristics of pedagogic practices employed by teachers within that nation. For example, listing the tasks set in a lesson does not reveal how teachers develop these in practice. What may look like a procedural task can give rise to cognitive activity that addresses pupils' understanding. Equally, a potentially 'open' task can be made procedural by the teachers' directions and interventions *(see Theme 12: Pedagogy, page 38)*.

Recommendations

There are a number of features widely held to be important in making a difference in attainment in mathematics, for example, setting, high expenditure on ICT (particularly interactive white boards) and specific types of lesson structure. Our survey found that such features either do not appear to make a difference to attainment or produce different results in different contexts (this is explored more fully in the Introduction). We therefore conclude by identifying a small number of areas that we consider *do* have an impact on mathematical attainment and where further investigation and development would be worthwhile.

1. Evidence is needed from detailed comparative studies of mathematics education with international 'culturally near-neighbours' to England, such as Scotland, Massachusetts and the Netherlands. Evidence from such studies could include teacher recruitment and training, teacher subject knowledge, curriculum and pedagogy.

2. Studies are needed into the pedagogies of schools that achieve high standards in mathematics for all pupils to help our understanding of the interplay between cultures of high achievement and individual pupil progress.

3. Research is needed into the extent to which attainment in mathematics in England is supported by shadow education. A particular question is whether or not pupils from lower socio-economic status backgrounds are being disadvantaged by lack of access to shadow education.

4. English textbooks or e-resources for pupils and teachers need to be improved. Textbooks or resources should be designed to have more mathematical coherence, connections and variation, as well as provide better support for teachers and support for pupils in independent learning.

5. Policy makers need to examine strategies for reducing the 'opportunity gap' in English schools in terms of the mathematics qualifications and experience of the teachers of pupils from different socio-economic backgrounds.

6. Pupils and their parents from all socio-economic groups need to be engaged in establishing appropriately challenging yet realistic expectations for learning.

7. Investigation is needed into how teachers can develop classroom tasks that encourage understanding through deeper thinking about mathematical concepts and inter-relationships as well as procedural fluency.

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