Seminar Series on *Mathematical Knowledge in Teaching:* Final Report to the Nuffield Foundation

Kenneth Ruthven & Tim Rowland
University of Cambridge Faculty of Education

Start date: 1 November 2006
Finish date: 30 October 2008

Intellectual aims and organisation

The overarching intellectual aim of this seminar series was to draw together current ideas and evidence about the forms and functions of the mathematically-related knowledge which enables teachers to support successful student learning of mathematics: more specific aims have been to achieve a critical conceptual synthesis, to establish significant professional implications, and to identify major research needs. Thus, the seminar series consisted of an opening 2-day conference followed by five 1-day conferences, each with a specific focus. Three of these events were concerned with taking a critical overview of existing thinking and research on the key issues of *conceptualising and theorising* mathematical knowledge for teaching, *auditing and assessing* such knowledge, and *developing and deepening* such knowledge. Two further events were concerned with closer analysis of a selection of research studies and teaching resources related to substantive aspects of mathematical knowledge for teaching. The specific aspects were *division and fractions*, and *argumentation and proof* chosen because they are well-researched and contrasting aspects of mathematics addressed at multiple levels of education. The final event focused on *formulating a research agenda*. Appendix 1 provides a summary overview of the seminar programme on each of these themes; in particular, of the papers and presentations prepared by speakers.

To encourage a focus on critical synthesis and professional implications, a specification was provided for presenting speakers (in a fairly standard form, adapted to fit the focus of the particular meeting), emphasising the importance of speakers explaining how and why ideas or methods represent a significant advance on, or alternative to, earlier ones, and identifying significant implications of bringing these ideas or methods to bear on the practices of teaching, and of teacher education and development. Similarly, the specification enjoined group discussions to identify commonalities and contrasts, complementarities and conflicts between different ideas and methods; and to identify any significant limitations of the ideas and methods in illuminating important practical issues, and any significant limitations of current policy and practice in acknowledging important insights from these ideas and methods.

Professional aims and organisation

The overarching professional aim of this seminar series was to build research capacity in this area: more specific aims were to establish a working network covering teacher educators and educational researchers, but also extending to potential research user groups with a direct professional interest in the area; and to provide particular opportunities for doctoral student researchers working in this area to participate in the network. Thus the network that has been established includes mathematics education researchers and teacher educators at various career stages (including 5 doctoral students). Recognising that future teachers develop much of their mathematical knowledge at school and university, prior to entering courses specifically devoted to teacher education, we recruited three persons active in designing and teaching undergraduate (specialist and service) mathematics

courses to join the core membership (Alcock, Sangwin, Siemons). To involve practitioner and policymaker communities concerned with mathematics teacher education and training, we were fortunate to be able to recruit as core members of the seminar series participants representing the Department for Education and Skills [DfES, now DCSF/DIUS] (Hoyles), the National Centre for Excellence in the Teaching of Mathematics [NCETM] (Sutherland, Joubert), and the Office for Standards in Education [OfStEd] (Jones). Although our attempts to recruit school-based subject mentors to core membership of the series proved unsuccessful, at each of the university-hosted meetings we were able to attract such colleagues from amongst those associated with local teacher education activities. Likewise, other doctoral students and academic staff of the host institution joined these seminars as local attendees. Finally, a number of other interested persons, primarily mathematics education researchers and teacher educators visiting from overseas, attended particular seminars. A full listing of core members of the seminar series, and of local and other attendees at individual seminars is provided in Appendix 2.

Findings

The main findings from the conceptual synthesis which has taken place over the course of the seminar series can be related to the research needs and professional implications identified at the final meeting.

One important need is for research on mathematical knowledge in teaching to take on a more programmatic character. This has a number of aspects. First, research to date has tended to focus on particular phases, topics and processes: on primary teachers and teaching rather than secondary or tertiary; on arithmetic rather than algebra, geometry or probability & statistics; on proof & proving rather than on models & modelling. Second, a range of approaches, drawing on different sources of evidence, have been developed in order to identify what mathematical knowledge is important for teaching, but insufficient attention has been given to triangulating – and potentially integrating – these approaches and understanding the relationships between them. A more systematic research programme would provide a stronger knowledge base for designing courses of initial teacher education and providing professional development for serving teachers.

