

Advisory Committee on Mathematics Education



Mathematical Needs – Summary

JUNE 2011

FOREWORD BY THE CHAIR



This report is about the mathematics that we need, as a nation, in order to ensure that we are able to fulfill the demands that we face, whether they be economic, intellectual or societal.

A few years ago, ACME decided to investigate the mathematics needed by various end users, including the learners themselves. It was felt that, in the face of a rapidly changing world, there was a need for a proper audit of the mathematical requirements of Higher Education and employers. This raised the related question of whether these end users' needs are matched by the mathematical knowledge, skills and understanding currently acquired by students at school and college. However these questions could not be isolated from the learning needs of students, as they progress through school. Are their needs different from those of employment and Higher Education? Can both be reconciled? And could this evidence be used to help inform education policy?

These are some of the questions we set out to answer at the beginning of this project. This short summary document provides an overview of two reports produced as part of this project: *Mathematics in the workplace and in Higher Education* and *The Mathematical Needs of Learners*. We hope that the findings from both reports will provide a solid evidence base to inform the government's education policy. In a world that is becoming increasingly reliant on Science, Technology, Engineering and Mathematical skills, we need to ensure young people are not only equipped with the requisite skills to function competently in the worlds of employment and higher education, but also that they are confident with their own mathematical knowledge and ability.

Our report highlights a range of findings and recommendations relating to mathematics which we hope the government and its agencies will take into account when deciding on the best education policy to implement not only relating to mathematics education, but to a whole range of other subjects, many of which are highly dependent on mathematics in various ways. Communication and concerted action between different stakeholders such as government, schools and colleges, employers, universities and learned societies will be crucial if we are to ensure that we meet our mathematical needs in the future.

4. S. Thogens

Professor Dame Julia Higgins FRS FREng, Chair of ACME



ABOUT ACME

The Advisory Committee on Mathematics Education (ACME www.acme-uk.org) is an independent committee, based at the Royal Society and operating under its auspices, that aims to influence Government strategy and policies with a view to improving the outcomes of mathematics teaching and learning in England and so secure a mathematically enabled population.

BACKGROUND

Mathematics is not only an important discipline in itself but it also underpins a very broad range of subjects and disciplines, from the obvious such as science and engineering to the less obvious including social sciences and nursing. In a world that relies more and more on science and technology, in all facets of employment and life in general, it is becoming increasingly important to be confident with and to be able to use mathematics in a wide variety of applications and develop and interpret quantitative information. Yet, young people in England are often unmotivated by the subject, employers have problems recruiting people with sufficient mathematical skills and universities find they often have to offer remedial courses in mathematics. It is against this background that ACME initiated this project, trying to qualify and quantify the expected mathematical needs of the nation.

This project therefore examines how both national and individual mathematical needs of 5–19 learners can best be met by a suitable modern curriculum and delivery policy and a corresponding implementation framework. Our aim is to make evidence on mathematical needs available in a form which will provide an agreed basis for future decision making, leading to a more appropriate provision from the start of school through to employment and higher education. The project looks at mathematical needs both from a 'top down', ie the mathematical needs of Higher Education and employment, and from a 'bottom up' perspective, ie the mathematical needs of the learners themselves. The 'top down' and 'bottom up' approaches were researched independently and their findings are outlined in two separate reports: *Mathematics in the workplace and in Higher Education and The Mathematical Needs of Learners*.

Mathematics in the workplace and in Higher Education

The first part of the project looks at the mathematical needs of Higher Education and employment. Supported by the Nuffield Foundation and the Clothworkers Foundation, this report aims to describe and quantify the mathematics skills and knowledge that employers across sectors and higher education need and expect from young people coming out of education. The research for this report was informed by interviews with a wide range of people, both in universities and across all sectors of employment, as well as a review of existing literature and data sources.

The Mathematical Needs of Learners

This part of the project looks at the mathematical needs of the learners themselves. It aims to identify what it is that learners need in order to be successful and proficient in mathematics, to learn mathematics well, and to engage in mathematics lessons. The report outlines the educational environment pupils and students need in order to better learn, enjoy and become more confident in mathematics. This report was informed by a review of existing literature and by a series of workshops, seminars and meetings with a broad range of stakeholders in mathematics and mathematics education, who contributed with their views and input of what young people need when learning mathematics.



SUMMARY OF KEY MESSAGES FROM THE PROJECT

 This is a summary report of the key messages that came out of this two-year long project. Key messages and findings were highlighted in the course of the project and ACME believes that these should become an important part of the evidence on which policy decisions on mathematics education are taken. In particular, the findings should impact on the ongoing National Curriculum review and on the development of a coherent suite of post-16 pathways.

Greater depth of information on the issues described here, as well as a detailed analysis and further recommendations, can be found in the main reports.

