Coastal erosion B: Integrating area

Activity description

Students use integration to approximate the area of a piece of land with an irregular coastline, and the area lost to coastal erosion over time.

Significant material is available online on coastal erosion to run more open-ended contextual variants touching on other aspects of modelling and calculus.

Suitability

Advanced (Level 3)

Time

1-3 hours

Resources

Student information sheet and worksheet
Optional: Slideshow, Excel spreadsheet

Equipment

Essential: either a graphic calculator or a computer with Excel
Optional: internet access

Key mathematical language

Estimating, area, area under a curve, modelling, integration, offset, rate, coordinates, limits, base line, cubic, quadratic, function

Notes on the activity

This activity uses integration to estimate the area under a curve in the context of coastal erosion. It is assumed that students are familiar with the formula for integrating a polynomial and also with evaluating its value for a given range.

The activity can be presented using the slideshow and related material on the internet. Students could use the accompanying spreadsheet to find functions to model the data.

Additionally, *Coastal erosion A: Estimating areas*, has the same example and exercises, but uses the Trapezium Rule to estimate areas, rather than integration. This is a natural precursor to the current activity and may be carried out alongside it.
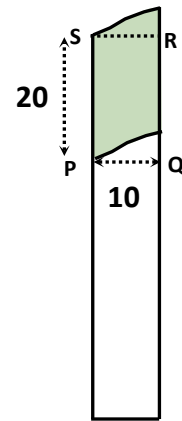
During the activity

Students should recognise that they do not have the data for the full coastline and hence any function they use will only be an approximation.

They also need to appreciate that they can subtract the later coastline function from the initial coastline function, and use this to evaluate the land lost due to erosion.

Points for discussion

The sketch shows the part of the graph between the first two offsets when a consistent loss of 20 m is assumed. The shaded section is the predicted loss of land. It can be seen that this is equal in area to the rectangle PQRS i.e. 10 x 20 = 200 m^2

Extending this idea to other sections of the graph gives the result that the predicted loss of land is equal to the length of the baseline multiplied by the expected reduction in the length of the offsets (see page 3 of the student sheets and the notes below).

Extensions

Coastal erosion A: Estimating area uses the same example and exercise, but the answers are evaluated using the Trapezium Rule. Each activity can be used by itself or the two activities can be combined, allowing comparisons to be made between the two methods. If you wish to use the two activities together, you will also need to provide each student with pages 2 and 3 from *Coastal erosion A: Estimating area*.

The Coastal Explorer tool www.coastalexplorer.eastriding.gov.uk provided by the East Riding local authority allows students to see various positions of the coastline over time, draw baselines and offsets, and also provides estimates of rate of land loss along a fixed set of points. Students could use these to draw initial and predicted coastlines, and then approximate the area lost to coastal erosion.

The British Geological Society www.bgs.ac.uk/ has a wealth of information on coastal erosion, including description of the use of modern laser-based technology (Terrestrial LiDAR Survey Techniques) that enables measurement of not just planar area but the calculation of volume and the recession rates of cliff faces.

A number of related links and further contextual information on the erosion of the Holderness Coast is available via the University of Hull www.hull.ac.uk/erosion. Teachers could also find examples for other coastal areas via the www.bgs.ac.uk/ website.

Answers

1 Quadratic model of initial position: $y = 29.71 + 0.8366x - 0.007529x^2$

a 4644 = 4600 m^2 (to 2 sf)

b 1800 m^2

Cubic model of initial position: $y = 0.00007129x^3 - 0.01822x^2 + 1.244x + 27.14$

a 4643 = 4600 m^2 (to 2 sf)

b 1800 m^2

2 Cubic model of initial position: $y = 61.83 - 1.026x + 0.01604x^2 - 0.00005863x^3$

Cubic model of position after 12 years: $y = 43.92 - 1.056x + 0.01623x^2 - 0.00005863x^3$

Loss of area $\approx$ 3678 = 3700 m^2 (to 2 sf)

3 a Either a quadratic or cubic model could be used.

b Quadratic model of initial position: $y = 0.0003619x^2 - 0.2476x + 75$

Quadratic model of position after 20 years: $y = 0.0001643x^2 - 0.1258x + 38.96$

giving:

Loss of area $\approx$ 7982 = 8000 m^2 (to 2 sf)

Cubic model of initial position: $y = 75.64 - 0.2810x + 0.0006165x^2 - 0.0000004848x^3$

Cubic model of position after 20 years: $y = 38.98 - 0.1272x + 0.0001749x^2 - 0.0000000202x^3$

Loss of area $\approx$ 7979 = 8000 m^2 (to 2 sf)