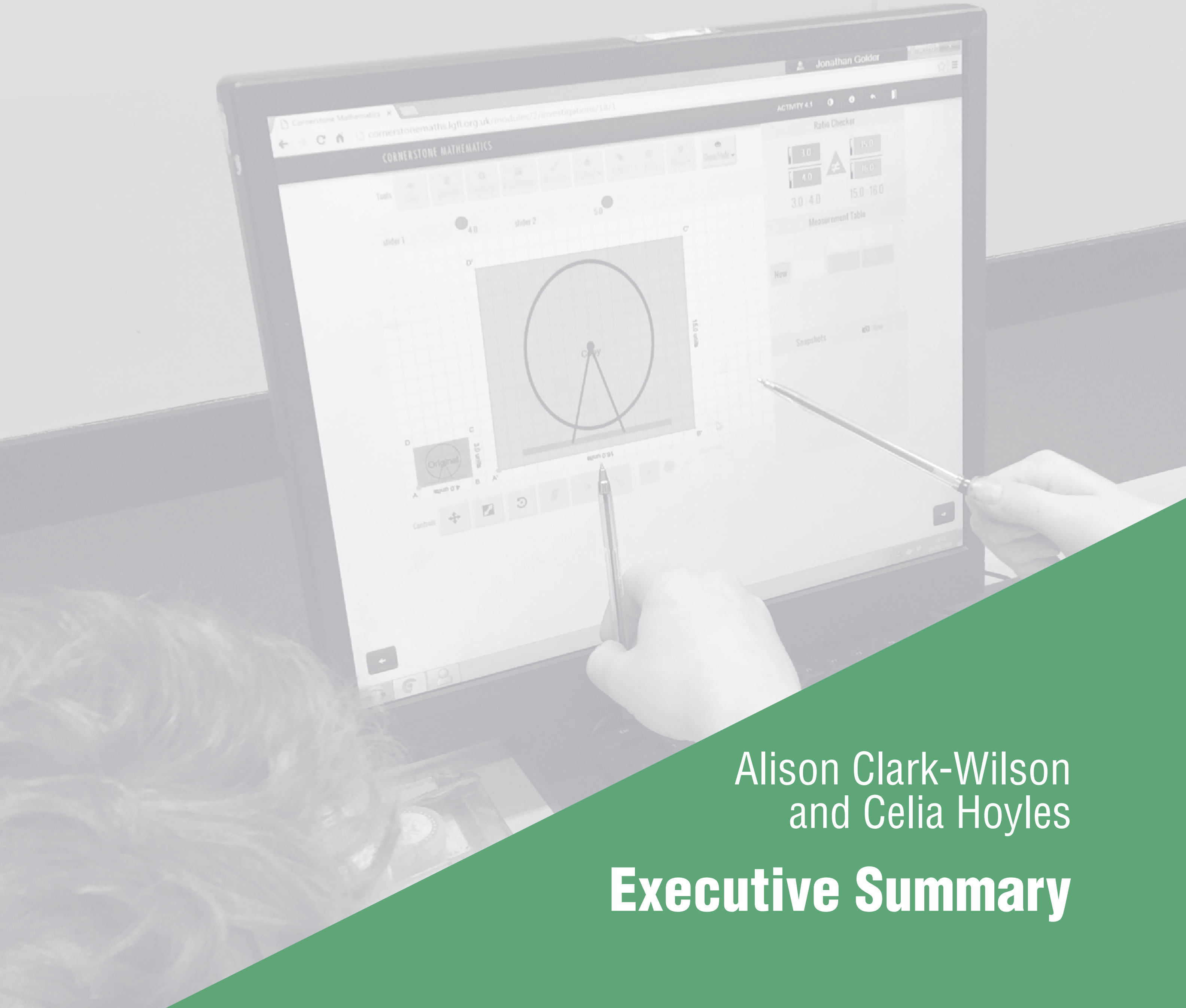
Dynamic Digital Technologies for Dynamic Mathematics

