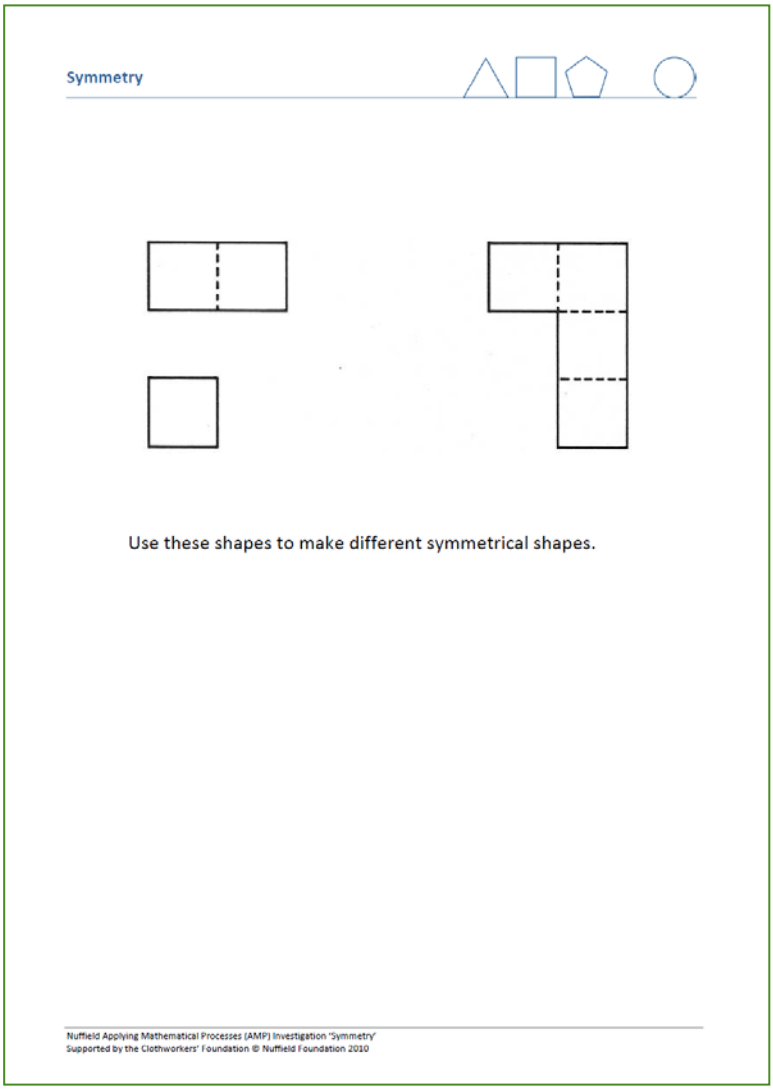
Activity description

Pupils make different symmetrical shapes, using one or more of three given shapes.



Suitability

Pupils working in pairs or small groups

Time

Up to 2 hours

AMP resources

Pupil stimulus, PDF interactive

Equipment

Squared paper
Templates of the shapes
Mirror
Tracing paper
Scissors
Glue
Computer software

Key mathematical language

Lines of symmetry, rotational symmetry

Key processes

Representing Identifying the mathematical aspects of the problem, deciding on constraints and freedoms.

Analysing Working logically, identifying and classifying patterns.

Interpreting and evaluating Making general statements based on findings.

Communicating and reflecting Communicating effectively, discussing results.

Teacher guidance

The activity can be introduced using an interactive whiteboard. Show the three shapes, and then ask pupils to work in small groups to decide how the shapes could be placed to make a symmetric design. Although the shapes could be joined in any way, joining them edge to edge, with the sides overlapping by full unit lengths, will help to prevent the task becoming overly complicated.

Invite a pupil to show their design on the whiteboard; ask them to show the line(s) of symmetry and/or centre of rotational symmetry.

Pupils are likely to query the 'rules' of this investigation, such as: 'Must all shapes be used?' and 'Can a shape be used more than once?' 'Must shapes be joined edge to edge?' Encourage them to decide on their own rules, but remind them to write their decisions down, such as 'I have decided that ….'

Tell pupils that they will need to think carefully about how to classify (group) their designs so that someone else can see what is the same and what is different about the designs. Communicating how they have chosen to classify their designs and why, and how complete a listing they have for their rule(s) is an essential part of the exercise.

Pupils need to be clear about when they will treat two designs as being equivalent, such as up to rotation, or whether they will allow reflection.

