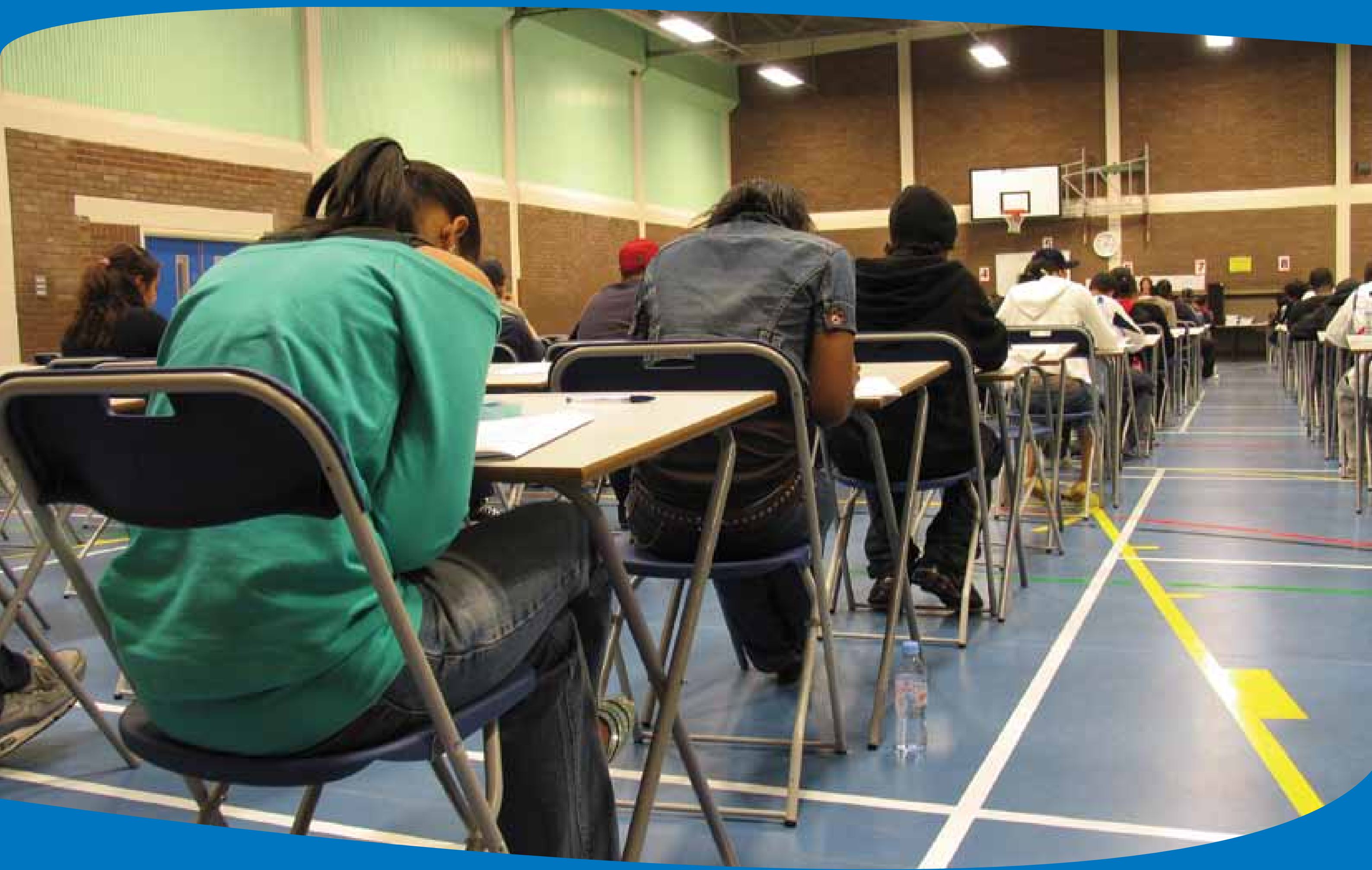
Nuffield Foundation

Mathematics in A level assessments

A report on the mathematical content of A level assessments in Business Studies, Computing, Economics, Geography, Psychology and Sociology



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Also available as an electronic publication at www.nuffieldfoundation.org

ISBN: 978-0-904956-85-6

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Foreword

In the UK fewer students engage with mathematics after the age of 16 than do their peers in other countries. This is a serious problem and the Foundation shares the Secretary of State's view that: "…within a decade the vast majority of pupils [should be] studying maths right through to the age of 18."[1]

One way of addressing this issue is to consider what kinds of free-standing mathematics provision might complement students' main courses of study. Another is to look at the mathematics embedded in those courses of study. Is it sufficient, and if not how might it be improved? That is the question that underlies this study. In a parallel exercise the SCORE (Science Community Representing Education) partnership has been looking at the mathematical content of science A levels. Our study examines six other subjects, chosen partly to reflect the Foundation's interests in the social sciences, and partly because they are taken by large numbers of students.

The study uses similar methods to the SCORE work, with teams of experts looking at examination papers in a controlled and systematic way. However, one important difference is that within the sciences there are national criteria for the mathematics that should be covered, but with most other subjects this is not explicit.

It is a common complaint from higher education that students come unprepared for the quantitative demands of their subjects, so the question of what mathematics should be required in A level teaching is important. Our hope is that this study will provide a useful platform for the subject communities, involving both the school and HE sectors, to address this issue.

A key finding of the study is that it is possible for students who are ostensibly following the same courses of study to have very different levels of engagement with the quantitative side of their subjects. This variety is driven by choices that are made at various stages; of exam board, of options, of examination questions and of ways of tackling those questions. Whether this wide diversity of experience is desirable is again something for the subject communities to discuss.

The project was led by Vinay Kathotia, the Foundation's Project Head for Mathematics and the author of this report. It would not have been possible without the collaboration of many partners including SCORE, the Awarding Organisations, subject associations and learned societies. We thank them and look forward to working with them to take this important matter forward.

Anthony Tomei

Anthony Tomei, Director

1 Michael Gove addressing the Royal Society, 29 June 2011, www.education.gov.uk/inthenews/speeches/a00191729/michael-gove-speaks-to-the-royal-society-on-maths-and-science

Introduction

This report presents the findings from a research project to map the mathematical and statistical content in the assessment of six A level subjects: Business Studies, Computing, Economics, Geography, Psychology and Sociology. Based on these findings, we make a number of recommendations.

The research consisted of analysis of the summer 2010 examination papers from AQA, CCEA, Edexcel, OCR and WJEC. The analysis was conducted by experienced teachers and examiners working in groups. Using a number of common measures, they established the *extent*, *difficulty*, and *type* of mathematics used in the assessments.

Our decision to undertake this project was reinforced by the knowledge that similar work was being done by SCORE (Science Community Representing Education)[2] to investigate the mathematical content of current Biology, Chemistry and Physics A level assessments. The aim of the SCORE project is to inform policy on the mathematics used in the criteria and assessments for science A levels, and to support its work on how the examinations system should operate to ensure science qualifications are fit for purpose.

We decided to extend the analysis by looking at the mathematical content of an additional six A level subjects, using similar, although not identical, methods. The subjects chosen are taken by large numbers of students and reflect the Foundation's interests in the social sciences. As far as quantitative approaches are concerned they make relatively high demands on students when it comes to study at higher education. The list is not exclusive and other subjects may wish to consider carrying out similar analyses. Our findings are presented separately to SCORE's, but both reports are being published at the same time.

Context

Existing evidence tells us that participation in post-16 mathematics in England, Wales and Northern Ireland is low compared to other countries[3]. Fewer than one in five students study mathematics after the age of 16. We also know that of the students studying university courses that require mathematical knowledge beyond GCSE level, around two-thirds do not have the required skills[4]. In other words, post-16 students need better mathematical skills even if they are not studying mathematics as a standalone subject.

2 SCORE is a partnership of organisations, which aims to improve science education in UK schools and colleges by supporting the development and implementation of effective education policy (www.score-education.org)

3 Hodgen J *et al* (2010) *Is the UK an outlier? An international comparison of upper secondary mathematics education,* London: Nuffield Foundation, www.nuffieldfoundation.org/uk-outlier-upper-secondary-maths-education

4 *Mathematical Needs: Mathematics in the workplace and Higher Education* (2011), London: ACME www.acme-uk.org/media/7624/acme_theme_a_final%20(2).pdf

There are two mutually supportive approaches to addressing this need. Firstly, by increasing availability and take-up of mathematics pathways outside A level mathematics, such as Free-Standing Mathematics Qualifications (FSMQs). Secondly, by understanding, and providing better support for, the mathematics used in other A level subjects. Our research aims to serve this second approach by providing evidence on the current mathematical content in the assessments of the six A level subjects selected, and making recommendations for change where appropriate.

Research approach

Using the SCORE methodology, we developed measures to map the extent, difficulty, and type of mathematics used in the assessments for each subject. For the purposes of this study, mathematics is defined as quantitative and/or logical approaches. This includes mathematics, statistics, and aspects of computing, financial literacy and problem-solving.

- To measure the *extent* of the mathematics, we looked at the proportion of the question parts that had mathematical content, and the proportion of the total marks that used mathematics. A number of questions allowed students to take quantitative or non-quantitative approaches, for example through their choice of evidence or case studies referenced. In light of this, we measured two sets of numbers: marks that **required** mathematics, and marks that **could be gained** using mathematics.