Implications for teachers' knowledge and practice



Alison Clark-Wilson
and Celia Hoyles

Executive Summary

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April 2017

UCL IOE Press

UCL Institute of Education Press

The Nuffield Foundation is an endowed charitable trust that aims to improve social well-being in the widest sense. It funds research and innovation in education and social policy and also works to build capacity in education, science and social science research. The Nuffield Foundation has funded this project, but the views expressed are those of the authors and not necessarily those of the Foundation. More information is available at www.nuffieldfoundation.org

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First published in 2017 by the UCL Institute of Education Press, University College London, 20 Bedford Way, London WC1H 0AL

www.ucl-ioe-press.com

British Library Cataloguing in Publication Data:
A catalogue record for this publication is available from the British Library

ISBNs
978-1-78277-224-8 (paperback)
978-1-78277-225-5 (PDF eBook)

Typeset by Quadrant Infotech (India) Pvt Ltd
Printed by CPI Group (UK) Ltd, Croydon, CR0 4YY
Cover image © the authors

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1 Aims and Background

This project set out to research the impact on teachers' mathematical knowledge for teaching (MKT) and associated mathematics pedagogical practice (MPP) on their teaching of key stage 3 topics with dynamic technology. We define such technology as that offering various mathematical representations that teachers and pupils can manipulate and link, and by doing so engage with the underlying mathematical concepts and relationships. The context chosen for the research is Cornerstone Maths, an existing, extensively researched resource for teaching key stage 3 mathematics topics with dynamic mathematical technology (DMT). Our work led us to adopt a comprehensive framework through which to interpret teachers' knowledge and practice: Mathematical Pedagogical Technology Knowledge (MPTK) (Thomas and Hong, 2005).

Furthermore, the project aimed to develop a professional development 'toolkit' (PD Toolkit), an online resource to support teachers in London with some prior experience of Cornerstone Maths as they engage and help other colleagues to also develop their use of DMT.

Cornerstone Maths is a set of curriculum units that aims to support teachers to develop their pupils' uses of carefully designed dynamic mathematical technology for the learning of three 'hard-to teach' topics in key stage 3 mathematics. We define dynamic mathematical technology as technology offering various mathematical representations (such as geometric shapes, graphs, tables and algebraic expressions) that teachers and pupils can manipulate and by doing so, engage with the underlying mathematical concepts and relationships. The three topic areas are: algebraic generalisation; geometric similarity; and linear functions.

The design of the Cornerstone Maths curriculum units is underpinned by the following design features:

- A DMT in which pupils explore, make conjectures and solve problems within sequential guided structured activities that have 'realistic' contexts.
- The dynamic environment makes the links between representations explicit to highlight the underlying mathematical concepts and relationships.
- Accompanying professional development through face-to-face professional development and an online community.

2 Research questions and methodology

RQ1 What is the impact on teachers' MPTK around the mathematical concepts of algebraic generalisation, geometric similarity, and linear functions, of their engagement with cycles of professional development and associated teaching that embeds DMT?

RQ2 What MPTK is desirable for teachers to integrate DMT into their teaching of these concepts?

RQ3 What are the design features of professional development activities for key stage 3 mathematics teachers that support them to use DMT in ways that become embedded in their practice and lead to effective learning?

The project took place between January 2014 and November 2016. It involved 48 self-selecting secondary schools from 23 London Boroughs, which were given free access to Cornerstone Maths via London Grid for Learning. At least 209[1] teachers participated in the project, and the number of participating teachers in each school ranged from 1 to 16. The teachers were either self-selecting or nominated by their school leadership team.

The participating teachers engaged in up to three cycles of professional activity, with each cycle focusing on one curriculum topic during which they:

- completed a pre-survey that captured contextual data; a pre-assessment of their MKT and practices with DMT; and an ethical agreement to give their consent as a participant in the study;
- participated in a one-day PD event;
- joined the online project community, hosted on the National Centre for Excellence in Teaching Mathematics (NCETM) portal;
- developed a plan to teach a nominated landmark activity as a 'research lesson' to a chosen key stage 3 class, which was shared by uploading to the NCETM portal;
- engaged in (optional) online PD webinars and tasks;
- taught the landmark activity as a 'research lesson' to their chosen class;
- attended a half-day PD meeting to feed back on their experiences;
- completed a post-PD cycle survey.

