Using manipulatives in the foundations of arithmetic:

Making Numbers

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What are manipulatives?

Objects that can be handled and moved and are used to develop learners' understanding of a mathematical situation

- everyday, 'home-made' or commercial
- not virtual manipulatives

The project

Focus on children aged 3 – 9 years (but expecting results to be useful for 3 to 12).

- How were teachers using manipulatives in the teaching of arithmetic?
- What guidance would be useful to improve this?

What were teachers doing?

Survey in 3 geographical areas in England (questionnaire with convenience sample)

- 35 primary schools; more than 450 teachers
- ‘What manipulatives have the children used in the last 4 weeks for work on number?’

Questionnaire results

	Low attainers	**Middle attainers**	**High attainers**	***Number of classes***
3-5 years	98	98	95	*87*
5-7 years	98	95	87	*152*
7-9 years	93	80	52	*114*
9-11 years	92	70	45	*131*
TOTAL	**94**	**85**	**68**	***484***

Results are given as percentages

Counters and interlocking cubes most common

Next steps...

Teachers said they lacked knowledge of

- how to use manipulatives
- the value of manipulatives for all learners

We used the questionnaires, interviews with focus groups, and workshops with an international group to identify priorities.

Focus on number sense not just standard methods

- number symbols, vocabulary and meaning
- systematic counting, ordinality and cardinality
- comparisons between different magnitudes
- different representations of number
- simple arithmetical operations
- number patterns

Back, Andrews & Sayers 2013

Number sense

Counting

Comparison

Composition

Task design

- Activities developed on the basis of research evidence, our combined teaching experience and observations of children
- Iterative trials with groups of children and teachers