- To measure the *difficulty* of the mathematics, we identified the number of steps in a calculation, the familiarity of the context used, and the complexity of the calculation.

- To measure the *type* of mathematics, we developed lists of mathematical requirements based on the content of GCSE mathematics, and the mathematical content implicit in the various subject specifications.

We used assessments from five different Awarding Organisations: AQA, CCEA, Edexcel, OCR, and WJEC. We have anonymised these organisations in the report, using letters of the alphabet. It should be noted that the letter used for each organisation is *not* consistent across the subjects included in the study. In other words, the Awarding Organisation referred to as A for Geography is not necessarily the same as the Awarding Organisation referred to as A in reference to Business Studies.

In order to maximise the impact of our research, we were keen to report alongside the SCORE project in spring 2012. Consequently, we did not consider all potential subjects, but chose six on the basis of the spread of mathematics within them, their take-up among the student population, and the feasibility of carrying out the study in late 2011.

While it was clear that the measures developed by SCORE for Biology, Chemistry and Physics were not a perfect fit for the subjects in our study, we decided to use them (with minor adjustments) to allow for cross-subject comparison. One measure in particular, the number of calculation steps, was ill-suited for some of the exam questions in our study. There were occasions where the mathematics used was not a calculation, but rather a bringing together of mathematical viewpoints and analyses, for example when considering the impact on sales of certain management decisions (Business Studies). In such cases, our analysts, comprised of experienced teachers and examiners, used their judgement.

For a small number of essays and field-work based questions we did not use the SCORE framework, as the mathematical content varied considerably from student to student. In addition, our study included examinations from summer 2010 only. For these reasons, our findings provide a partial picture of the mathematical nature of the assessments analysed, but one we believe is substantial enough to provide a base for further analysis.

Full details of the research methodology are given on page 51.

Acknowledgements

We are grateful to the many individuals and organisations who made this project possible, particularly to those who participated in the analysis of the assessments. While we cannot name them all here, we extend our thanks to each of them for their time and expertise.

We are also grateful to SCORE for sharing their approach and collaborating with us on the coordinated release of our respective reports.

Organisations that provided materials and/or put us in touch with experts include AQA, CCEA, Edexcel, OCR, WJEC, the Association of Teachers of Psychology, the British Sociological Association Teaching Group (formerly the Association of Teachers of Social Sciences), Computing at School, the Economics Business and Enterprise Association, the Geographical Association, the Royal Geographical Society (with IBG).

Clare Green played a pivotal role throughout the study by developing the methodology, conducting all six of the analysis meetings, and initiating the data analysis. Particular thanks are due to Karen Duffy for helping orchestrate the Psychology and Sociology analyses. A number of Nuffield Foundation staff contributed to the project, including Fran Bright, Sarah Codrington, Steve Steward and in particular Kim Woodruff, who did the lion's share of the work.

Findings

In this investigation we set out to map the extent, difficulty and type of mathematics assessed in the six subjects. We found it was possible for students to follow different pathways through their subjects and for these pathways to vary significantly in the amount of mathematics they contain. For this reason it is difficult to quantify the mathematical experience of the typical student. Some students may have had significant mathematical experience and others may have had none, even within the same subject.

Variations in assessments

We found significant variation among the different Awarding Organisations' assessments on all the attributes that were studied. Further variation is added through student choice of different units and questions. Some A levels give students choice over the combination of units studied, but the mathematical content of different units can vary significantly.

Within each unit, different routes are available to students based on their choice of questions. As the mathematical content of questions is not equal, this provides an additional level of variation in mathematical experience. In a number of cases, students could completely avoid mathematics, whereas others could gain over 50% of their total marks for mathematical work. Consequently, students may have very different experiences depending on the choices they make (or that are made for them) at different stages.

Finally, five of the six subjects offered flexibility in the way questions were answered, meaning that students could answer the same question without using any mathematics, or by using rich and complex mathematical approaches in their answer. The exception to this was Computing.

Type and difficulty of mathematics

For subjects other than Computing, much of the mathematical content concerned statistics. The most common questions needed complex subject-specific reasoning combined with simple mathematics. Most calculations involved arithmetic and proportional reasoning, and there was little algebra outside of the Computing A level assessments. Use of geometry was minimal.

We found significant opportunities for the use of graphs and moving between text, graphical, numerical and algebraic forms, but these opportunities were not always taken. For example, some Business Studies and Geography case studies were rich in numerical and graphical data, but the actual questions and the mark schemes did not take advantage of this opportunity.

Not many assessments had extended calculations and there were few questions that could be deemed mathematically complex. Where questions with mathematical content were demanding, they were set in familiar contexts. Occasionally we found that the mark scheme was not recognising valid and relevant mathematical work.

In a number of subjects, some content and approaches went beyond what is covered in GCSE mathematics.

Ofqual criteria

The Ofqual subject criteria list mathematical content for science subjects[5]. Psychology is categorised as a science and is therefore covered by these criteria. The Ofqual subject criteria for all the other subjects in the study do not explicitly list any mathematical requirements, nor is there any mention of the words mathematics or statistics (or variants). Sociology has a reference to numerical data in a section on methods of sociological enquiry.

Given the significant variation in the subject specifications of the different Awarding Organisations, the lack of core quantitative guidelines in the Ofqual subject criteria hinders any definitive mapping of the mathematics that students need in their various subjects.

5 http://www.ofqual.gov.uk/downloads/category/191-gce-as-and-a-level-subject-criteria

Recommendations

Our starting point for this study is the analysis in the ACME report *Mathematical Needs*[6]. This shows that the mathematical demands of virtually all subjects being taught at higher education (HE) level are increasing, and that large numbers of students entering HE are insufficiently prepared for those demands.

This is an important issue that will require a range of measures and national leadership if it is to be addressed satisfactorily. One strand is to look at the mathematics and statistics embedded in A levels other than mathematics itself. The extent to which it is possible and desirable for the needs of students to be addressed by these means is a matter for HE, subject associations, learned societies and other stakeholders to determine. However, it seems reasonable to suggest that the demands of those subjects should be reflected in A level teaching (and assessment), and that there should be some consistency across different assessments of the same subject. In light of that we make these recommendations:

- HE, subject associations, learned societies and other stakeholders should work with Ofqual on a better definition of the quantitative skills needed by students studying their subjects. Stakeholders could track typical student trajectories and examiner reports to obtain a more detailed picture of the type, extent and complexity of mathematics and statistics currently assessed and how this aligns with these identified needs. In turn, this could inform discussions around inclusion and assessment of relevant mathematics and statistics, appropriate ways to spread mathematical content over the assessment cycle, and the regulation of this process.

- Stakeholders should consider the implications, advantages and disadvantages of variation in the coverage of mathematics and statistics across the different Awarding Organisations revealed by this study. There is scope to work with the Awarding Organisations and the regulatory regime to ensure that quantitative requirements are better delineated.[7]

- The report shows that it is possible for students who are ostensibly following the same course of study to have widely different levels of exposure to quantitative approaches to their subject. Stakeholders should consider the implications of this, and its advantages and disadvantages.

6 *Mathematical Needs: Mathematics in the workplace and in Higher Education* (2011), London: ACME www.acme-uk.org/media/7624/acme_theme_a_final%20(2).pdf

7 See exchange of letters between Ofqual and the Secretary of State on proposed A level reform: www.ofqual.gov.uk/news-and-announcements/83-news-and-announcements-news/873-ofqual-statement-on-proposed-changes-to-a-levels

- Awarding Organisations are currently required to assess the quality of written communication, and indicate to students which questions will include such assessment. A similar requirement could be introduced for the assessment of quantitative approaches where they are considered important or desirable.

- HE should improve its signalling of what is required for progression to different disciplines within HE. The strong message coming from HE is that students are on the whole not well equipped to meet these demands and that the demands are increasing. Clearer and more robust statements of this message would assist in the design of post-16 courses and pathways, and help students make better informed choices on relevant mathematics and statistics modules that complement their programmes of study.

- In some subjects the mathematical content or approaches are beyond those currently covered in GCSE. Where this is the case, it needs to be made explicit so that subject teachers can provide appropriate support for students and/or students can consider taking relevant post-16 mathematics qualifications.

- Exemplification and sharing of good practice could help develop better ways for students to acquire the necessary quantitative skills. The analysis indicates there is significant scope for supporting and strengthening mathematical and statistical skills via the subjects we have studied (for example by use of other subject contexts to anchor mathematical learning). We recognise this is a complex area that would need further study, not least because of the issues around teaching and examining capacity.

- A common complaint from the HE sector is that students find it difficult to use and apply mathematics in unfamiliar contexts. In this study we found very few instances where mathematics had to be applied in unfamiliar contexts, which is perhaps unsurprising given the high stakes nature of A level assessment. If this is an aspect of mathematics that is required in HE then we need further thought about how it can be taught and assessed in advance of HE. Current examination structures are not well designed to foster such skills, but if the skills are not assessed they are not likely to be taught.

Four dimensions of variation – some illustrations

1. Variation by Awarding Organisation

Using Psychology as an example, the percentage of marks requiring mathematics ranged from 10% to over 40% across the different Awarding Organisations. This variation was also evident in the difficulty and type of mathematics used. In Economics for example, extended calculations accounted for one fifth of all calculations used by one Awarding Organisation, but were not used at all by another.