A further need is for research which probes some of the assumptions which have pervaded the work undertaken to date, and explores the viability of alternative conceptualisations of mathematical knowledge in teaching and corresponding ways of investigating its operation. For example, much work to date has conceived mathematical knowledge in teaching as strictly individual knowledge capable of being elicited in isolation from the actual teaching situation, and as independent of the tools and resources available to support subject teaching. Equally, most research, even when it has compared educational systems, has proceeded on the assumption that mathematical knowledge in teaching is independent of particular contexts for (and cultures of) mathematics education. Finally, the field has tended to emphasise those aspects of mathematical knowledge which are specific to teaching, perhaps at the expense of more generic aspects.

A final need is for a stronger emphasis on research aimed at developing and validating tools to support the enhancement of mathematical knowledge for teaching within the initial and continuing professional education of teachers; research and tools locating such knowledge more strongly within everyday processes of teaching, and relating it more directly to providing effective support for student learning of mathematics. Examples of such tools

examined during the seminar series include diagnostic 'mathsmaps' to help teachers reflect on, and develop, their mathematical knowledge; and the 'knowledge quartet', an analytic framework for the identification and discussion of primary teachers' mathematical knowledge as evidenced in their teaching.

The last stage of the seminar series coincided with publication of the interim and final reports of the Williams review of primary mathematics teaching (raising many issues equally applicable to mathematics teaching in other phases). During the final seminar, several papers examined the main recommendations of the Williams review, endorsing the main recommendation that professional development for teachers should focus on "the three interrelated strands of mathematical content, mathematical pedagogy and embedded practice", but also noting some of the challenges of implementing the review's proposals. The review's emphasis on more informal modes of teacher development through in-school mentoring and coaching calls for investigation of how best to embed professional learning around teachers' everyday work in school communities in ways which enhance mathematical knowledge, for example through forms of lesson-focused peer interaction. Equally, the review's emphasis on peer interaction may underestimate the potential contribution of what have been termed 'educative' curriculum materials, specially designed to support the mathematical learning of teachers as well as their students.

Dissemination

Three main avenues of dissemination from the seminar series are being pursued. First, an open website was established at the start of the project (http://www.maths-ed.org.uk/mkit/index.html) to make public details of the series, and provide access to documents prepared in the course of it. Now that the seminar series has been completed, this website provides a valuable and readily accessible archive of its work and findings.

Second, to further develop the synthesis undertaken by the seminar series, and make it more widely available, an edited book is now in preparation. We expect this to appear in 2010 in the Springer *Mathematics Education Library*, the leading international series in the mathematics education field. The main chapters will develop papers and presentations given during the seminar series. The editorial guidelines emphasise the same concern as the seminar specifications that authors should make clear what intellectual progress and professional implications are represented by the research that they are reviewing and presenting. This represents a valuable opportunity to refine the work undertaken during the seminar series, and to further strengthen critical synthesis, particularly through the inclusion of discussion chapters at the end of each section of the book which will draw on the seminar discussions to explicitly address that brief.

Third, and more immediately, at the February 2009 day-conference of the British Society for Research into Learning Mathematics, we intend to organise a presentation and discussion session focusing on the research needs which the seminar series has identified, with a view to getting feedback on these from a wider audience, and encouraging a wider group of researchers to develop an interest in this important area. A report of this session will also appear in the online Informal Proceedings of the Society, and in the Society's journal, *Research in Mathematics Education*.

Finally, following on the involvement of DfES [now DCSF/DIUS], NCETM and OfStEd colleagues in the seminar series, we hope that its work will make some contribution to informing future initiatives undertaken by these bodies on developing mathematical knowledge and confidence amongst the teacher workforce.