Counting

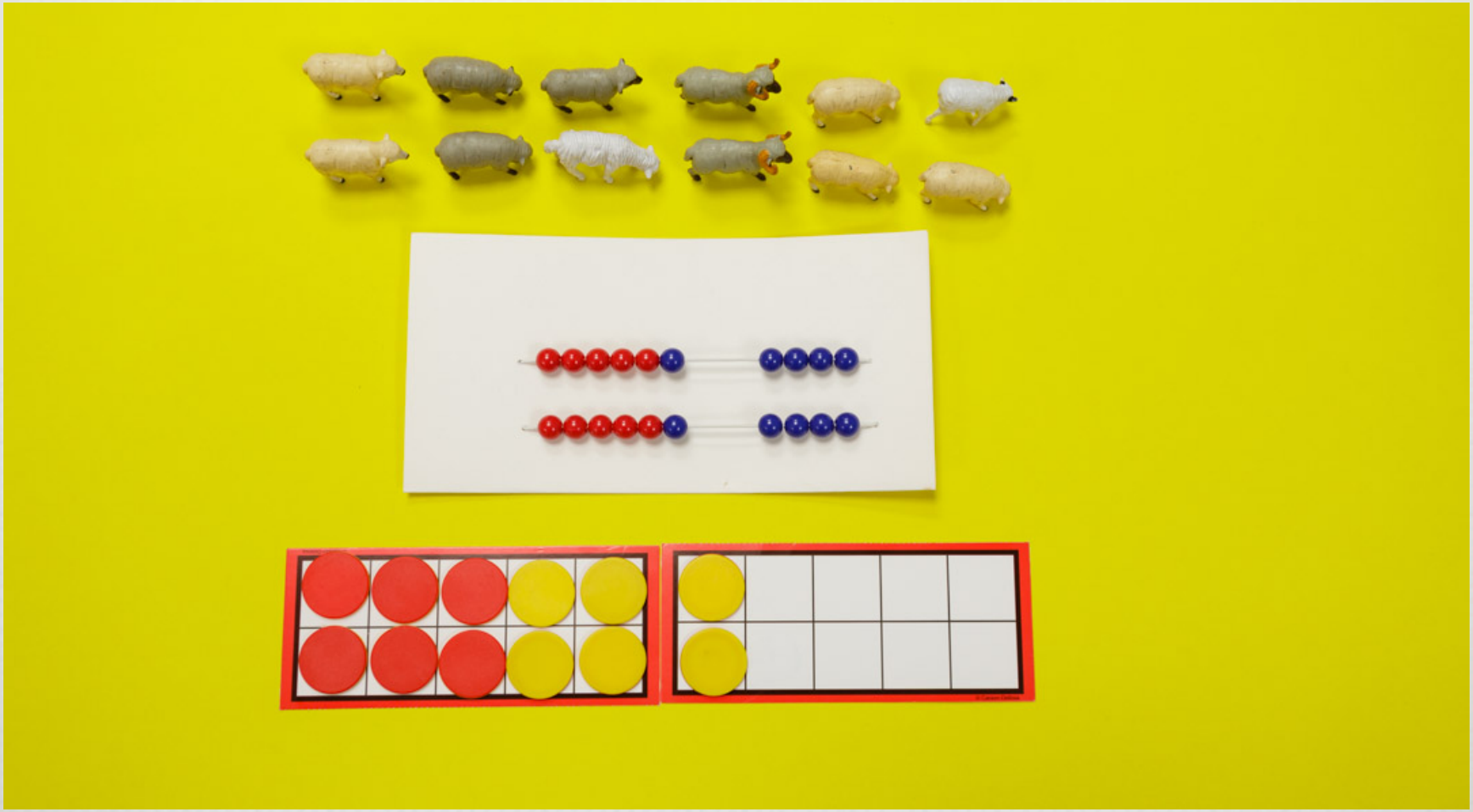
Counting large numbers

Comparison:
Who has the most?

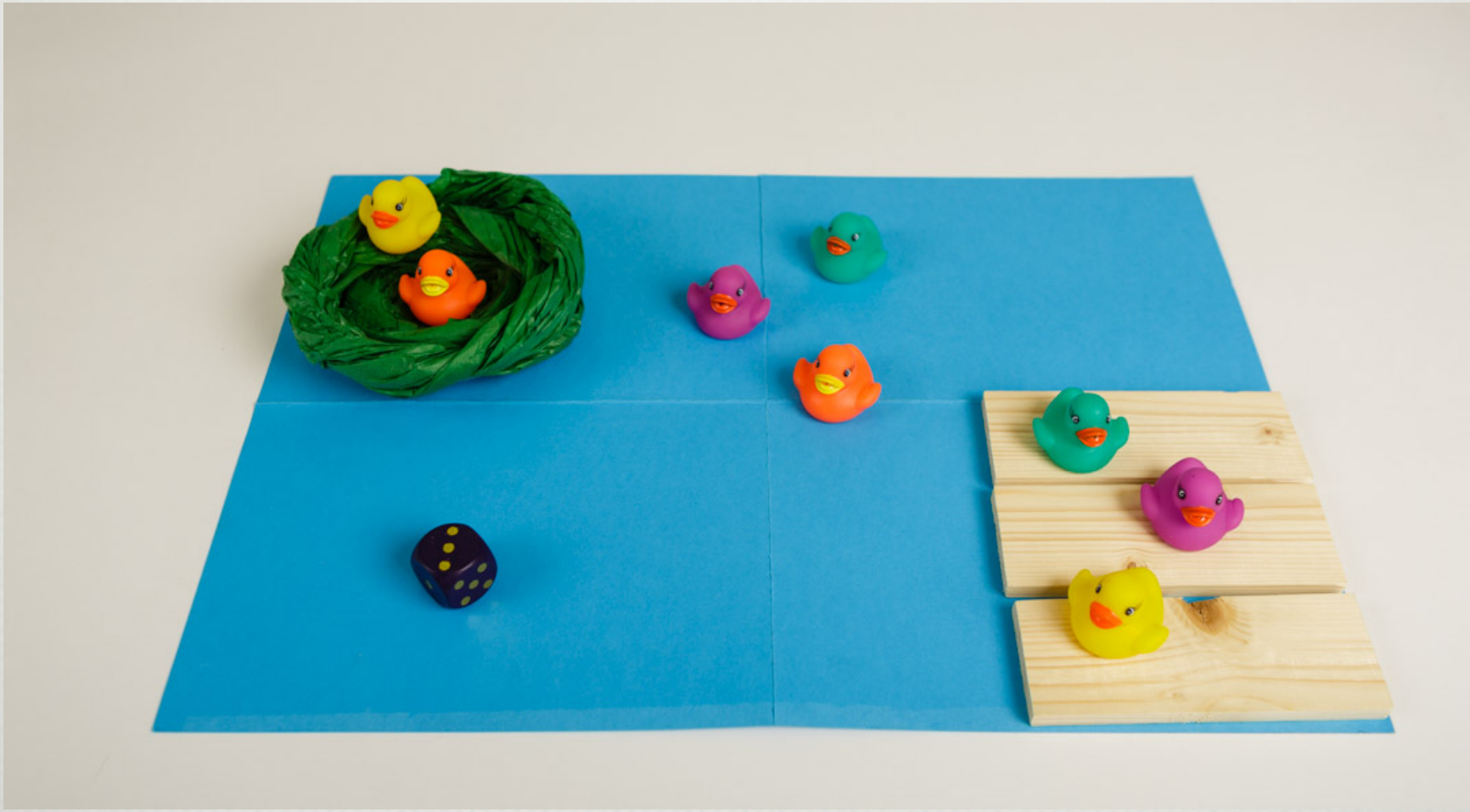
Comparison: how many in a handful?