- 2. While this report has looked at mathematical 'needs' in a utilitarian sense, understanding of mathematics depends on an engagement with abstract structures and underlying concepts; without this, young people are neither able to apply their knowledge in new situations nor have the confidence to function mathematically. Furthermore, they will not develop the intellectual satisfaction that such an understanding engenders.
- 3. Both aspects of the project confirm that simply focusing teaching on utilitarian aims rather than the underlying concepts and abstract structures will not achieve even these limited aims. To do so produces students who may learn one or two recipes but who will not be able to transfer this knowledge to progress in mathematics or apply it in unfamiliar ways. It also deprives them of the full opportunity for intellectual development which mathematics can provide and reduces the chance that they will appreciate and enjoy mathematics and wish to take it further.
- 4. One of the main messages to come out of the two reports is simple. We need more young people to know more mathematics and to be confident, robust and fluent in their use of it. Not only are university courses in many disciplines increasingly quantitative in content, but there is also a steady shift in the employment market away from manual and low skill jobs and toward those requiring higher levels of management expertise and problem-solving skills, many of which are mathematical in nature. In addition, we estimate that of those entering Higher Education in any year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 will have done so.
- 5. The research highlighted concerns that the current high stakes assessment system, where institutions are more accountable for results than for the mathematical understanding of their students, has a detrimental effect on the ability of young people to apply mathematics. This creates problems both in the workplace and in higher education, as young people are unable to use the mathematics they have learned at school or college. There is a concern that some areas of mathematics which are more difficult to assess, such as problem-solving, reasoning and communication, are not given sufficient teaching time and are often replaced in the

classroom by teaching routines and procedures necessary to pass the test. Mathematics is a highly interconnected subject that involves understanding and reasoning about concepts, and the relationships between them. Young people need time and multiple experiences in order to develop the mathematical confidence to tackle unfamiliar tasks.

- 6. Students have to learn to use methods of reasoning, so it is impossible to talk about learners' needs without talking about mathematically-appropriate pedagogy and the environment for learning. Good mathematics learning needs knowledgeable teachers, who can draw on students' understanding, involve them in discussion, and engage all students in a variety of complex tasks in which mathematics is presented as a subject with many aspects.
- 7. Both reports highlight that not only do young people need to be able to use and apply their mathematics, but they also need to be able to communicate mathematically. Learners need to read, talk and interpret mathematical text in order to engage successfully in mathematical lessons. Similarly, employers and universities need young people to be able to communicate the results of the applications of mathematics effectively to the appropriate audience.
- 8. Changes in commonplace technology also affect the kinds of mathematical questions that can be asked and answered, and the way that mathematics is used in the workplace. Learners need to understand ideas and problems that earlier generations could not even have formulated, and to become adept at answering them by using, and developing, 21st century tools. For example, computer software is used throughout the workplace across all sectors; often this involves working with mathematical models, which people need to be able to understand, interpret, interrogate and use advantageously.
- 9. One interesting issue highlighted by employers is that to use mathematics confidently at one level, experience of it at a higher level is required. However, a common concern is that the demands of 'performance tables' may be forcing schools to take low risk options and discourage students from taking higher levels of mathematics – either at GCSE or at A-level. In addition, the ability of students to make good choices about the mathematics they might study can be compromised by a lack of information from universities about the mathematics content of their courses in all disciplines. Teachers and careers officers need to know about the mathematical needs of employers and what is desirable on courses in Higher Education.
- 10. Another common concern is transition at all levels. Universities feel they need to run catch up programmes, as do 6th forms, secondary schools and middle schools. This arises in part from the perceived need for short-term learning in order to meet high-stakes assessment goals. There is a need for clear strategic leadership applied to mathematics from early years through to university or employment.



ACTIONS

The evidence gathered in both reports and summarized here suggests that the school (and college) education system is not providing anywhere near sufficient people with the mathematical skills that the nation needs. The discrepancy between the number of students going on to do mathematics post-16 and the mathematical knowledge required in a variety of university courses is telling. The wide range of mathematical skills used in all sectors of employment nowadays is set to continue to increase as the world becomes more reliant on technology. Both reports find that students coming out of schools are poorly equipped to deal with the mathematical demands of work and employment.

Changes in the mathematics education landscape are required to ensure that the mathematical needs of the learners as well as those of the end users are fulfilled. ACME has identified 15 recommendations to ensure that those changes happen. This includes some recommendations for the ongoing National Curriculum review, which should also apply to any further review of the curriculum.