APPENDIX 1: The seminar programme

Conceptualising and theorising mathematical knowledge in teaching (11-12 January 2007, Cambridge).
All precirculated speaker papers and speaker presentations available at http://www.maths-ed.org.uk/mkit/seminar1.html

Speaker	Title	Summary
Maria Goulding	Mathematical Subject Knowledge in Primary Teacher Training - A View from England and Wales	Contextualises contemporary E&W concern to develop prospective teachers' mathematical knowledge during initial teacher education [ITE], and summarises recent studies of this. Examines Shuman and subsequent conceptualisation of subject knowledge in teaching. Reviews British research which has investigated how –and what forms of – mathematical knowledge and belief contribute to effective mathematics teaching. Relates this to issues of recruitment and provision in ITE.
Jeremy Hodgen	The Situated Nature of Mathematics Teacher Knowledge	Reviews development of ideas about mathematical knowledge in teaching to recognise its embedding in social practice. Illustrates significance of this perspective through case study contrasting teacher's knowledge of proportional reasoning in the context of developing lessons and leading professional development sessions with her knowledge in the context of a structured mathematics interview. Draws implications of this perspective for research on teacher knowledge, the role of relationships and emotion, and issues of collaboration.
Heinz Steinbring	Changed Views on Mathematical Knowledge in the Course of Didactical Theory Development.	Shows how the German didactical tradition has evolved in response to new theoretical ideas and new – empirical – research approaches in mathematics education, changing the basic views on the roles that mathematical knowledge, teacher and student have to play in teaching-learning processes to focus on the cultural system of mathematical interaction between teacher and students. In this view, mathematical knowledge is interactively constructed within the social teaching-learning system on the basis of its epistemological constraints.
Dina Tirosh & Ruhama Even	Teachers' Knowledge of Students' Mathematical Learning: An Examination of a Commonly Held Assumption.	Reviews three aspects of students' mathematical learning that have been at the centre of researchers' attention: student conceptions, different forms of knowledge and kinds of understanding, and classroom culture. Examines assumption that teacher knowledge of these dimensions of students' mathematical learning is essential for good teaching in light of different theoretical perspectives: behaviourism, constructivism, and situationism. Discusses what it might mean for teacher education to focus on the three aspects of student mathematical learning.
Kenneth Ruthven	Synthesis	Synthesises papers by examining key issues identified for teacher mathematical knowledge, and shifts from earlier received perspectives to productively reconceived perspectives.

Auditing and assessing mathematical knowledge in teaching (27 September 2007, London)
All precirculated speaker papers and discussion group summaries available at http://www.maths-ed.org.uk/mkit/seminar3.html

Speaker	Title	Summary
Julian Williams	Audit and Evaluation of Pedagogy - Towards a Sociocultural Perspective	Develops a critical sociocultural analysis of the forces at work on audit. Draws on empirical studies of teachers understanding/knowledge of student mathematical knowledge to show that (i) teachers sometimes mis-judge their students' knowledge; (ii) their judgments are influenced by their own mathematical knowledge, and by their teaching experience; (iii) their knowledge of their students can be strongly 'task-situated' and 'tool-mediated' rather than 'conceptual' and 'in the head'. Suggests the need to examine methodology for studying teacher-knowledge with due recognition of the danger (or opportunity) that 'teacher knowledge' is a particular, politically and socially mediated construct of audit methodology.
Tim Rowland	Auditing the Mathematics Subject Matter Knowledge of Elementary School Teachers	Reports a project initiated in 1998 in the context of UK government policy to introduce subject content knowledge as an explicit dimension of the 'standards' for the award of Qualified Teacher Status in England. Project investigated the mathematics subject knowledge of prospective elementary school teachers in the UK, and how this relates to classroom teaching performance. Reviews findings about topics that trainees found difficult and indications that the extent and security of their subject matter knowledge is related to their teaching competence.
Julie Ryan & Julian Williams	Mathsmaps for Diagnostic Assessment with Pre-Service Teachers	Reports an innovative assessment feedback tool – the mathsmap. Describes how pre-service teachers made sense of this personalised diagnostic profile of their attainment and errors across the mathematics curriculum. Reflective use of the mathsmap to examine learning on a personal level provokes 'accounts' or 'stories' that might inform pedagogical content knowledge: in making their mathsmap comprehensible to themselves, teachers account for their own knowledge- troubles; that is, narrate their metacognition.
Marilena Petrou	Michigan Research on Developing a Practice-Based Theory of Content Knowledge of Teaching	Summarises efforts of Michigan University research team to develop an empirical approach to identify what teachers need to know in order to teach mathematics effectively. Particularly, the team has been devising instruments to measure both teachers' Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge PCK, that can give good predictions of classroom effectiveness and student learning. Moreover, these measures helped the team to develop a practice-based theory of content knowledge for teaching by testing and refining categories of knowledge needed to teach mathematics effectively.