In addition, at least 10 per cent of the teachers within each cycle were selected for classroom observation of their teaching of the research lesson, and were interviewed to gather their reflections. The sample was chosen across a range of classroom and school contexts.

The research data set comprised: teacher contextual data; teacher responses to pre- and post-MKT survey items; teacher responses to PD tasks; lesson plans for landmark activity (planned in pairs/trios) and associated reflections; lesson observations of landmark activities and associated reflections. The data was analysed using a mixed-methods approach that involved both qualitative (deductive and thematic) and quantitative (frequencies, measures of spread and effect size) techniques. Three teacher case studies were developed that triangulated multiple data sources to result in narrative accounts of the development of their MPTK.

During the second year of the study we worked with a group of project teachers to design a 'professional development toolkit' that would support teachers who had some knowledge of the CM approach in sustaining and scaling their use of the DMT in the longer term in their own school and/or in leading PD for teachers from other schools.

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1 We suspect that this is a minimum number as some schools did not inform us of their school-based PD activity.

3 Summary of findings

3.1 Development of teachers' Mathematical Pedagogical Technology Knowledge

3.1.1 Algebraic variable

- **Defining and naming variables**. Teachers developed more precise mathematical responses of the meaning of a variable in terms of recognising its domain and range. Teachers required carefully mediated PD support to ensure that they fully appreciated the representation of the variable provided by the DMT, which was a slider. This mediation included drawing attention to the values on the slider and discussing the 'point' of dragging the slider.
- **Building a general expression**. There was limited improvement among one-third of the teachers in being able to relate the algebraic expression for a figural pattern to its geometrical structure. The remaining teachers were able to do this, as indicated by responses to the pre- and post-surveys. This indicates that, for some teachers, MPTK is resistant to change and that further PD cycles are needed, possibly in school.
- **Understanding letters as variables**. Teachers reported that their predominant teaching approach at key stage 3 centred on the treatment of 'letters as unknowns' rather than as variables. The pre- and post-survey data again suggest little change in teachers' responses, although some teachers did give fuller explanations in their later responses.

3.1.2 Linear functions

- **The meaning of m and c in y=mx+c**. The teachers' knowledge of the meaning of m and c in $y=mx+c$ showed improvements, particularly concerning the interpretation of the value of c as a variable that can take a negative value.
- **Defining a linear function**. Most teachers' initial definitions of a 'linear function' were partial and tended only to refer to the characteristics of the graphical output of the function, with few teachers making reference to the one-to-one nature of a function or its domain and range. The teachers' subsequent professional discussion was a significant factor in the teachers' refinement and expansion of their earlier responses.
- **Editing graphs of functions**. Teachers required carefully mediated PD support to ensure that they fully appreciated the rationale, role and effects of draggable 'hotspots' on the graph. Initially, only half of the teachers paid attention to the specific affordances of these 'hotspots', which controlled the resulting animation.
- **Interpreting multiple representations of motion**. Teachers used a limited number of strategies to read the multiple representations of the DMT when faced with a problem-solving task.

3.1.3 Geometric similarity

- **Properties of geometrically similar polygons**. The task to state the properties of geometrically similar shapes was challenging, with just over two-thirds of the teachers stating no, or only one, correct property (e.g. failing to mention angles, restricting definitions to triangles). By the end of the project many teachers could produce more general and complete responses.

- **Defining geometric similarity**. Teachers' definitions of geometrically similar shapes showed notable improvement, although not statistically significant. This improvement was also matched by an increase in their reported confidence in their definitions.
- **Within polygon and between polygon ratios**. The most significant improvement in the teachers' MPKT related to their understanding of the 'within polygon' invariant for geometrically similar polygons, an 'invisible' property that had been made visible by the DMT.