2. Variation in choice of units and options

The most obvious example of the differing mathematical content of unit options is Business Studies. For one A level, mathematics was required for over 80% of the marks available for the A2 Accounting option, but this figure was under 10% for the alternative options (Marketing, People in Organisations, or Business Production).

3. Variation via choice of questions

Students are often given a choice as to which questions to answer. For one A2 unit in Psychology for example, students have four options with four questions each (16 questions in total) and have to choose two options and answer two questions for each (four questions to be answered). The mathematical content of each pathway can vary considerably, from none to about one third.

4. Variation in marks that require mathematics or marks that could be gained using mathematics

With the exception of Computing, there can be significant variation in the amount, type and complexity of the mathematics that different students bring to the same question. On one Sociology A2 unit for example, students have to answer two questions from a choice of 12. Two thirds of questions involved mathematics, but for each of these questions, marks for mathematics ranged from 2% to 20% of the total. This variation was somewhat stark for Sociology where the baseline mathematical content was low. It was less pronounced for Geography and Psychology compared with Business Studies and Economics.

Summary of findings by subject

Business Studies

The vast majority of the mathematics required involved basic arithmetic operations and the use of formulae, but with minimal algebraic manipulation if any. Questions also required students to interpret data presented in tables and to use percentages, proportions and ratios. We found some instances of the use of line graphs and statistical evidence. There was a handful of occurrences of business-specific techniques, such as use of critical path analysis and decision trees.

With the exception of one Awarding Organisation (B), few, if any, questions were judged to be mathematically complex. In cases of high complexity, the context was familiar and the mathematics did not account for a significant proportion of the marks. In most cases, the assessments required subject-specific reasoning combined with simple mathematics.

In some questions, there was more than one way that a student could develop his or her response. One path might involve little or no mathematical content, while another might use or include a high level of mathematical content. We captured these differences by recording the lowest (marks that *required* mathematics) and highest number of marks (marks that *could be gained* using mathematics). This variation was present in all Awarding Organisations, though it was more pronounced for some.

We found a number of examples where the case studies used in assessments presented multiple opportunities to assess a range of mathematics relevant for business, but the questions, and on occasion the mark schemes, did not make use of these opportunities.

Computing

Throughout the exercise, there was debate as to whether content areas relating to logical and data structures were mathematics or computing. Skills such as the use of Boolean operators, graph theory and numerical arrays, which are not part of GCSE mathematics, were added on as 'computing related mathematics', while skills such as ordering strings alphabetically were not deemed to be mathematical.

The most frequent mathematical activity was algebraic, involving the development, use and analysis of algorithms. Aspects of number were also covered, especially the use of binary and other non-decimal representation. Computing-related mathematics was the third most common type of mathematics found, largely around understanding and manipulating arrays and some use of Boolean operators. We found very few instances of geometry, statistics, and graphical representation of data, though we noted that much of this could occur in practical/project work.

The type, extent and difficulty of the mathematics in practical/project work could vary considerably, depending on the nature of the project and the student's aptitude for mathematics. Higher grades could be achieved if the student includes complex mathematics, and the mathematical content was likely to be higher if a student was writing a programme from scratch.

In addition to the preponderance of algebra, and relative absence of statistics, Computing contrasted strongly on two additional measures with the other subjects analysed in this study. Firstly, there was no variation between marks that required mathematics and marks that could be gained using mathematics. Second, on the complexity of calculations, Computing had a majority of multi-step (and a number of extended) calculations, although we found variation on these measures between various AS and A2 units and between Awarding Organisations.

Beyond the multi-step calculations, the mathematics was not complex in terms of the mathematical domains that were accessed, and the contexts used would have been familiar to students. Almost all the questions involving mathematics required computing as well as mathematical comprehension.

Economics

Economics had mathematical content areas that were not formally assessed, but which need to be taught in order for students to understand certain economic concepts. Examples include *elasticities,* which require knowledge and understanding of percentage changes, and *production possibility curves* which require knowledge and understanding of tangents and gradients. We did not explicitly map these content areas.

Most of the mathematics required in the assessments involved the use of graphs – sketching diagrams and models and interpreting and presenting line graphs, and to a lesser extent, translating information between textual, graphical and numerical form. The second most common type of mathematics was the use of percentages, proportions and ratios and basic arithmetic operations. We found some use of formulae and equalities/inequalities, but with minimal algebraic manipulation. In a small number of instances students were required to interpret statistical evidence presented in tables and charts. We also found a handful of occurrences of business-specific techniques, such as using and interpreting indices, and analysing the relationship between variables over time.

We found instances where the mark scheme from Awarding Organisations A and B did not acknowledge or use the mathematics (a diagram or graph) that was present in the question. This may penalise students who have chosen to use a mathematical approach in their response.

There was considerable variation between Awarding Organisations on mathematical coverage, but we found examples of moderately complex problems across all of them, although the contexts used were familiar. In most cases, the mathematical aspects of the assessments required both subject-specific reasoning and mathematical understanding. For all Awarding Organisations, we found significant variation between marks that required mathematics and marks that could be gained using mathematics. Coupled with the additional variation given by the choice of questions, there is potential for a significant difference between the mathematics used by two students achieving the same grade.

Geography

Most of the mathematics required in the assessments involved statistical methods. This encompassed field experiments (from setting up to drawing conclusions from summary data and graphs); translating information between text, graphical, and numerical form; and interpreting and presenting bar charts and line graphs. The second most common type of mathematics required was the use of percentages, proportions and ratios, and basic arithmetic operations. We found considerable use of maps and scales and some use of equalities/inequalities, but no explicit algebraic manipulation. There were instances of geography-specific mathematics, such as use of *4 and 6 figure co-ordinates* and use of *triangular graphs* and *choropleths*. While these were recorded, some geographical techniques that have mathematical aspects, e.g. cost-benefit analysis and risk assessments, were not separately mapped. Rather, the mathematical skills that they contained were recorded using the mathematical requirements list. We also identified the use and interpretation of spatial data through the application of geographical information systems (GIS) as a specifically geographical technique.

We found considerable variation between Awarding Organisations on mathematical coverage. With the exception of one Awarding Organisation (B), we found few, if any, questions which were mathematically complex. In cases of high complexity, the context was familiar and the mathematics did not account for a significant proportion of the marks. In most cases, the assessments required subject-specific reasoning combined with a fair understanding of graphical and statistical methods. Although it appears there was greater mathematical coverage at AS than A2, this is because the units and most of the essay questions that were excluded from analysis were from A2.

With the exception of Awarding Organisation C, we found significant variation between marks that required mathematics and marks that could be gained using mathematics. The valid use of number enhanced a student's response, as opposed to purely descriptive responses which could gain only lower marks. It was possible to gain higher marks by increasing the use of comparisons or proportional changes, percentages, data manipulation, and by using specific numbers to illustrate a point. Our analysts felt that inclusion of these gave precision to an answer.

Some of the question parts had up to 50 marks allocated to them. While some specified, for example, that 8 marks were for a graph, beyond that, our analysts made judgements as to the range of marks that may have been awarded for the mathematical content of the students' responses. Some questions referred to students using their own work, data or fieldwork, but we could not map this in our analysis.

Psychology

We found a fair range of variation between the Awarding Organisations on all the attributes studied, although hardly any had extended calculations and there were few questions that could be deemed mathematically complex.

While there was considerable variation between Awarding Organisations on the extent of the mathematical coverage, the variation between marks that required mathematics and marks that could be gained using mathematics for individual Awarding Organisations in Psychology was much smaller than the comparable variation in Business Studies, Economics and Sociology.

Overall, we found A level Psychology candidates needed to have an ability in a few specific mathematical content areas. The most prevalent of these concerned aspects of statistics, especially relating to research. A couple of the Awarding Organisation specifications went into significant detail on statistical research methods, and these were used to supplement the statistical approaches used for mapping the type of mathematics. There were some elements of number, in particular percentages, arithmetic operations, and proportions and ratios. We also found question parts involving interpreting scales, recognising correlations, interpreting and presenting tables, and scatter graphs.

The vast majority of calculations were not complex, though Awarding Organisation A had some instances of complex calculations. We found a fair number of examples where the mathematics was set in contexts with some novel aspects, and the analysts noted that the assessments demanded a fair amount of complex scientific reasoning combined with simple mathematics.

Sociology

We found the predominant mathematics across all Awarding Organisations involved statistics and number, arising from sociological studies referenced in student responses. Some of these were highly statistical in their nature, while others were purely theoretical. Similarly, both qualitative and quantitative research was used. Both these factors affected the level of mathematical content in students' responses, and it was possible to achieve a high mark using references to studies with high mathematical content **or** to achieve the same mark using references to studies with *no* mathematical content. In other words, the number of marks that *required* the student to use mathematics was very low (1–3%). Marks that could be gained using mathematics were higher (12% to 19%).

We found a fair range of variation between the Awarding Organisations, though the overall mathematical content was limited and none had extended calculations or mathematically complex tasks. All the mathematics was set in contexts that would be familiar and required sociology-specific understanding combined with basic statistical methods and/or proportional reasoning.