- Estimate
- Count and label
- Put in order

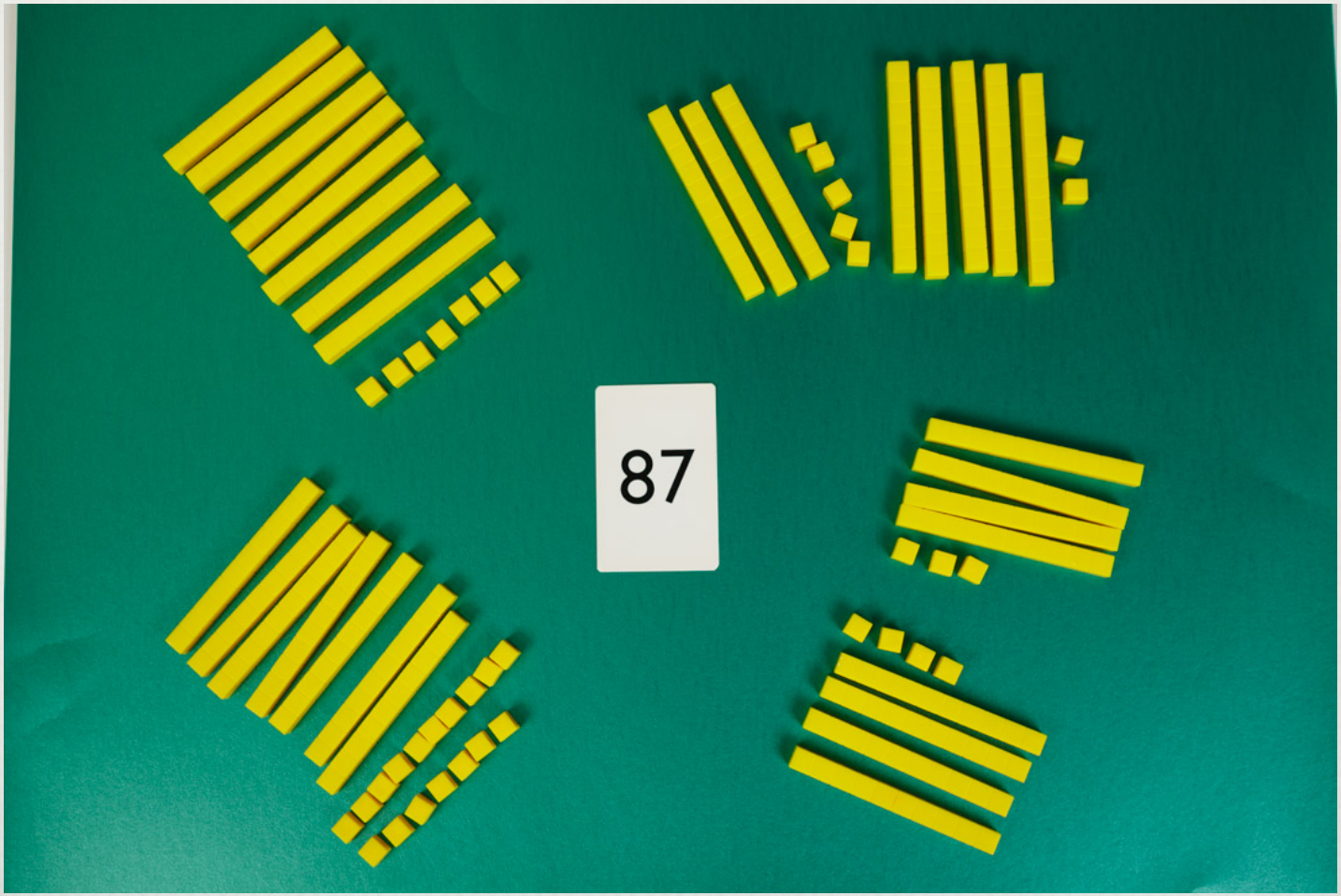
Composition



Composition: Eight ducks in the lake



Ways of making 87



Power of manipulatives

- Give children and teachers tools to explore and explain mathematics – supporting understanding
- Encourage visualisation – supporting shift to abstract reasoning and the use of symbols