3.1.4 Teachers' lesson planning

- The teachers' lesson plans became increasingly detailed with respect to planning what they were going to do (especially with the DMT) and say during the landmark activity.
- The plans showed how teachers became more mindful of the need to provide support for pupils in making sense of the DMT, such that they could use it in mathematically productive ways beyond their initial experiences.
- Over time, the teachers' plans became more explicit about the inclusion of whole-class plenaries to focus on the mathematics at stake, with more teachers incorporating how they would use the DMT to support this.

3.1.5 Lesson observations of landmark activities

- The majority of teachers were initially reluctant to use the DMT 'live' during lessons, particularly during whole-class teaching. It is clear that this practice takes time to develop. When it was used effectively, the focus was on its use to: show counter-examples, extend the mathematical ideas, promote the use of appropriate vocabulary, and support or refute the pupils' mathematical predictions.
- Where teachers had taken the time to rehearse their 'curriculum script', they were noticeably more confident during whole-class use of the DMT and in scaffolding the pupils' work with the DMT. In such cases, the emphasis was on encouraging pupils to see the mathematical connections between the different representations and to describe the impact of the dynamic elements on these representations.
- In more effective lessons, the mathematical focus for the activities was maintained throughout. For example, in the *linear functions* landmark activity, the mathematical focus remained on the value of the coefficient of x, and the impact of using the DMT to change its value on the linked mathematical representations. In these lessons it was apparent that both teacher and pupils had sufficient prior knowledge and experience of the DMT to enable this to happen. Furthermore, teachers sustained the predict–check–explain pedagogical approach, and encouraged the use of correct vocabulary to support pupils' oral and written explanations.
- Some teachers developed practices whereby the pupils took on key roles in the use of the DMT to share with others, for example by

demonstrating or talking through their own strategies, which in turn widened the teachers' own knowledge and understanding of its use.

3.1.6 Teacher engagement

- Teachers overcame their initial apprehension about discussing the mathematical concepts in depth, reporting high levels of motivation and value in such professional discussions with colleagues, particularly when stimulated and supported by the PD resources embedding DMT.
- The alignment of the project goals to individual, departmental and school-level goals was a crucial factor with respect to both individual teachers' engagement and the potential sustainability of their use of the DMT.
- The participating teachers' roles and responsibilities were varied. An ideal pairing seemed to be the combination of a less-experienced teacher alongside a more-experienced teacher (in terms of general teaching experience, not necessarily the use of DMT) with some departmental responsibility with respect to the project goals.

3.2 Design features of the professional development toolkit (PD toolkit)

- Professional development to support teachers to implement DMT in their classrooms needs to blend face-to-face sessions that involve first-hand experiences with the DMT alongside PD tasks that deepen teachers' understandings of the mathematics involved and promote lesson planning for common activities. The adapted lesson study design, which offers a cyclical PD approach over a period of 6–8 weeks, was reported to be successful model by the participating teachers.
- The PD Toolkit resources extended beyond resources for PD sessions for other teachers of key stage 3 to include resources to support the wider communication of research findings that underpinned the use of DMT, and case studies of successful departmental implementations of DMT in various schools.
- Early data on the use of the PD Toolkit resources by schools suggests that there is sufficient content to enable further scaling within the CM project schools and to support the associated sustainability of new practices with DMT.

4 Discussion

4.1 Development of teachers' Mathematical Pedagogical Technology Knowledge

The project findings highlight how the process of integrating DMT within secondary mathematics teaching presents considerable potential for learning but also a significant challenge for teachers, as they rethink the underlying mathematical concepts, undergo their own instrumental genesis, develop their curriculum scripts and learn to support pupils' technological experiences.

Initially, the teachers reported very low level of use of DMT in both their whole-class teaching and by their pupils, and indicated a range of professional development needs in this respect. This suggests that both initial teacher education and in-service professional development opportunities are not currently enabling teachers to develop sufficient knowledge and use of DMT.

Teachers reported that their MPTK had developed in ways that directly supported their teaching of the topics within and beyond the Cornerstone Maths lessons. It was a key finding that teachers' mathematical knowledge for teaching algebraic variable and linear functions, as assessed by the items in the pre- and post-surveys, was resistant to change. However, as the majority of teachers performed well in these pre-survey items, it is possible that the intervention was too short to impact strongly on those teachers with weaker mathematical starting points. There was some evidence of more significant impact in relation to mathematical knowledge for teaching geometric similarity, which concerned more robust definitions of geometric similarity for a broader range of polygons and the appreciation of the invariant ratio property for pairs of corresponding sides within similar polygons.