Developing and deepening mathematical knowledge in teaching (18 March 2008, Loughborough)
All precirculated speaker papers and discussion group summaries available at http://www.maths-ed.org.uk/mkit/seminar5.html

Speaker	Title	Summary
Anne Watson	Developing and Deepening Mathematical Knowledge in Teaching: Being and Knowing	Develops a view of mathematical knowledge in teaching [MkiT] as a way of being and acting, avoiding categorisation and acquisition metaphors of knowledge. Presents MKiT as participation in mathematical practices in the classroom, and also during preparation for teaching. Thus development and deepening of knowledge take place through doing mathematics and being mathematical in social contexts in which mathematical habits of mind are embedded, recognised and valued. Explains how some of the tasks of teaching can be seen as particular contextual applications of mathematical modes of enquiry.
Fay Turner & Tim Rowland	The Knowledge Quartet: A Means of Developing and Deepening Mathematical Knowledge in Teaching	Describes a practice-based framework for the identification and discussion of prospective elementary school teachers' mathematics content knowledge: 'The Knowledge Quartet' emerged from intensive scrutiny of 24 videotaped lessons, taught by novice teachers. Illustrates application of the Knowledge Quartet in lesson observation and teacher education with reference to a particular lesson taught by one trainee teacher. Introduces subsequent longitudinal study of beginning primary school teachers, suggesting that mathematical content knowledge for teaching is most usefully identified in the act of teaching, and developed through reflection on teaching: here through an approach based on reflection, using the Knowledge Quartet, and framed by socio-cultural theories of learning.
Birgit Pepin & Linda Haggarty	Making Connections and Seeking Understanding: Mathematical Tasks in English, French and German Textbooks	Contends, from literature review, that connectivity and the making of 'connections' can be used as an analytical tool to analyse tasks with respect to potential pupil understanding of mathematics. Identifies ways of making connections in mathematical tasks in textbooks, exemplified by treatment of 'Negative Numbers' in selected English, French and German lower secondary textbooks. Concludes that particular tasks provide different (mis)represen-tations of mathematics for their students in school textbooks, in particular with respect to connectivity: despite perhaps gaining proficiency at certain kinds of procedures and tasks, some students may well have gained at best a fragmented sense of the mathematics and understood few if any connections that tie together the procedures they had studied.

Mathematical knowledge in teaching: the case of division and fractions (18 April 2007, Manchester)
All speaker presentations and discussion group summaries available at http://www.maths-ed.org.uk/mkit/seminar2.html

Speakers	**Precirculated study introduced by speakers**
Dolores Corcoran Johannes Siemons	Ma, L. (1999) *Knowing and teaching mathematics: teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum.
Ray Huntley Lara Alcock	An, S., Kulm, G., & Wu, J. (2004). The Pedagogical Content Knowledge of Middle School Mathematics Teachers in China and the U.S. *Journal of Mathematics Teacher Education* 7(2), 145-172.
Peter Huckstep Sandy Pepperell	Borko, H., Eisenhart, M., Brown, C. A., Underhill R. G., Jones, D. & Agard P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education* 23 (3) 194-222.
Discussion group	**Theme discussed in relation to studies**
1	Conceptualising and theorising mathematical knowledge in teaching
2	Auditing and assessing mathematical knowledge in teaching
3	Developing and deepening mathematical knowledge in teaching

Mathematical knowledge in teaching: the case of argumentation and proof (10 January 2008, Cambridge)
All speaker presentations and discussion group summaries available at http://www.maths-ed.org.uk/mkit/seminar4.html