During the activity

Ideally, provide templates of the three shapes to support pupils in generating results. Multiple copies of the shapes can allow for them to be glued down.

Pupils may benefit from using mirrors to check line symmetry, and/or using tracing paper to check rotational symmetry.

Remind pupils to write down their rules; for instance 'I have decided that my designs must include one copy of all three shapes'.

Encourage them to change their rules to extend the activity, and remind them of the need to sort their designs into sets.

Probing questions and feedback

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- Tell me about the symmetry in this design. Does it have rotational symmetry? Why? Why not?
- What rule are you using? What could you change? What could you keep the same?
- Have you generated all possible designs (for the rule currently being used)? How do you know? Convince me that there are no others.
- Is there any relationship between the number of lines of symmetry in a design and the order of its rotational symmetry?
- Where there is more than one line of symmetry, is there a connection with the rotational symmetry of the design?
- Show me a design that has only one line of symmetry.
  Or two …. or four.
  Is it possible to create a design that has three lines of symmetry, or … ? Why? Why not?
- How many designs do you think there might be in total? Why?

Extensions

- Pupils describing their designs and putting them into a database.
- Choosing or inventing a different set of shapes to start with.
- Investigating the symmetry of pentominoes or other polyminoes.
- Using multilink cubes for a related investigation in 3 dimensions.

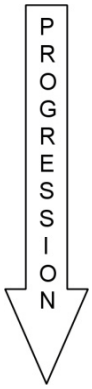
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

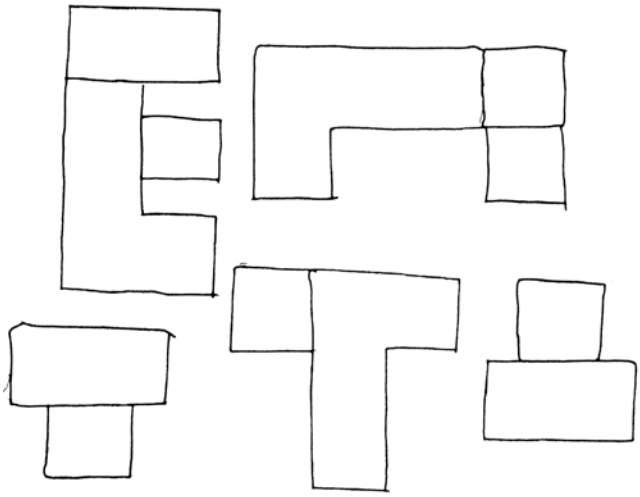
The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

Representing *Decides constraints and freedoms*	**Analysing** *Identifies and classifies patterns*	**Interpreting and evaluating** *Makes general statements*	**Communicating and reflecting** *Communicates and discusses results*
Selects and uses a simple rule, e.g. decides to use all 3 shapes once only Pupils B, C, D	Creates simple symmetric designs Pupils A, D	Recognises or avoids duplicates Pupil B	Draws lines of symmetry or describes where they are Pupil B
Extends or refines the rule to extend their investigation Pupil E	Creates designs that have more than one line of symmetry and/or recognises rotational symmetry, even if expressed informally Pupil B	Starts to interpret possible relationships between reflection and rotation symmetries Pupil G	Describes both reflection and rotation symmetry for their shapes Pupils C, D, E
Works systematically to produce new designs Pupils F, G	Works confidently with both reflection and rotation symmetry Pupil C	Tests generalisations, finding counter-examples or attempting to justify findings Pupil F	States generalisation(s) and/or describes counter examples Pupils F, G
Chooses means of representing that enables efficient classification of designs they have found	Works on classifying or sorting designs Pupils E, F, G	Attempts to justify patterns emerging from the constraints, e.g. that joining shapes edge to edge means that certain outcomes are impossible	Summarises outcomes effectively**,** relating these back to the choice of rules