The linear functions PD cycle impacted most on teachers' instrumental genesis, which we hypothesise is due to the more familiar set of representations (graphs, tables and equations), alongside the ease of the initial access to the context by playing the animation. Of the three topics, the highest proportion of teachers went on to teach this lesson and to disseminate this unit to their colleagues.

On the whole, teachers did not seem accustomed to discussing mathematical concepts in great depth but these opportunities were appreciated and perceived as worthwhile professional activity. This was further evidenced by teachers' requests to include some of the MPKT survey items and PD tasks within the PD Toolkit.

Teachers' lesson plans for the landmark activities improved in terms of their explicit focus and accompanying detail relating to the following eight quality features:

- includes teachers' actions and questions (not involving the DMT);
- includes pupils' actions on the DMT;
- provides support for pupils' instrumental genesis;
- maintains focus on the mathematical concept involved;
- privileges actions on representations to explore mathematical concepts;
- incorporates mathematical vocabulary;
- incorporates technical and/or contextual vocabulary; and
- includes planned plenary phases involving the DMT.

There was particular impact on the teachers' planning with respect to increased frequencies of the intended use of the DMT by teachers and pupils during the lesson. Furthermore, there was evidence that the practice of sharing plans within and across the different PD groups through the online community supported the teachers to learn from each other.

Teachers developed deeper appreciation of each of the three mathematical concepts (algebraic variable, linear functions and geometric similarity), which included greater rigour in definitions, representations, vocabulary, and relationships to other areas of mathematics. Enhanced teacher knowledge was more apparent where the teacher had engaged with two or more of the CM units of work.

The development of the teachers' MPTK happens over a significant amount of time (our hypothesis is 2–3 years), which is in part dependent on the need for teachers to assimilate use of the DMT into their personal mathematical experience, a necessary precursor to being able to incorporate it confidently within their classroom practices. Our case studies show that teachers' classroom practices developed over a 15-month period and improved with each PD cycle as they reflected on previous experiences and incorporated this learning into their future planning and practice.

The majority of teachers were initially very reluctant to use the DMT 'live' during lessons, particularly during whole-class teaching. It is clear that this practice takes time to develop. Where teachers had taken the time to rehearse their curriculum script, they were noticeably more confident during whole-class use of the DMT.

In the more effective lessons, the mathematical focus for the activities was maintained throughout. In these lessons, it was apparent that the teacher and pupils had sufficient prior knowledge and experience of the DMT to enable this to happen.

Some teachers developed practices whereby the pupils took on key roles in the use of the DMT, for example by demonstrating or talking through their own strategies, which in turn, widened the teachers' own knowledge and understanding of its use.

Maintaining a balance between pupils' uses of the DMT, their mathematical discussions and written recording was key to effective pedagogy.

The project revealed a number of effective classroom practices involving the use of the DMT, sometimes in conjunction with generic classroom management software available in school computer suites. Central to this was the use of the DMT 'live' during lessons, by both the teacher and pupils, in order to:

- support pupils to become familiar with the DMT for mathematical purposes;
- share, discuss and critique pupils' responses;
- highlight mathematical concepts through planned and thoughtful interactions with the DMT, such as dragging sliders and encouraging pupils to observe carefully and explain or justify the effects; and
- develop mathematical and technical vocabulary to support shared understandings.

Each curriculum topic revealed rich exemplar practices that were particular to the mathematical concept involved. This highlights the need for a mathematics-focused approach to professional development as the representations, mathematical connections and tangible objects are specific to the design of the DMT for each topic. Hence the nature of effective practice is at the level of the curriculum unit, although there are some more general uses of the technology that teachers need to master to establish familiar classroom routines.

Not all teachers wanted to be involved in all three PD cycles. Of the 27 who did, all have become PD Champions and are now leading the PD of other colleagues in their schools. However, we also have some teachers who only engaged in one PD cycle who chose to focus on embedding their first unit in their practice before engaging with another topic. A small number of schools (5) withdrew from the project altogether.