Annex 1: Detailed findings

Description of measures

For each of the six assessments, we mapped the extent, difficulty, and type of mathematics using the following measures.

1. Extent

1a. Proportion of question parts with mathematical content
The number of question parts that had mathematical content calculated as a percentage of the total number of question parts, to give an indication of the extent of the mathematical content across the papers.

1b. Number of marks for mathematical content
The number of marks, where part or all of the mark was awarded for mathematical content. For some of the questions, students could gain the marks through more than one pathway, one that might include mathematics and one which may not. Therefore, two sets of data were recorded:

1. *analysis of the number of marks that required using mathematical methods; and*
2. *analysis of the number of marks that could be gained by using mathematical methods.*

2. Difficulty

2a. Number of steps
A single step calculation involves only one step. Multi-step calculations involve more than one step, and an extended calculation requires using the result from one section of a calculation to find a solution to a subsequent section.

2b. Complexity of task
The complexity of the task was defined according to whether it required straightforward recall of common procedures or whether it involved application or synthesis in one or more mathematical domains. A domain is a mathematical content area, for example, algebraic equations, ratios or tangents. Each task was assigned one of four levels of complexity.

1. *Straightforward/routine – requires recall of procedures and relatively straightforward application.*
2. *Requires application and understanding of mathematics within one domain.*

3. *Requires understanding/use of mathematics across domains and necessitates a decision about the direction in which to proceed.*
4. *Complex activity requiring synthesis across a number of domains with structuring/ decision-making being necessary.*

2c. Familiarity of Context

Each task was to assigned one of three categories relating to the familiarity of the context. The three categories correspond to whether the mathematics:

1. *would be typically met through learning programme;*
2. *had some novel aspects; or*
3. *was in a situation unlikely to have been met before.*

Applying mathematics in unfamiliar contexts is a higher order skill than applying mathematics in a situation repeatedly met in the classroom, so categories 2 and 3 could be judged to be more difficult mathematically, as they require more application of knowledge.

2d. Mathematical skills or subject-specific comprehension

This measure considers the balance between comprehension of the subject and mathematical skills on question parts with mathematical content. The three categories correspond to whether the marks were awarded purely for mathematical skills (MS), purely for subject-specific comprehension (SSC), or for both (BOTH).

3. Type of Mathematics

The type of mathematics found in the questions was categorised against mathematical content areas on a list of requirements (see Page 53), taking into account the content of GCSE mathematics, and the mathematical content implicit and explicit in the Awarding Organisation specifications.

Business Studies

All five of the Awarding Organisations in our study offer an A level in Business Studies (AQA, CCEA, Edexcel, OCR and WJEC). There are four externally assessed units: two AS and two A2. Awarding Organisation E offers a choice of four units for one of the two A2 units: Marketing, Accounting, People in Organisations, or Business Production. The mathematical component of the Accounting unit is significantly larger than that of the other units. In light of this, we have presented two variations of this particular qualification – one where Accounting is the chosen option, and one where the data is averaged over the other three options. We have anonymised the five Awarding Organisations, using letters A, B, C, D and E.

Extent – proportion of question parts with mathematical content

We found a marked difference between Awarding Organisations, with the percentage of question parts with mathematical content ranging from 19% to 79%. There was a higher proportion of question parts with mathematics in the A2 papers than the AS papers, with one exception.

The percentage of question parts with mathematical content.

Awarding Organisation	A	B	C	D	E (with Acc)	E (non Acc)
AS unit 1	10	13	29	70	13	13
AS unit 2	20	36	0	100	42	42
AS average	15	26	15	84	30	30
A2 unit 3	14	40	0	100	100	38
A2 unit 4	40	50	33	40	80	80
A2 average	25	47	17	67	92	54
A level average	19	35	15	79	53	39

Extent – number of marks for mathematical content

On a number of question papers, students have a choice of questions. We calculated percentages by taking into account all options and giving an overall indication of the extent of mathematics. The actual percentage for a particular choice of questions would be higher or lower depending on the questions the student chooses to answer.

There was a spread from 2% to 36% for the marks requiring mathematics, with 36% being the case for the A level that included an Accounting A2 unit. The maximum for the other Awarding Organisations was 13%.

The number of marks that could be gained using a mathematical route, was between 9% and 49% of the total. Again, the maximum was for the Accounting option A level, with 35% being the maximum for the other Awarding Organisations. Additional variation given by the choice of questions meant there could be a marked difference between the mathematics used by two students achieving the same or similar grades.

Measure 1: Percentage of marks that required mathematics

Awarding Organisation	A	B	C	D	E (with Acc)	E (non Acc)
AS unit 1	0	8	7	10	0	0
AS unit 2	5	21	0	9	16	16
AS average	3	16	4	9	9	9
A2 unit 3	3	8	0	13	82	7
A2 unit 4	0	13	10	0	50	50
A2 average	1	11	5	4	63	33
A level average	2	13	4	9	36	21

Measure 2: Percentage of marks that could be gained using mathematics

Awarding Organisation	A	B	C	D	E (with Acc)	E (non Acc)
AS unit 1	10	8	23	48	10	10
AS unit 2	10	27	0	51	32	32
AS average	10	19	11	50	23	23
A2 unit 3	3	22	0	49	100	10
A2 unit 4	20	22	13	8	59	59
A2 average	11	22	6	20	75	39
A level average	11	21	9	35	49	31

Difficulty – Number of steps

With the exception of one Awarding Organisation (B), the vast majority (between 55% and 83%) of the calculations required only a single step. Between 15% and 30% were multi-step calculations. Awarding Organisations A and C did not require students to complete any extended calculations at all. For Awarding Organisation B, 10 of the total 12 questions involving mathematics required extended calculations, but only 13% of the total marks required mathematics.

The number of single (S), multi-step (M) and extended (E) calculations for each unit, and the overall A level percentage of each type.

Awarding Organisation	A			B			C			D			E (with Acc)			E (non Acc)		
Calculation steps	S	M	E	S	M	E	S	M	E	S	M	E	S	M	E	S	M	E
AS unit 1	1	0	0	1	0	0	3	3	0	2	4	1	1	0	0	1	0	0
AS unit 2	1	1	0	1	0	3	0	0	0	7	0	2	4	1	0	4	1	0
AS total	2	1	0	2	0	3	3	3	0	9	4	3	5	1	0	5	1	0
A2 unit 3	1	0	0	0	0	2	0	0	0	0	2	0	2	4	1	2	1	0
A2 unit 4	2	0	0	0	0	5	3	0	0	2	0	0	3	0	1	3	0	1
A2 total	3	0	0	0	0	7	3	0	0	2	2	0	5	4	2	5	1	1
A level percentage	83	17	0	17	0	83	67	33	0	55	30	15	59	29	12	77	15	8

Difficulty – Complexity of task

With the exception of Awarding Organisation B, the vast majority of calculations were of level one or two in complexity (84% to 100%). Awarding Organisation A had the highest proportion of level one calculations, but C was the only Awarding Organisation to use *only* level one calculations. Awarding Organisation A had some level two calculations, but none that were level three or four. Awarding Organisation B had the largest percentage of level three and four calculations (42% and 33% respectively).

The number of calculations by level of complexity, and the overall A level percentage of each level.

Awarding Organisation	A				B				C				D				E (with Acc)				E (non Acc)			
Complexity	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
AS unit 1	1	0	0	0	0	0	1	0	6	0	0	0	2	5	0	0	1	0	0	0	1	0	0	0
AS unit 2	1	1	0	0	1	0	3	0	0	0	0	0	6	2	1	0	4	0	1	0	4	0	1	0
AS total	2	1	0	0	1	0	4	0	6	0	0	0	8	7	1	0	5	0	1	0	5	0	1	0
A2 unit 3	1	0	0	0	0	0	0	2	0	0	0	0	1	1	0	2	2	5	0	0	2	1	0	0
A2 unit 4	2	0	0	0	0	2	1	2	3	0	0	0	2	0	0	0	0	3	0	1	0	3	0	1
A2 total	3	0	0	0	0	2	1	4	3	0	0	0	3	1	0	2	2	8	0	1	2	4	0	1
A level percentage	83	17	0	0	8	17	42	33	100	0	0	0	50	36	5	9	41	47	6	6	54	30	8	8

Levels of complexity

1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of context

Between 64% and 100% of the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). Awarding Organisations D and E used some mathematics where the context had novel aspects (category two), but none of the Awarding Organisations used mathematics set in a context that the students were unlikely to have met before (category three).

The number of calculations by familiarity of context and the overall A level percentage for each category.

Awarding Organisation	A			B			C			D			E (with Acc)			E (non Acc)		
Familiarity of Context	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
AS unit 1	1	0	0	1	0	0	6	0	0	5	2	0	1	0	0	1	0	0
AS unit 2	2	0	0	4	0	0	0	0	0	5	4	0	4	1	0	4	1	0
AS total	3	0	0	5	0	0	6	0	0	10	6	0	5	1	0	5	1	0
A2 unit 3	1	0	0	2	0	0	0	0	0	2	2	0	3	4	0	3	0	0
A2 unit 4	2	0	0	5	0	0	3	0	0	2	0	0	3	1	0	3	1	0
A2 total	3	0	0	7	0	0	3	0	0	4	2	0	6	5	0	6	1	0
A level percentage	100	0	0	100	0	0	100	0	0	64	36	0	65	35	0	85	15	0

Familiarity of context: 1 = typical, 2 = some novel aspects, 3 = unlikely to have been met before

Mathematical skills or subject-specific comprehension

Four of the six A levels required both subject-specific comprehension and mathematical skills for *all* the question parts with mathematical content. The majority of the question parts in the other two A levels also required both (78% and 85%). None of the Awarding Organisations gave marks relating to mathematics purely for subject-specific comprehension.