- Make children's thinking and understanding more transparent
- Increase children's engagement and enjoyment

Issues from the Literature

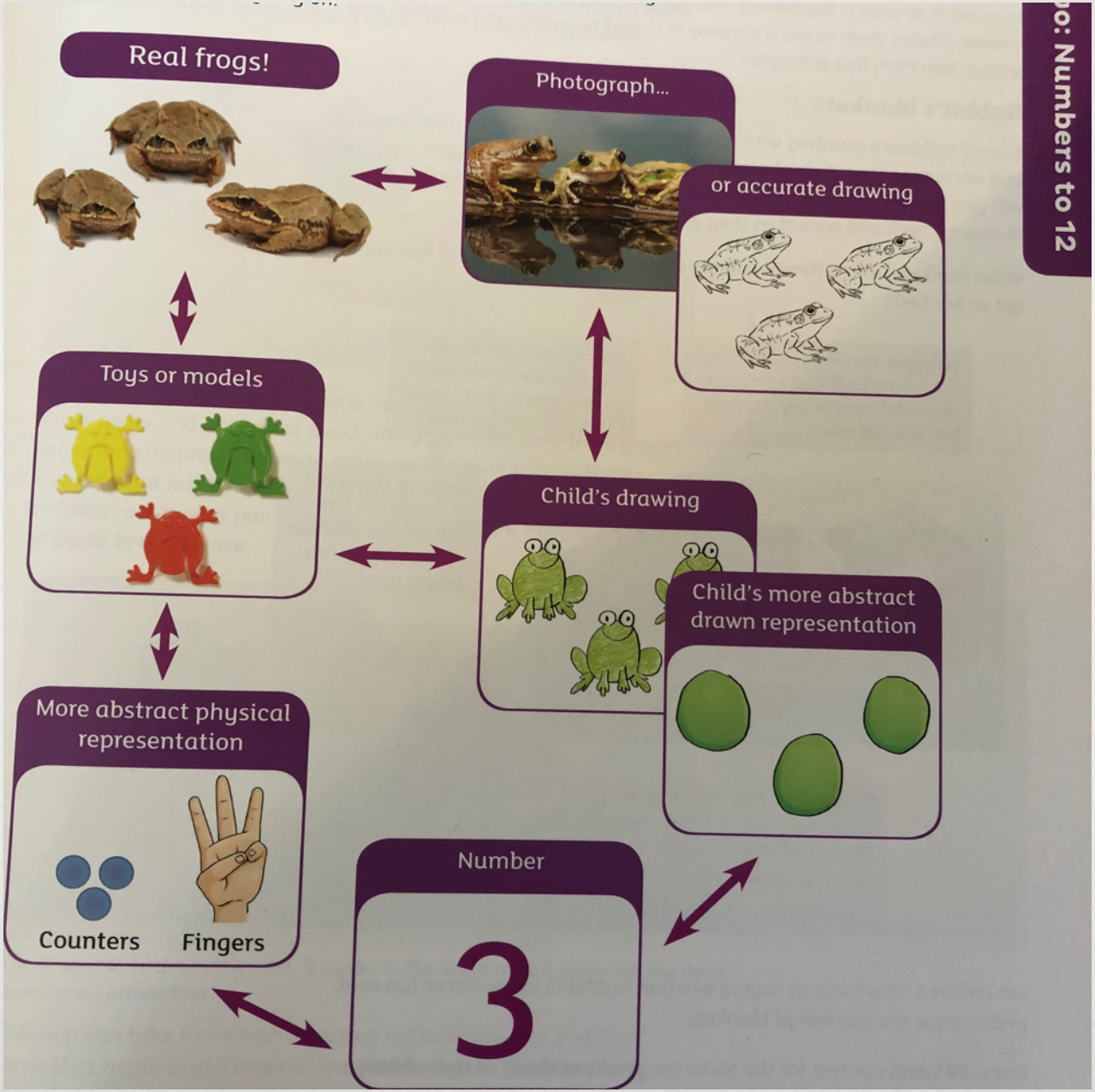
- How does the manipulative represent the mathematics?
- Are some representations too opaque or complex for some children?
- How does children's understanding move from concrete to abstract relationships?