Speakers	**Precirculated study introduced by speakers**
Maria Goulding Marie Joubert	Yackel, E. (2002) What we can learn from analyzing the teacher's role in collective argumentation? *Journal of Mathematical Behavior* 21 (4) 423-440
Andreas Stylianides Cathy Smith	Stylianides, A. J. & Ball, D. L. (in press) Understanding and describing mathematical knowledge for teaching: Knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education*.
Johannes Siemons	Knuth, E. J. (2002) Teachers' conceptions of proof in the context of secondary school mathematics. *Journal of Mathematics Teacher Education* 5(1) 61–88
Discussion group	**Theme discussed in relation to studies**
1	Conceptualising and theorising mathematical knowledge in teaching
2	Auditing and assessing mathematical knowledge in teaching
3	Developing and deepening mathematical knowledge in teaching

Formulating a research agenda on mathematical knowledge in teaching (17 June 2008, London)
All precirculated speaker papers available at http://www.maths-ed.org.uk/mkit/seminar6.html

Speaker	Title
	Towards a programmatic framework
Maria Goulding & Marilena Petrou	Conceptualising teachers' mathematical knowledge for teaching
Andreas Stylianides	Towards a research programme for identifying what mathematical knowledge is important for teaching
Johannes Siemons	Mathematics knowledge in teaching: Formulating research
Julian Williams	Towards a conceptualisation of a 'collective teachers' mathematical knowledge
	Towards more informed provision
Lara Alcock	The relative impact and teacher perceptions of different kinds of professional development
Julie Ryan & Julian Williams	Teachers' stories of mathematical knowledge
Ray Huntley & Peter Huckstep	The place of exemplification in mathematical knowledge
Anne Watson	How can learning more maths impact on teaching?
	Towards a comparative perspective
Paul Andrews	The cultural location of teachers' mathematical knowledge: another hidden variable in research on mathematical knowledge for teaching?
Birgit Pepin	What kinds of knowledge help teachers to become effective teachers of mathematics? What kinds of choices do teachers have? – a comparative perspective
	Towards a broadened agenda
Dolores Corcoran & Sandy Pepperell	Mathematical knowledge in teaching for social justice
Marie Joubert	Using ICT in mathematics classrooms
	Into the post-Williams era
Tim Rowland & Fay Turner	Research into how deep knowledge of mathematics may be developed through ITE and PPD
Kenneth Ruthven	The need for a programme of research on educative curriculum materials as a mechanism for the diffusion of mathematical knowledge in and for teaching