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

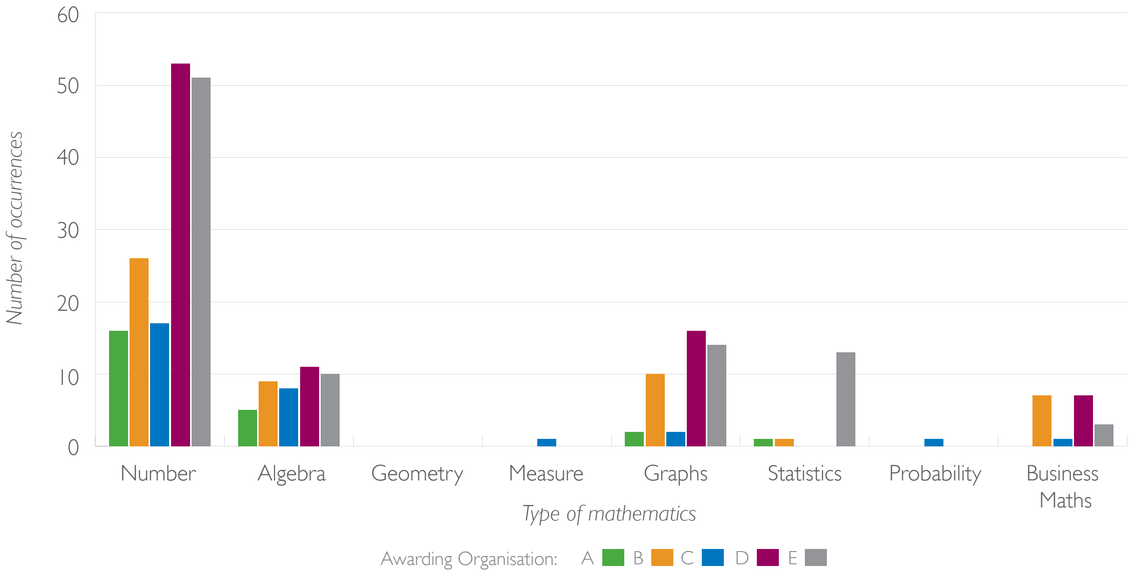
Awarding Organisation	A			B			C			D			E (with Acc)			E (non Acc)		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
AS unit 1	0	0	1	0	0	1	2	0	4	0	0	7	0	0	1	0	0	1
AS unit 2	0	0	2	0	0	4				0	0	9	0	0	5	0	0	5
AS total	0	0	3	0	0	5	2	0	4	0	0	16	0	0	6	0	0	6
A2 unit 3	0	0	1	0	0	2				0	0	4	0	0	7	2	0	1
A2 unit 4	0	0	2	0	0	5	0	0	3	0	0	2	0	0	4	0	0	4
A2 total	0	0	3	0	0	7	0	0	3	0	0	6	0	0	11	2	0	5
A level percentage	0	0	100	0	0	100	22	0	78	0	0	100	0	0	100	15	0	85

Type of mathematics

In addition to categorising the type of mathematics according to the general list of requirements (see page 53), we added four requirements specific to Business Studies:

a. understand, construct and use critical path analysis
b. use of bracket notation
c. use of negatives to denote inverse relationships
d. construct and interpret decision trees

The number of occurrences of each type of mathematics in A level Business Studies assessments (summer 2010)



Number

The most frequently found tasks across all Awarding Organisations were on the four basic operations (1a: addition, subtraction, multiplication and division). Many question parts required use of percentages (1e), and a significant number used proportions and ratios (1f). Papers for Awarding Organisation B had no number content. Awarding Organisation C had the greatest spread of number content, with all requirements found across the A level.

Graphs and Tables

Most of the graph/table work involved interpreting data in tables (7i). There were also instances of interpreting and presenting line graphs (7g). Three of the Awarding Organisations used a question requiring the translating of information between graphical, numerical and algebraic forms (7a). There was a single instance of use of pie charts.

Statistics (5a – 5g) and Probability (6)

In some questions, students can use prior knowledge of data or statistics to enhance their responses. This prior knowledge may involve quantitative reasoning, and for the purpose of our analysis was categorised under requirement 5c (interpret statistical data).

Awarding Organisation E used a number of questions requiring the interpretation of statistical data (5c), but no explicit computations of statistics. The other Awarding Organisations had at most one question part involving interpreting statistical data, and one Organisation had a question that touched on probability. Overall, the dearth of explicit statistical analysis was striking. It was notable that our analysis did not require us to use the more detailed statistical and experiment design criteria necessary for analysing the Psychology assessments.

Algebra

There were some instances of the use of formulae (2d), and of the use of equalities and inequalities (2a).

Geometry and Measure

Apart from a single occurrence of conversion of units, there were no questions relating to geometry and measure.

Business-specific mathematics

Awarding Organisation D had two questions involving use of critical path analysis. Awarding Organisations B and C each had a question relating to the use of decision trees. There were uses of notation and terminology specific to business studies (as opposed to the other subjects in this study), such as bracket notation to denote negative numbers, debits, outflows or losses, and the use of negatives to denote inverse relationships.

Investment Appraisal

There were some mathematical methods specific to business studies, for example, DCF, ARR, and payback methods. As these methods could be broken down into mathematical requirements listed under number, they were not recorded separately.

Computing

Three of the Awarding Organisations in our study offer an A level in Computing (AQA, OCR and WJEC). For each of the Awarding Organisations, the fourth unit of the A level assessments consists of a practical project (and the second AS unit for WJEC is also a practical task). Given the possible variations in the practical work, we did not analyse them using the framework, but make some observations at the end of this section. We have anonymised the three Awarding Organisations, using letters A, B, and C.

Extent – proportion of question parts with mathematical content

We found some difference between Awarding Organisations, with the percentage of question parts with mathematical content ranging from 30% to 44%.

The percentage of question parts with mathematical content.

Awarding Organisation	A	B	C
AS average	31	42	43
A2 average	35	47	15
A level average	33	44	30

Extent – number of marks for mathematical content

Among the various AS and A2 units, the marks for mathematics ranged from 14% to 65%. Awarding Organisation C had both the highest (AS) and lowest (A2) percentages of marks available for mathematics. Across the whole A level, the percentage ranged from 19% to 36%, Computing was the only subject for which it was not necessary to differentiate between the marks that required mathematics and the marks that could be gained using mathematics.

Percentage of marks for mathematics

Awarding Organisation	A	B	C
AS average	15	46	41
A2 average	23	21	14
A level average	19	36	31

Difficulty – Number of steps

In contrast to the other subjects in this study, most calculations were multi-step (51% of those across all A levels). There were more complex calculations on AS papers than on A2 papers, and a fair number of extended calculations (ranging from 5% to 20%).

The number of single (S), multi-step (M) and extended (E) calculations for each unit, and the overall A level percentage of each type.

Awarding Organisation	A			B			C		
Calculation steps	S	M	E	S	M	E	S	M	E
AS total	3	8	0	10	18	12	8	15	2
A2 total	7	3	1	7	14	0	4	2	2
A level percentage	45	50	5	28	52	20	36	52	12

Difficulty – Complexity of task

With the exception of Awarding Organisation C, the vast majority of calculations were of level one or two in complexity (93% to 96%). One quarter of the calculations used by Awarding Organisation C were classified as level three, requiring work across more than one domain.

The number of calculations by level of complexity and the overall A level percentage of each level.

Awarding Organisation	A				B				C			
Complexity	1	2	3	4	1	2	3	4	1	2	3	4
AS total	7	5	0	0	14	16	4	0	11	7	6	1
A2 total	10	0	1	0	13	8	0	0	5	1	1	0
A level percentage	74	22	4	0	49	44	7	0	50	25	22	3

Levels of complexity

1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of Context

Between 76% and 100% of the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). Awarding Organisations B and C used some mathematics where the context had novel aspects (category two: 11% and 24% respectively), but none of the Awarding Organisations used mathematics set in a context that the students were unlikely to have met before (category three).

The number of calculations by familiarity of context and the overall A level percentage for each category.

Awarding Organisation	A			B			C		
Familiarity of Context	1	2	3	1	2	3	1	2	3
AS total	12	0	0	29	5	0	17	8	0
A2 total	10	0	0	20	1	0	8	0	0
A level percentage	100	0	0	89	11	0	76	24	0

Familiarity of context: 1 = typical, 2 = some novel aspects, 3 = unlikely to have been met before

Mathematical skills or subject-specific comprehension

Two of the three A levels required both subject-specific comprehension and mathematical skills for *all* the question parts with mathematical content. For the third, Awarding Organisation B, 85% of question parts required both. None of the Awarding Organisations gave marks relating to mathematics purely for subject-specific comprehension.