The alignment of the project goals to individual, departmental and school-level goals was a crucial factor for both individual teachers' engagement and the potential sustainability of their use of the DMT. The participating teachers' roles and responsibilities were varied. An ideal pairing seemed to be the combination of a less-experienced teacher alongside a more-experienced teacher (in terms of general teaching experience, not necessarily the use of DMT) with some departmental responsibility with respect to the project goals.

Our methodological decision to focus the PD cycles around nominated landmark activities as mediating constructs to reveal important mathematical knowledge and prompt teachers' reflections of their pedagogic practices proved to be a highly successful feature of the project. The landmark activities made the purpose of the DMT tangible for the teachers, although some compromises had to be made. For example, some teachers struggled to find sufficient time and access to the DMT to teach the lessons that preceded the landmark activity, resulting in both the teacher and pupils being less prepared to use the DMT. Other teachers were unable to teach the landmark activity during the period of the PD cycle, although during the second PD session in each cycle most teachers benefited from listening to the experiences of others even though they were yet to teach the lesson for themselves.

4.2 Design of professional development activities and support

The challenges associated with learning to engage with mathematics through the medium of a DMT, however well-designed, lead us to conclude that some initial face-to-face professional development is essential to support early activities such as discussion of the mathematics, hands-on experiences with the DMT, and preparation for classroom-based uses.

The project adopted a design-based methodology (which involved project teachers) to produce a PD Toolkit, accessible from the UCL-hosted website at http://ucl.ac.uk/cornerstone-maths.

The components and content of the PD Toolkit are:

Component	Content
Overview	› Video introduction to the CM Project.
Getting started	› Video overview of CM and its design principles. › Overview of the PD Toolkit and suggestions for how it might be used. › Technical information. › Project history and funders.
Curriculum units *This component is replicated for all three curriculum topics: algebraic variable; linear functions; and geometric similarity.*	PD resources › Video overview of the curriculum unit. › Pupil workbook and teacher guide. › Structured PD session, including: ○ MPKT items; ○ instrumentation tasks; ○ MPTK tasks to analyse pupils' digital productions; and ○ introduction to the landmark activity; leading to a: ○ lesson-planning task of the landmark activity. › Resources to support planning and reflection on the landmark activity: ○ video of the landmark activity; ○ video clips of 'typical' classroom enactments; and ○ examples of typical pupil responses. › Suggestions for assessing pupils' mathematical understanding.
Departmental case studies (embedding DMT)	› Outline of their approach for scaling/ sustainability. › Examples of within-school PD activities.
Evidence base	› Research summary for senior leaders and headteachers. › More detailed research summary for heads of department (including references to departmental case studies). › Links to published research.
Project community	› Link to the online project community – where teachers can share resources they create and discuss their implementations.

The take-up of the PD Toolkit in schools is still in its early stages but early data from CM Champions in three schools indicates that:

› it offers sufficient initial resource for the design of school-based PD in London schools that fits with the prevailing PD culture in the school concerning frequency and timing (from one 2-hour session to three 2-hour sessions spread over a few weeks).

- the overviews of the curriculum units (in video and document formats) were reported to be particularly useful.

5 Implications for policy and practice

We conclude by stating the implications of our findings, making our recommendations for both policy and practice and highlighting some next steps that would support both the dissemination of the project's outcomes and the future research agenda.

5.1 Implications

The mathematics education community does need to keep revisiting the role of technology within teaching and learning and the nature of its associated pedagogy. Although DMT has been around for over 20 years it is continually evolving and, while access to DMT may be easier, the main barrier is still opportunities for teachers' sustained PD that maintains a strong focus on the mathematics. Without this level of support for teachers to engage in cycles of PD that scaffold their uses of DMT, PD is unlikely to be effective.

The project outcomes make a substantial contribution to knowledge of more effective features of teachers' classroom practices with DMT that might underpin future large-scale PD initiatives in this area and offer a tighter framework within which such opportunities can be designed to be more effective and sustainable. This is even more important when teachers are choosing to move schools frequently or are leaving the profession early in their careers.