Sample responses

Pupil A

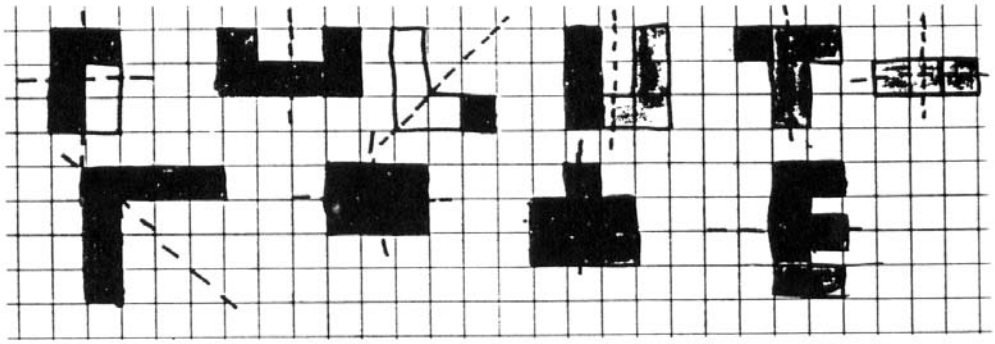


Pupil A has chosen to work on plain paper so the designs are not accurate. However, it is clear that the pupil appreciates mirror symmetry. Duplicates have not been recognised.

Probing questions and feedback

- Show me where the lines of symmetry are on your diagrams.
- Are any of your shapes the same? Why?
- As this pupil has not produced any designs with rotational symmetry, it would be worth asking the pupil what 'being symmetric' means to them.

Pupil B

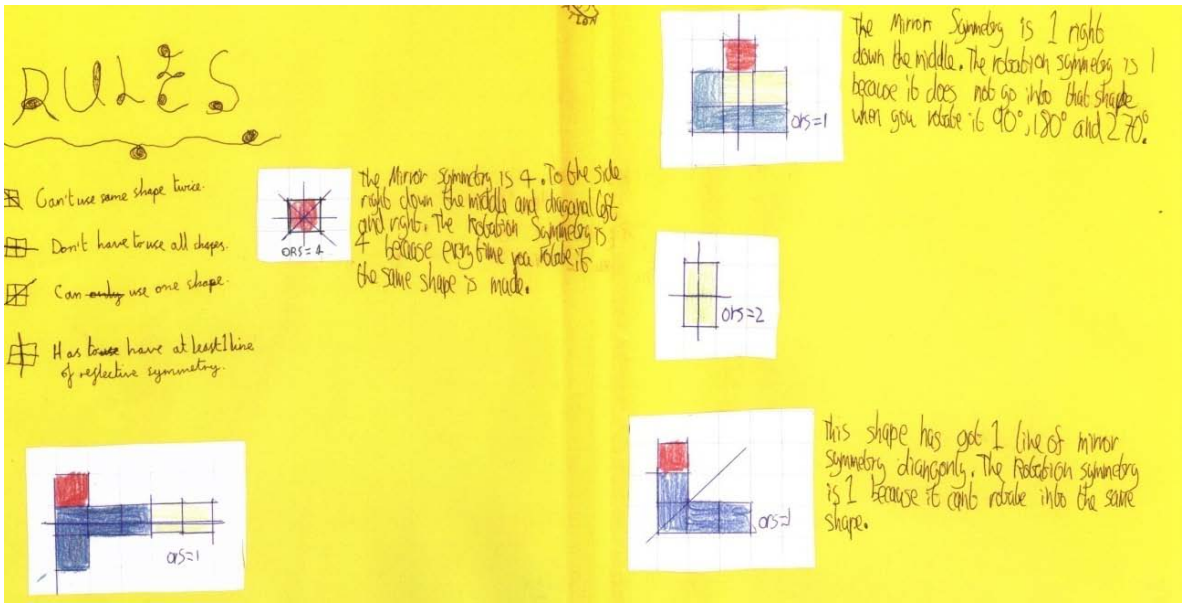


Pupil B started the investigation by choosing to combine two shapes, and then progressed to three shapes. However, there is insufficient evidence to suggest an attempt to control variables or to find all possible designs made with only two shapes. Lines of symmetry are shown clearly.

Probing questions and feedback

- Which of your designs have two lines of symmetry? Do they have anything in common? Can you create another design that has two lines of symmetry?
- Do any of your designs have rotational symmetry? What is the order of the symmetry? Can you create a design that has rotational symmetry of order 4?
- One can discuss how the pupil has chosen to approach the problem, and encourage them to describe this and their findings using words and images.

Pupil C



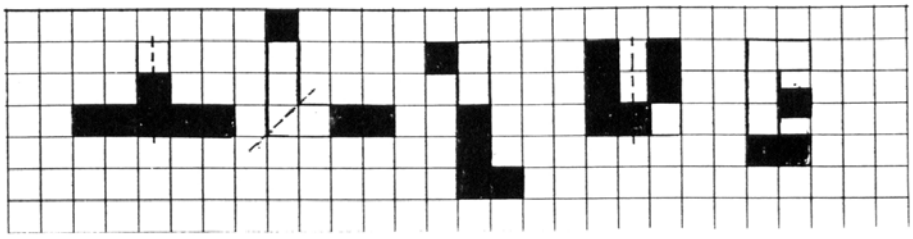
Pupil C has chosen some rules for how the shapes may be combined, and has started to produce symmetric designs that satisfy these rules. Rotation and reflection symmetry have been recorded.