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

Awarding Organisation	A			B			C		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
AS total	0	0	12	8	0	26	0	0	25
A2 total	0	0	9	0	0	21	0	0	8
A level percentage	0	0	100	15	0	85	0	0	100

Mathematical skills or computing skills?

There was considerable debate about whether some content areas should be considered mathematics skills for the purposes of our research, or purely computing skills. As a result, we established a new category in the list of mathematical requirements called: *computing related mathematics*. By doing this, we aimed to capture those skills found in the computing assessments which were considered to be mathematical skills, but which did not feature in the mathematics GCSE specification.

Computing related mathematics

a. understand and use the set and Boolean operators AND, OR, NOT, UNION, INTERSECTION, IF, THEN, ELSE
b. understand and manipulate arrays
c. understand and use graph theory

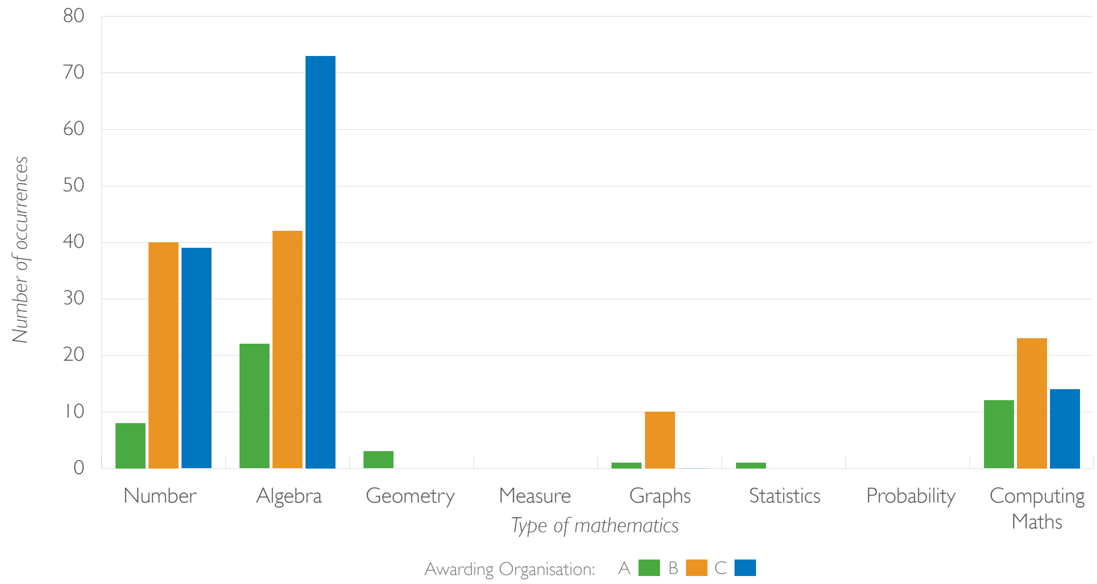
Requirements that were not considered to be mathematics skills and therefore not included in the analysis of the assessments were:

- recognising that a string is a numerical value; and
- placing data in alphabetical order.

On classifying certain counting
When working with binary sequences, students have to count the instances of the number 1, and this was classified as counting (1a) under mathematical requirements.

Type of mathematics

The number of occurrences of each type of mathematics in A level Computing assessments (summer 2010)



Algebra
The most frequent mathematical activity was algebraic, involving developing and using algorithms (2c), followed by use of equalities and inequalities (2a) and formulae (2d).

Number
The preponderance of algebra was followed by the use of number – binary and other non-decimal and decimal number systems (1g, 1b), and the use of arithmetic operations (1a).

Computing-related mathematics
Understanding and manipulating arrays was quite prevalent and more frequent than the use of set and Boolean operators. Awarding Organisation A had some questions relating to graph theory.

Geometry, Measure and Graphing

There were very few instances of geometry and none of measure. There were a number of instances of translating between text, graphical, numerical and algebraic forms (7a) for Awarding Organisation B, but little else relating to graphs.

Statistics and Probability

Apart from a single statistical computation (5d), there were no questions requiring the use of statistics and probability.

Practical Projects

The fourth unit of the A level assessments for all Awarding Organisations was a practical project. We did not analyse these units using the framework, but developed a description of the nature of the mathematical content in them based on analysts' observations and discussions. The following points were noted:

- The type, extent and difficulty of the mathematics in the practical project varies greatly as it depends on the nature of the project and the student's mathematical aptitude.
- The mathematical content in a project is likely to be higher if the student is writing a programme from scratch.
- Higher grades can be achieved if the student includes complex mathematics.
- The understanding of algorithms is key to a successful project, and algorithms and functions are at the heart of all programmes.
- Number work, algebra and computing-related mathematics are frequently included in the project.
- There is no need to include any extended calculations in the project, although some students may do so.
- The mathematics found in the projects may be different from that found in the theory papers. For example scale drawing (measure), using summary data (statistics), 2D and 3D visualisation (geometry) were not found in units 1, 2 and 3, but may feature in the unit 4 projects.
- Mathematics in the projects may be of complexity 3 or even complexity 4 in some cases.
- The context of the projects may mean that the mathematics is set in a familiar context or it may be totally new to the student.

Economics

All five of the Awarding Organisations in our study offer an A level in Economics (AQA, CCEA, Edexcel, OCR and WJEC). There are four externally assessed units: two AS and two A2. For its first A2 unit, OCR offers a choice between Work & Leisure, or Transport. As the extent and difficulty of mathematics in each of these options was broadly similar, we averaged the measures for these two options, and consequently all five A levels are listed as comprising four units. We have anonymised the five Awarding Organisations, using letters A, B, C, D and E.

Extent – proportion of question parts with mathematical content

The overall percentage of question parts with mathematical content was relatively consistent across Awarding Organisations, ranging from 58% to 73%. With the exception of Awarding Organisation D, the spread across AS and A2 units was roughly equal (60% to 70%). While these percentages may look substantial, they are a measure of the spread of the mathematics rather than the volume (see next measure of extent).

The percentage of question parts with mathematical content.

Awarding Organisation	A	B	C	D	E
AS unit 1	60	64	85	55	70
AS unit 2	64	63	64	30	60
AS average	62	63	76	42	65
A2 unit 3	91	77	56	100	69
A2 unit 4	30	55	81	100	71
A2 average	62	67	69	100	70
A level average	62	65	73	58	68

Extent – number of marks for mathematical content

On a number of question papers, students have a choice of questions. We calculated percentages by taking into account all options and giving an overall indication of the extent of mathematics. The actual percentage for a particular choice of questions would be higher or lower depending on the questions the student chooses to answer.

In a multi-mark question, students with limited mathematical skills/understanding may include a diagram in their response, but without relating it to further analysis and evaluation. However, use of good mathematical skills/understanding would enable students to gain the marks for the analysis and possibly for the evaluation. This variation contributes to the gap between marks that require mathematics and marks that could be gained using mathematics.

There was a spread from 4% to 14% for the marks requiring mathematics. For marks that could be gained using a mathematical route, the spread was between 27% and 57% of the total marks. On average, the percentage of marks that could be gained by using mathematics was four times greater than the percentage of marks that required mathematics, a variation more marked than in Business Studies for example. Additional variation given by the choice of questions meant there could be a marked difference between the mathematics used by two students achieving the same or similar grades.

Measure 1: Percentage of marks that required mathematics

Awarding Organisation	A	B	C	D	E
AS unit 1	4	14	16	30	23
AS unit 2	6	10	10	13	17
AS average	5	11	13	22	20
A2 unit 3	6	8	3	9	15
A2 unit 4	0	4	1	9	0
A2 average	3	5	1	9	11
A level average	4	8	6	14	14

Measure 2: Percentage of marks that could be gained using mathematics

Awarding Organisation	A	B	C	D	E
AS unit 1	57	34	45	59	57
AS unit 2	33	25	26	26	53
AS average	45	27	34	42	55
A2 unit 3	29	41	27	57	61
A2 unit 4	7	16	43	54	53
A2 average	18	27	38	55	58
A level average	31	27	36	50	57

Difficulty – Number of steps

Awarding Organisations B and C had predominantly single-step calculations (74% and 72% respectively), and around half the calculations for the others (A, D and E) were multi-step. Extended calculations ranged from none (D) to 20% (B), though it should be noted that the volume of possible marks for mathematics was low for Awarding Organisation B.

The number of single (S), multi-step (M), and extended (E) calculations for each unit, and the percentages of each type for the overall A level.

Awarding Organisation	A			B			C			D			E		
Calculation steps	S	M	E	S	M	E	S	M	E	S	M	E	S	M	E
AS unit 1	6	3	0	5	1	1	9	8	0	9	9	0	3	4	0
AS unit 2	0	9	0	11	0	1	6	2	1	10	0	0	0	2	4
AS total	6	12	0	16	1	2	15	10	1	19	9	0	3	6	4
A2 unit 3	7	2	1	6	0	4	9	2	0	2	10	0	2	7	0
A2 unit 4	3	0	0	4	1	1	12	0	1	2	10	0	3	2	0
A2 total	10	2	1	10	1	5	21	2	1	4	20	0	5	9	0
A level percentage	52	45	3	74	6	20	72	24	4	44	56	0	30	55	15

Difficulty – Complexity of task

None of the mathematical tasks were of level four complexity, but there were significantly more level three complexity tasks in Economics than for other subjects in this study. Although this varied across different Awarding Organisations (ranging from 15% to 52%), the Economics assessments provided good opportunities for students to apply mathematics beyond routine application of procedures.