For policy-makers:

- Policy-makers should ensure that the less easy-to-test aspects of mathematical proficiency are not reduced to procedures in highstakes assessments. The assessment regime should be revised to incorporate all aspects of mathematical knowledge and should encourage proficiency instead of short-term teaching to the test, which hinders understanding. (Recommendation 2 – *The Mathematical Needs of Learners*)
- A wider mathematical curriculum and provision than exists at present should be developed in order to ensure that all young people are well placed to benefit from their studies in mathematics. (Recommendation 2 – *Mathematics in the workplace and in Higher Education*)
- Subject-specific values, knowledge and methods of enquiry (including reasoning and application) should be upheld throughout the curriculum, assessment methods and materials, and teaching methods and resources. (Recommendation 7 – *The Mathematical Needs of Learners*)
- There should be an emphasis on building students' confidence, and their ability to use mathematics, in a range of familiar and unfamiliar contexts. (Recommendation 10 – Mathematics in the workplace and in Higher Education)

For experts and policy-makers working on the National Curriculum review

- Every review of the National Curriculum should take into account that doing mathematics involves a wide range of components and that learners need all components. These components include (Recommendation 1 – *The Mathematical Needs of Learners*):
 - facts, methods, conventions and theorems.
 - mathematical concepts and structures.
 - connections between concepts.
 - notations, models and representations of situations within and outside mathematics.
 - symbols that are defined by formal rules of combination.
 - numerical, spatial, algebraic and logical reasoning within and outside mathematics.
 - mathematical ideas and contextual problems and applications.
 - deductions from axioms, hypotheses, generalizations and proofs.
 - generalizations from mathematical results and abstract higherorder concepts.

- 6. The value of being able to communicate mathematics should be given more prominence as this is an essential skill in employment and in HE. (Recommendation 9 *Mathematics in the workplace and in Higher Education*)
- Teaching should be done by knowledgeable teachers and presented as a conceptually coherent and cognitive progression of ideas that enables learners to develop all aspects of mathematical proficiency. This implies a curriculum review cycle that is long enough to develop a coherent and informed package of assessment, textbooks and teacher knowledge. (Recommendation 13 – The Mathematical Needs of Learners)
- The 2011 National Curriculum review should ensure that in the new curriculum students are familiarized from an early stage of their education with the concept of a mathematical model. These should be drawn from a wide range of areas. Students also need to experience the power of computer software when applied to problems that can be formulated mathematically. (Recommendations 4 and 5 – Mathematics in the workplace and in Higher Education)

For policy-makers and government departments responsible for post-16 education

- Additional courses need to be developed for the post-16 cohort, so as to extend the current provision to cover the full range of students both in terms of their career aspirations and also their prior attainment in mathematics. The major elements in such new courses should include statistics, problem-solving and working with mathematical models. Sufficient time also needs to be allocated for study and assimilation of fundamental concepts. (Recommendation 7 – Mathematics in the workplace and in Higher Education)
- The new post-16 courses should include statistics beyond the descriptive methods of GCSE to meet the needs of the large number of students who progress to a range of courses such as those in social and life sciences. (Recommendation 8 – Mathematics in the workplace and in Higher Education)

For teacher training and Continuing Professional Development (CPD) providers

- All teachers of mathematics should be entitled to subject-specific CPD. In particular, incentives and funding should be found for non-specialists to undertake subject courses at an appropriate level. Such courses should focus on key mathematical ideas, the latest research on teaching and learning, and the nature of mathematics. (Recommendation 6 – *The Mathematical Needs of Learners*)
- 12. School leaders should be informed about the subject-specific needs of learners and the implications for teaching. (Recommendation 5 *The Mathematical Needs of Learners*)
- 13. Teachers should be provided with information about the wider uses and value of particular mathematical ideas. Teachers need to know about the mathematical needs of employers and what is desirable on courses in HE; they must be encouraged to include non-routine and unfamiliar situations, and opportunities for reasoning, in their teaching. (Recommendation 16 – Mathematics in the workplace and in Higher Education)

For Universities

- 14. Universities should make clear the level and extent of mathematics encountered within each of their degree programmes.
 (Recommendation 14 Mathematics in the workplace and in Higher Education)
- 15. Universities should state explicitly where they have preferences for qualifications in mathematics. (Recommendation 15 *Mathematics in the workplace and in Higher Education*)

IN CONCLUSION:

The recommendations set out in this report will require significant changes, not only in terms of curriculum, qualifications and assessment but also in terms of teaching and learning. The Government and its agencies and associated nongovernmental organizations, industries, universities and schools and colleges will need to collaborate in order to arrive at a situation where young people receive a mathematics education that not only gives them a solid knowledge base of mathematics, but also the ability and confidence to use that knowledge to progress and thrive in employment and/or higher education, thus enabling the nation as a whole to fulfill its mathematical needs.

For further information about the Advisory Committee on Mathematics Education:

ACME

The Royal Society 6–9 Carlton House Terrace London SW1Y 5AG Tel +44 (0)20 7451 2574 Fax +44 (0)20 7451 2693 Email acme@royalsociety.org Web acme-uk.org

ISBN No: 978-0-85403-906-7





JINC Joint Mathematical Council Supported by the Department for Education, the Royal Society, the Wellcome Trust and a range of other organizations across the STEM landscape.