Probing questions and feedback

- One can explore why the pupil has chosen that a design must have reflective symmetry, how they can ascertain that they have found all designs satisfying their rules, and what would be a next step.
- Are there any more designs that satisfy your rules?
- Can you make a design with rotational symmetry but no lines of symmetry?

Pupil D

Pupil D chose to use one copy of each of the three shapes, and the diagrams show how the shapes have been combined.



Almost all lines of symmetry have been identified clearly, and the third shape was described in discussion as having 'rotation symmetry but not reflection symmetry'.

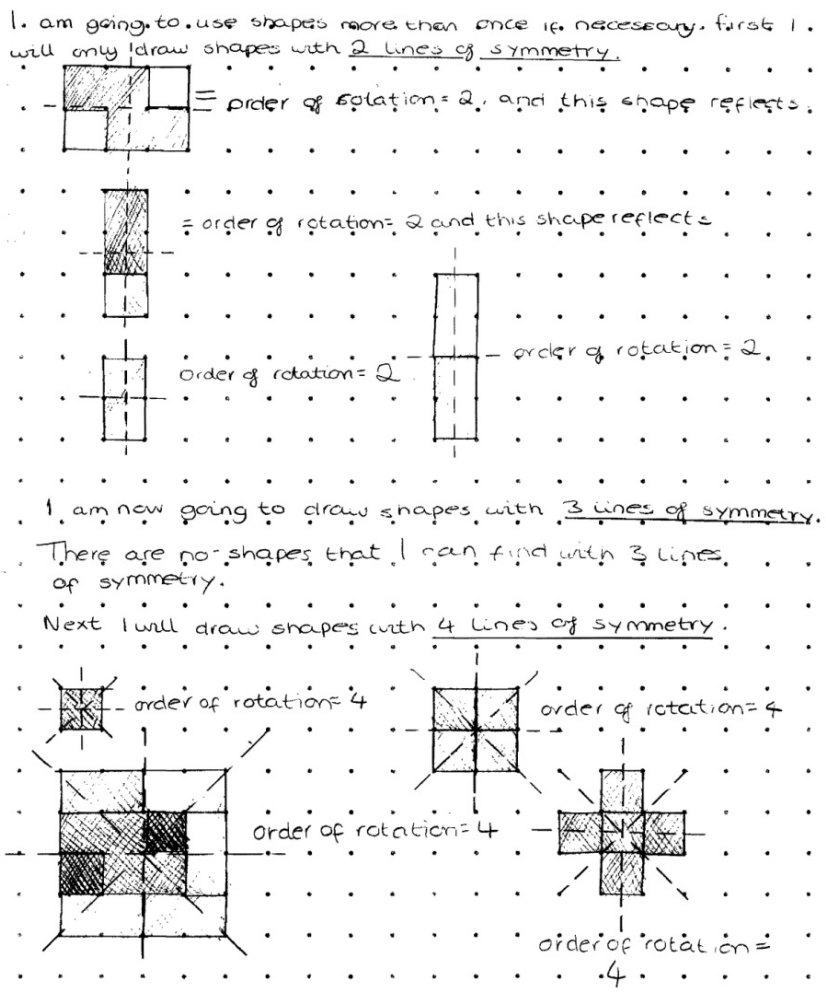
Probing questions

- What other rules could you use?
- See if you can produce a design with both rotation and reflection symmetry.
- Can you create a design that has rotation symmetry of order 4?

Pupil E

Pupil E has attempted to work systematically by trying to produce designs with two, three, or four lines of symmetry.

The pupil has noticed that there are no designs with three lines of symmetry, although they have not attempted to explain this observation.