CPA (Concrete, Pictorial, Abstract)

- eg cubes - number line – equation

- **Bruner:** *not a linear progression*
  - enactive: muscle memory
  - iconic: mental imagery
  - symbolic: language & signs

- **Mason:** *different worlds of experience*
  - manipulating, specialising, exemplifying
  - getting a sense of, diagramatising to capture relationships
  - articulating generalisations, formal expressions, reification


Real frogs!
Photograph...
or accurate drawing
Toys or models
Child's drawing
Child's more abstract drawn representation
More abstract physical representation
Counters
Fingers
Number
3

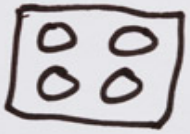
Do, talk..

..and record

How many packs of 4 yoghurts are needed for 135 children?

it's the 4x tables 4, 8, 12

30 x 4 = 120

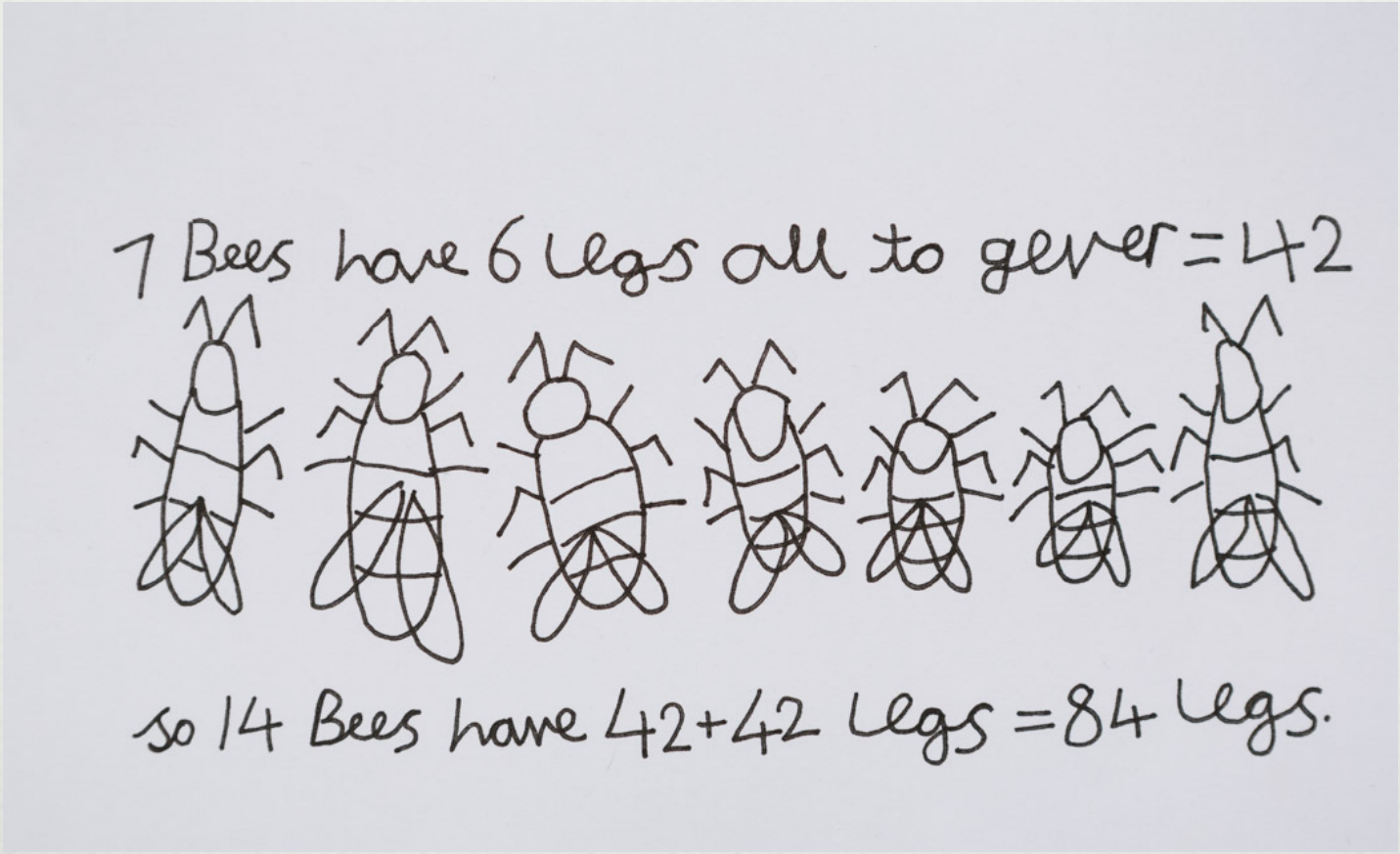
120 +  +  = 128

128 +  = 132

extra 3 left

33 packs

What else do you know?



Pedagogical implications

- Check children's prior knowledge
- Choose appropriate manipulatives for the maths
- Give opportunities for familiarisation
- Provide varied and open-ended challenges
- Develop maths discussion for reasoning and generalizing
- Imagine the materials - visualisation