The number of calculations by level of complexity and the overall A level percentage of each level.

Awarding Organisation	A				B				C				D				E			
Complexity	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
AS unit 1	4	0	5	0	5	0	2	0	7	6	4	0	8	8	2	0	3	0	4	0
AS unit 2	0	5	4	0	9	0	3	0	6	3	0	0	7	3	0	0	2	1	3	0
AS total	4	5	9	0	14	0	5	0	13	9	4	0	15	11	2	0	5	1	7	0
A2 unit 3	7	2	1	0	6	1	3	0	10	1	0	0	1	9	2	0	2	4	3	0
A2 unit 4	2	1	0	0	4	1	1	0	10	1	2	0	0	8	4	0	0	1	4	0
A2 total	9	3	1	0	10	2	4	0	20	2	2	0	1	17	6	0	2	5	7	0
A level percentage	42	26	32	0	69	6	26	0	66	22	12	0	31	54	15	0	26	22	52	0

Levels of complexity

1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of Context

Between 90% and 100% of the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). Awarding Organisations A, B and E had one or two question parts where the context had novel aspects (category two), Awarding Organisation D had five, and Awarding Organisation C did not have any. None of the Awarding Organisations used mathematics set in a context that the students were unlikely to have met before (category three).

The number of calculations by familiarity of context and the overall A level percentage for each category.

Awarding Organisation	A			B			C			D			E		
Familiarity of Context	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
AS unit 1	8	1	0	7	0	0	17	0	0	16	2	0	6	1	0
AS unit 2	9	0	0	11	1	0	9	0	0	8	2	0	6	1	0
AS total	17	1	0	18	1	0	26	0	0	24	4	0	12	2	0
A2 unit 3	9	0	0	9	1	0	11	0	0	11	1	0	9	0	0
A2 unit 4	3	0	0	6	0	0	13	0	0	12	0	0	5	0	0
A2 total	12	0	0	15	1	0	24	0	0	23	1	0	14	0	0
A level percentage	97	3	0	94	6	0	100	0	0	90	10	0	93	7	0

Familiarity of context: 1 = typical, 2 = some novel aspects, 3 = unlikely to have been met before

Mathematical skills or subject-specific comprehension

For all five Awarding Organisations, the great majority of question parts with mathematical content required both subject-specific comprehension and mathematical skills (85% to 100%). There were only a handful of questions where the marks related to mathematics were purely for mathematical computation. Awarding Organisations C and D had some questions in their AS papers which used mathematics but for which the marks allocated were for the understanding of economics.

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

Awarding Organisation	A			B			C			D			E		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
AS unit 1	0	0	9	0	0	7	0	3	14	0	7	11	0	0	7
AS unit 2	0	0	9	2	0	10	1	1	7	0	1	9	0	0	6
AS total	0	0	18	2	0	17	1	4	21	0	8	20	0	0	13
A2 unit 3	1	0	9	0	0	10	0	0	11	0	0	12	0	0	9
A2 unit 4	0	0	3	2	0	4	0	0	13	0	0	12	0	0	5
A2 total	1	0	12	2	0	14	0	0	24	0	0	24	0	0	14
A level percentage	3	0	97	11	0	89	2	8	90	0	15	85	0	0	100

Type of mathematics

When it came to categorising the type of mathematics according to the general list of requirements (see the annex to this report), we made some clarifications specific to Economics:

3a) Identify the area of standard shapes. This concerns triangles and quadrilaterals found within graphs. They represent the area bounded by one axis or both axes and lines representing economic variables within the graph.

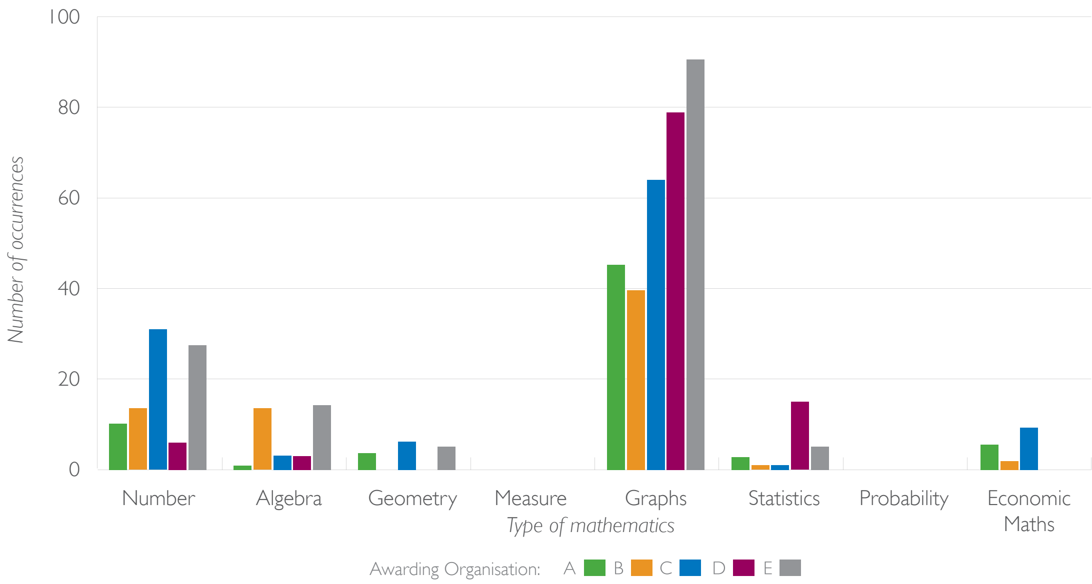
7k) Sketch diagrams or models. Sketching is similar to presenting line graphs, however, it involves no plotting of specific values on axes or correlations between changes on the two axes, so this distinction was noted on the requirements.

9a) Use and interpret indices. Line graphs relate to actual values whereas indices use a base year value (always 100) to establish the y-axis values. This allows for percentage comparisons to be made between the base year and any other year.

9b) Understand and use the relationship between variables over time. For example, a change in the rate of inflation and a change in the rate of unemployment are usually inversely related.

For Awarding Organisations A and B there were instances where the mark scheme did not acknowledge or use the mathematics (a diagram or graph) that was present in the question. The analysts felt this may penalise students who choose to use a mathematical approach in their response.

The number of occurrences of each type of mathematics in A level Economics assessments (summer 2010)



Graphs

The most common mathematical activity was sketching diagrams and models (7k), followed by interpreting and presenting line graphs (7g: on average 5-6 occurrences in each paper). These occurred most frequently in Awarding Organisations C, D and E. Awarding Organisation E had the most instances of translation between various forms of representation (7a).

Number

The most frequent tasks involved using percentages (1e) and basic arithmetic operations (1a). There was also a significant amount of work on the use of proportions and ratios (1f).

Statistics

The only statistics formally evaluated involved interpreting statistical data (5c). Awarding Organisation D had a good spread of this, and Awarding Organisation E used it in the final A2 unit, but it was seldom encountered elsewhere.

Algebra:

There was some algebra, largely the use of formulae (2d), and some understanding and use of equalities and inequalities (2a).

Geometry and Measure:

We found a handful of instances identifying the area of standard shapes (3a), essentially in the first AS unit from Awarding Organisations A, C and E.

Economics-specific mathematics

We found instances of using and interpreting indices (9a) in Awarding Organisations A and C. Organisations A, B and C had more numerous instances of analysing the relationship between variables over time (9c).

Geography

All five of the Awarding Organisations in our study offer an A level in Geography (AQA, CCEA, Edexcel, OCR and WJEC). The assessments could involve coursework and/or essay questions that build on students' research and fieldwork. We did not include these questions/units in our analysis, as the mathematical content can vary considerably from student to student. AQA had two options for its final A2 unit, but as the mathematical elements within these were comparable, we have used an average over the two units for our analysis. We have anonymised the five Awarding Organisations, using letters A, B, C, D and E.

Fieldwork and use of prior work in responses

Some questions refer to students using their own work, data or fieldwork. The completion of fieldwork provides students with opportunities to develop a range of mathematical skills, in particular surrounding the use of primary data, but we could not measure this in our analysis and further research is needed.

Our analysis also excludes essay questions for which students were able to write about their own research or fieldwork. This is because some essays did not require mathematical content in the responses, whereas others leant themselves more to using mathematical techniques and statistical research methods. The type and extent of mathematics used would depend on the context of the fieldwork, the teacher's approach and the individual student's aptitude for mathematics.

Extent – proportion of question parts with mathematical content

We found a fair variation between Awarding Organisations, with the percentage of question parts with mathematical content ranging from 25% to 65%.

The percentage of question parts with mathematical content.

Awarding Organisation	A	B	C	D	E
AS average	64	56	21	45	28
A2 average	11	100	38	39	93
A level average	32	65	25	43	47

Extent – number of marks for mathematical content

On a number of question papers, students have a choice of questions. We calculated percentages by taking into account all options, excluding coursework and essay questions where mathematical content was indeterminate, to give an overall approximate indication of the extent of mathematics. The presence of multiple essay options in Awarding Organisations, A, B, C and D may have had a downward impact on the A2 averages, in which case the AS averages are more indicative of the mathematical content. The actual percentage of mathematical content for a particular choice of questions could be higher or lower depending on the questions the student chooses to answer.