APPENDIX 2: The seminar participants

Type	Forename	Surname	Organisation/Institution
Core	Jill	Adler	University of Witwatersrand/ King's College London
Core	Lara	Alcock	Loughborough University
Core	Paul	Andrews	University of Cambridge
Core	Dolores	Corcoran	St Patrick's College, Dublin/ University of Cambridge
Core	Maria	Goulding	University of York
Core	Jeremy	Hodgen	King's College London
Core	Celia	Hoyles	DfES/Institute of Education
Core	Peter	Huckstep	University of Cambridge
Core	Ray	Huntley	University of Gloucestershire
Core	Jane	Jones	OfStEd
Core	Marie	Joubert	NCETM/University of Bristol
Core	Birgit	Pepin	University of Manchester
Core	Sandy	Pepperell	Roehampton University
Core	Marilena	Petrou	University of Cambridge
Core	Tim	Rowland	University of Cambridge
Core	Kenneth	Ruthven	University Of Cambridge
Core	Julie	Ryan	Manchester Metropolitan University
Core	Chris	Sangwin	Birmingham University
Core	Johannes	Siemons	University of East Anglia
Core	Andreas	Stylianides	University of Oxford
Core	Ros	Sutherland	NCETM/University of Bristol
Core	Fay	Turner	University of Cambridge
Core	Anne	Watson	University of Oxford
Core	Julian	Williams	University of Manchester
Local	Julia	Anghileri	University of Cambridge
Local	Mike	Askew	Kings College London
Local	Khaled	Ben-Motreb	University of Manchester
Local	Laura	Black	University of Manchester
Local	Rod	Bond	North Leics. Further Maths Cent.
Local	Jeremy	Burke	King's College London
Local	Penny	Coltman	University of Cambridge
Local	Tony	Croft	Loughborough University
Local	Pauline	Davis	University of Manchester
Local	Mark	Dawes	Comberton Village College
Local	Rosemary	Deaney	University of Cambridge
Local	Richard	Gadsen	Loughborough University
Local	Zurina	Haji-Harun	University of Manchester
Local	Anne	Haworth	University of Manchester
Local	Barbara	Jaworski	Loughborough University
Local	Clive	Kanes	King's College London
Local	David	Mallabone	Swavesey Village College
Local	Sian	Mawditt	St. Felix CE School, Newmarket
Local	Olwen	McNamara	University of Manchester
Local	Su	Nicholson	Loreto School
Local	Maria	Pampaka	University of Manchester
Local	Carol	Robinson	Loughborough University
Local	Barbara	Rundle	South Notts. Further Maths Cent.
Local	Cathy	Smith	University Of Cambridge
Local	Geoff	Wake	University of Manchester
Local	Michelle	Webb	Northolt High School
Local	Elaine	Wilson	University of Cambridge
Other	Bill	Barton	University of Auckland
Other	José	Carrillo	University of Huelva
Other	João Pedro	da Ponte	Universidade de Lisboa
Other	Heinz	Steinbring	Universität Duisburg-Essen
Other	Max	Stephens	University of Melbourne
Other	Dina	Tirosh	Tel Aviv University
Other	Pessia	Tsamir	Tel Aviv University
Other	Linton	Waters	Nuffield Foundation

APPENDIX 3: The book in preparation

Mathematical Knowledge in Teaching

1 Introduction
Tim Rowland & Kenneth Ruthven (University of Cambridge)

Conceptualising mathematical knowledge in teaching

2 Conceptualising teachers' mathematical knowledge in teaching
Maria Goulding (University of York) & Marilena Petrou (Open University)

3 Teacher knowledge for developing students' mathematical thinking: A conceptual framework and its professional implications
Dina Tirosh (Tel Aviv University) & Ruhama Even (Weizmann Institute of Science)

4 Changed views on mathematical knowledge in the course of didactical theory development – independent corpus of scientific knowledge or result of social constructions?
Heinz Steinbring (University of Duisburg-Essen)

5 Knowing and identity: A situated theory of mathematics teacher knowledge
Jeremy Hodgen (King's College London)

6 Teaching mathematics as the contextual application of mathematical modes of enquiry
Anne Watson (University of Oxford) & Bill Barton (University of Auckland)

7 Conceptualising mathematical knowledge in teaching: Discussion and synthesis
Kenneth Ruthven (University of Cambridge)

Understanding the cultural context of mathematical knowledge in teaching

8 The cultural location of teachers' mathematical knowledge: exemplary teaching of linear equations in five European countries
Paul Andrews (University of Cambridge)

9 How educational systems and cultures mediate teacher knowledge: case studies from England, France and Germany
Birgit Pepin (University of Manchester)

10 Audit and evaluation of pedagogy: towards a sociocultural perspective
Julian Williams (University of Manchester)

11 Understanding the cultural context of mathematical knowledge in teaching: Discussion and synthesis
Andreas Stylianides (University of Cambridge) & Séan Delaney (University of Michigan)

Building mathematical knowledge in teaching by means of theorised tools

12 The Knowledge Quartet as an organising framework for developing and deepening teachers' mathematical knowledge
Tim Rowland & Fay Turner (University of Cambridge)

13 Learning to teach mathematics using lesson study
Dolores Corcoran (St Patrick's College, Dublin City University) & Sandy Pepperell (Roehampton University London)

14 Teachers' stories of mathematical knowledge: Accounting for the unexpected
Julie Ryan (Manchester Metropolitan University) & Julian Williams (University of Manchester)

15 Building mathematical knowledge in teaching by means of theorised tools: Discussion and synthesis
Jill Adler (University of Witwatersrand/King's College London