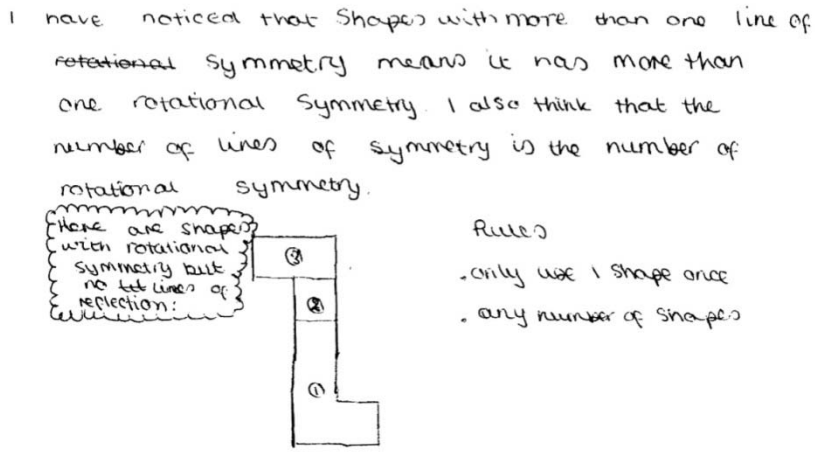


Probing questions

- Why do you think you couldn't find designs with three lines of symmetry?
- Is there a connection between the number of lines of symmetry and the order of the rotation symmetry?

Pupil F

Pupil F worked systematically to produce designs using first two of the three shapes and then all three. The pupil has made observations about the connections between lines of symmetry and rotational symmetry, and produced an example of a design that does not fit the pattern.



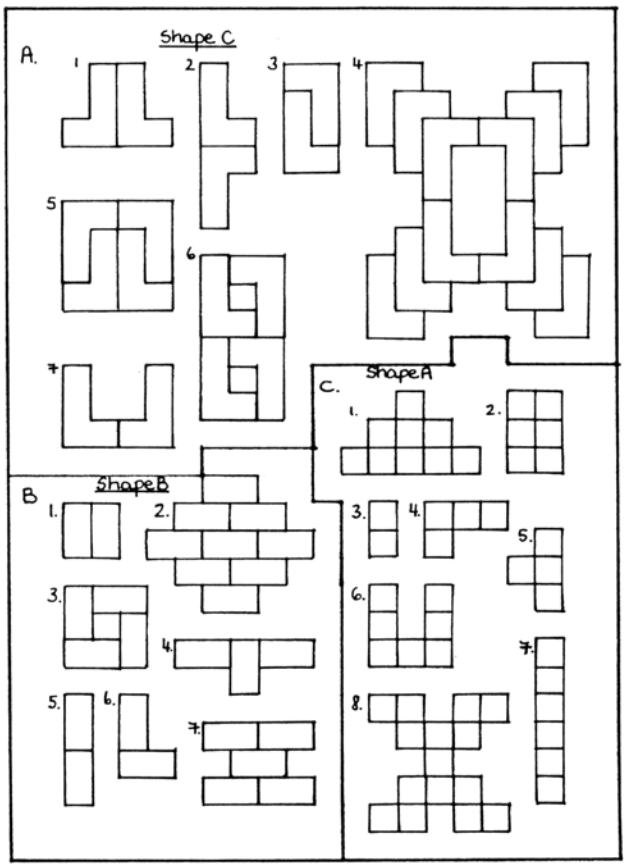
Probing questions

- Can you explain your observation about the number of lines of symmetry and the order of the rotation symmetry?
- Are there any more designs with rotation symmetry but no lines of symmetry?

Pupil G

Pupil G has used a systematic approach to generate a range of designs by using multiple copies of one shape at a time. The method for generating and ordering more complex shapes is not clear from the work. A table of results is used to record the symmetries.

The pupil explains:
'I think that the order of rotation symmetry is always the same as the number of lines of symmetry, unless it didn't have any lines in which case the order is 1 because you don't have order 0.' Example 4 for Shape A and Example 6 for Shape B are included in support of this observation.'



Shape C				Shape B				Shape A		
	Lines	Rotational			Lines	Rotational			Lines	Rotational
1	1	1		1	4	4		1	1	1
2	1	1		2	2	2		2	2	2
3	2	2		3	4	4		3	2	2
4	2	2		4	1	1		4	0	1
5	4	4		5	2	2		5	1	1
6	2	2		6	0	1		6	1	1
7	1	1		7	2	2		7	2	2
								8	2	4 ~~2~~

Probing questions and feedback

- What approach/pattern have you used for generating more complex shapes?
- The pupil would benefit with a discussion on relations between rotation and reflection symmetry. One starting point would be to have the pupil construct a shape with rotation symmetry of order 2 or greater but no reflection symmetry, and then revisit their conjecture.