- Record informally and symbolically
- Maintain an inclusive learning community

And so...

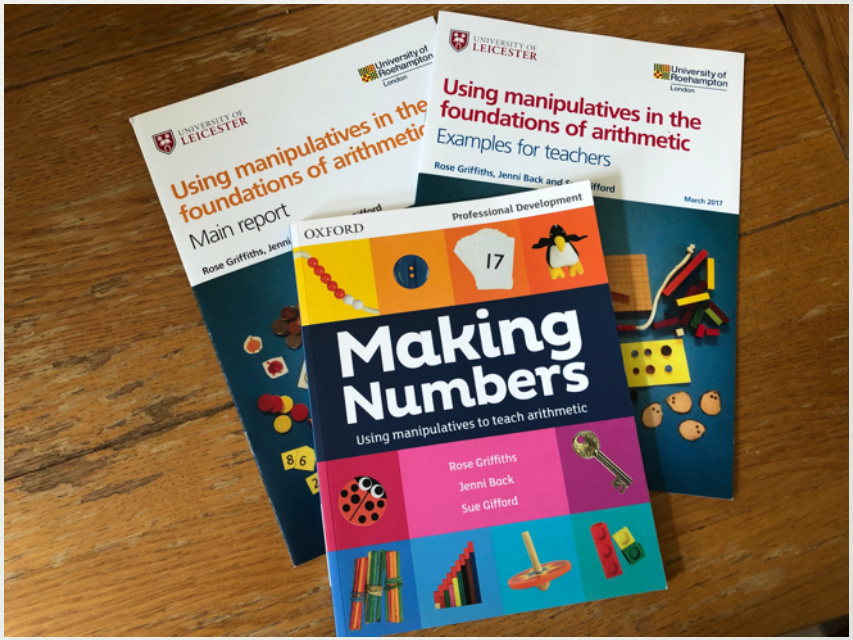
Children do not learn from manipulatives. They learn from the activities which they do with manipulatives.

(Sfard interviewing Skemp 1990)

Outputs

Nuffield website:

- Main report
- Literature review
- Examples for teachers



Oxford University Press:

- *Making Numbers*
- Stop frame animations
- Talking heads videos with demonstrations

Animations

- *Five friends counting*
- *Ten fishes in the sea*
- *Half a hundred hedgehogs*
- *Ten tens and a dragon*

Available free of charge on *Oxford Owl*: *(* go to *Professional Development,* then *PD books)*

https://www.oxfordowl.co.uk/for-school/default

Further questions

- Has our guidance made a difference? Which aspects influenced teachers' activity in the classroom? Which aspects most improved children's learning?
- What characterises teachers' use of manipulatives which enhances understanding for all children?
- What other areas of mathematics teaching need better use of practical activity?
- What else could benefit from research-informed task design?

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