Initial teacher education seems an obvious starting point. However, as all teachers' first experiences with DMT are tentative, they need to be well supported (with realistic expectations of the time required) and involve productive collaboration with more-expert practitioners. All too often teachers reject DMT based on early lessons that they deem to have been unsuccessful, without necessarily having tools for reflection that enable them to learn from the experience.

Our experiences within the previous Cornerstone Maths-funded project phases alongside this one lead us to suggest that it takes at least two years for teachers to develop more confident teaching practices with DMT, which means that for a department, the timeline to 'embed' DMT across key stage 3 is likely to be four to five years. A key element of this is the commitment to sustain the department-wide efforts alongside other demands on departmental time and energy. Hence leadership at the school level is key. The PD Toolkit promises to play a major role in this respect, as it aims to improve the quality of in-school professional development activities and enable new teachers to become involved through the inclusion of PD resources that can be revisited over time.

5.2 Recommendations for policy

There is a role for government, its agencies and affiliated organisations to provide better sign-posting to more effective teaching practices with DMT in secondary mathematics. This role may fall within the remit of the National Centre for Excellence in Teaching Mathematics and the Department for Education-funded Maths Hub programme, in which case a targeted effort is needed over time to share the project's outcomes, develop capacity in the school system and disseminate successful models for school-based professional development.

The findings from this study have fed into the Joint Mathematical Council's ongoing work on 'developing mathematics-specific pedagogy in Initial Teacher Education' with respect to the use of digital resources has been disseminated to ITE providers in collaboration with the NCETM and the National College for School Leadership. This may lead to an opportunity to develop a specific publication aimed at disseminating models for the introduction of DMT within mathematics teacher training routes.

We reiterate the recommendation from the recent Advisory Committee on Mathematics Education report that 'personalised and sustained professional learning within a supportive professional environment, with time for self-reflection' is an essential opportunity for teachers of key stage 3 mathematics to enable them to deepen and strengthen the quality of their teaching with DMT (Advisory Committee on Mathematics Education, 2016). Furthermore, the severe teacher shortage and staff turnover rates for key stage 3 mathematics in some parts of the country present additional systemic challenges for which the CM PD Toolkit may offer a possible solution as schools are increasingly unable to release teachers for externally led professional development.

5.3 Recommendations for practice

There should be an expectation that teachers of secondary mathematics on training routes are supported to use a DMT from the outset and the expectation that such practice is continued in the early years of teaching. These early experiences should then be built upon such that a range of (research-informed) DMTs are experienced, ideally supported by reflective PD cycles in collaboration with more expert colleagues. While it is likely that a minority of these teachers may become innovative designers of new teaching tasks and approaches with DMT, it is more likely that they will draw on their earlier experiences to enable them to select appropriate DMT for their teaching purposes using a more research-informed set of criteria. It is important that both school leaders and institutional structures provide a sustainable set of conditions within which this can be achieved.

6 References

Advisory Committee on Mathematics Education (2016) *Professional Learning for all Teachers of Mathematics*. London: The Royal Society.

Thomas, M.O.J. and Hong, Y.Y. (2005) 'Teacher factors in integration of graphic calculators into mathematics learning'. In Chick, J.L.V.H.L. (ed.), *Proceedings of the 29th Annual Conference for the Psychology of Mathematics Education*, vol. 4. Melbourne: University of Melbourne. 257–64.

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This report summarises the outcomes of the Nuffield Foundation funded 2014–17 project 'Developing teachers' mathematical knowledge for teaching and classroom use of technology through engagement with key mathematical concepts using dynamic digital technology'. The Nuffield Foundation is an endowed charitable trust that aims to improve social well-being in the widest sense. It funds research and innovation in education and social policy and also works to build capacity in education, science and social science research.

The full Final Report is available as a separate publication, ISBN 9781782772217.

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ISBN 978-1-78277-224-8

UCL Institute of Education Press
20 Bedford Way
London
WC1H 0AL
www.ucl-ioe-press.com