There was a spread from 10% to 20% for the marks requiring mathematics. This was a higher percentage than for Business Studies and Economics, but lower than Psychology.

The number of marks that could be gained using a mathematical route, was between 19% and 35% of the total. With the exception of Awarding Organisation C, we found a substantial variation between marks that required mathematics and marks that could be gained using mathematics. This variation was comparable with that found in Psychology but less than that found in Business Studies and Economics. Additional variation given by the choice of questions meant there could be a marked difference between the mathematics used by two students achieving the same or similar grades.

Measure 1: Percentage of marks that required mathematics

Awarding Organisation	A	B	C	D	E
AS average	26	26	17	20	8
A2 average	6	10	15	17	12
A level average	16	20	16	19	10

Measure 2: Percentage of marks that could be gained using mathematics

Awarding Organisation	A	B	C	D	E
AS average	43	38	20	27	12
A2 average	7	29	16	24	40
A level average	25	35	19	26	26

Difficulty – Number of steps

The vast majority of the calculations across all Awarding Organisations required only a single step (ranging from 79% to 100 % for A and B). Awarding Organisation D had the greatest number of multi-step calculations (21%), but no extended calculations were used by any of the Awarding Organisations.

The number of single (S), multi-step (M) and extended (E) calculations for each unit, and the overall A level percentage of each type.

Awarding Organisation	A			B			C			D			E		
Calculation steps	S	M	E	S	M	E	S	M	E	S	M	E	S	M	E
AS total	23	0	0	10	0	0	9	0	0	18	3	0	9	1	0
A2 total	6	0	0	5	0	0	4	1	0	4	3	0	14	0	0
A level percentage	100	0	0	100	0	0	93	7	0	79	21	0	96	4	0

Difficulty – Complexity of task

Awarding Organisation B had three mathematical tasks of level four complexity, a rare occurrence across all subjects in our study. Awarding Organisations A and B each had three tasks of level three complexity. Awarding organisations C, D and E only had tasks of level one or two complexity. While there is fair variation across the Awarding Organisations, Geography assessments provide good opportunities for students to apply mathematics beyond routine application of procedures.

The number of calculations by level of complexity and the overall A level percentage of each level.

Awarding Organisation	A				B				C				D				E			
Complexity	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
AS total	18	3	2	0	4	4	1	1	2	7	0	0	15	6	0	0	6	4	0	0
A2 total	1	4	1	0	0	1	2	2	0	5	0	0	2	6	0	0	13	1	0	0
A level percentage	66	24	10	0	27	33	20	20	14	86	0	0	59	41	0	0	79	21	0	0

Levels of complexity

1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of context

Between 82% and 96% of the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). All Awarding Organisations had at least one mathematical task where the context had some novel aspects, with D having five such tasks (category two). None of the Awarding Organisations' used mathematics set in a context that the students were unlikely to have met before (category three).

The number of calculations by familiarity of context and the overall A level percentage for each category.

Awarding Organisation	A			B			C			D			E		
Familiarity of context	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
AS total	22	1	0	8	2	0	9	0	0	19	2	0	10	0	0
A2 total	2	4	0	5	0	0	4	1	0	4	3	0	13	1	0
A level percentage	83	17	0	87	13	0	93	7	0	82	18	0	96	4	0

Mathematical skills or subject-specific comprehension

The majority of question parts for all Awarding Organisations required both subject-specific comprehension and mathematical skills (58% to 93%). Marks gained purely through mathematical computation were found across all Awarding Organisations (7% to 21%). Awarding Organisations, A, D and E, each had an AS question which used mathematics, but for which the marks allocated were for geographical understanding. Awarding Organisation E had a further six such questions at A2, representing around a third of the questions with mathematical content. Overall, the pattern was different to other subjects, and a more detailed comparative analysis across Awarding Organisations would be interesting.

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

Awarding Organisation	A			B			C			D			E		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
AS total	6	1	16	2	0	8	1	0	8	3	2	16	0	1	9
A2 total	0	0	6	0	0	5	0	0	5	0	0	7	3	6	5
A level percentage	21	3	76	13	0	87	7	0	93	11	7	82	13	29	58

Type of mathematics

When it came to categorising the type of mathematics according to the general list of requirements (see page 53), we made some clarifications specific to Geography:

4a) interpret scales on a range of measuring instruments and in a range of situations – this was introduced to cover interpretation of scales in settings that did not necessarily involve instruments.

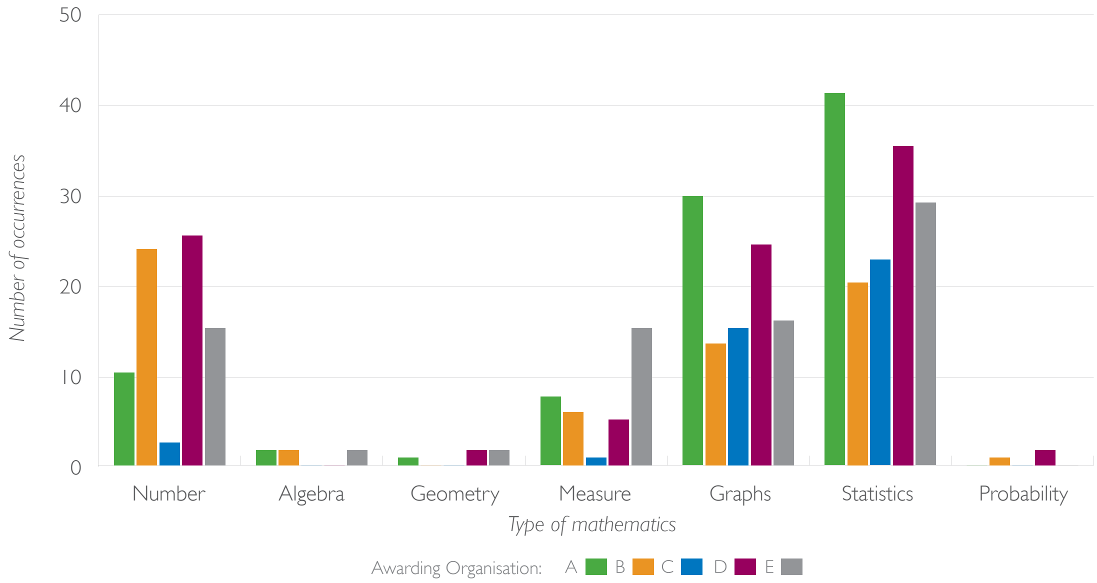
4d) use and interpret 4 and 6 figure co-ordinates

7k) use and interpret triangular graphs

Both 4d and 7k are geography-specific mathematical elements. There are also some geographical techniques that have mathematical aspects, for example cost-benefit analysis and risk assessments, but we did not specifically highlight these as techniques. Instead, we recorded the component mathematical skills using the mathematical requirements list.

There was variation across the Awarding Organisations on the different mathematical elements covered and their frequency, but there was fair coverage of the topics (other than algebra, geometry and probability) across Awarding Organisations. Awarding Organisation C had somewhat fewer instances of mathematical tasks.

The number of occurrences of each type of mathematics in A level Geography assessments (summer 2010)



Statistics

We found significant use of statistical research methods and analysis in the Geography assessments and therefore the more detailed 8a – 8s listing of statistical requirements was used in the mapping exercise instead of the 5a – 5g listing. Common requirements found were: knowing and understanding statistical research methods including field and natural experiments observations; sampling and self reporting techniques (8a: i-iii); drawing conclusions from summary data and graphs (8o); and, collectively across all Awarding Organisations, instances of all the other statistical criteria.

Graphs

Use of graphs was the second most common type of mathematics: interpreting and presenting bar charts (7e); translating between text, graphical, and numerical form (7a); and interpreting and presenting line graphs (7g).

Number

The most common number tasks involved using percentages (1e), followed by basic arithmetic operations (1a), and the use of proportions and ratios (1f).

Geometry and Measure

We found considerable use and interpretation of maps and scale drawings (4b) across all Awarding Organisations. There were also instances of geography-specific mathematics, use of *4 and 6 figure co-ordinates* and use of *triangular graphs* and *choropleths*.

Algebra

There was some use of equalities/inequalities but with no explicit algebraic manipulation.

Additional notes

Use of primary or secondary data

How students interpret or interrogate data differs according to whether it is primary or secondary data. Students may use mathematical techniques in primary data collection that we were not able to assess in the paper assessments. However, analysis and presentation of data are skills that are assessed in the paper assessments.

Fieldwork

The completion of fieldwork provides students with the opportunity to develop a range of mathematical skills, in particular surrounding the use of primary data.

Psychology

Edexcel, OCR and WJEC each offer an A level in Psychology, while AQA offers two. We have included both AQA A levels in our study and therefore the letters A, B, C, D and E refer to the five different A levels rather than Awarding Organisations. There are four externally assessed units, two AS and two A2.

Extent – proportion of question parts with mathematical content

We found a marked difference between A levels, with the percentage of question parts with mathematical content ranging from 37% to 91%. There was a higher proportion of question parts with mathematics in the A2 papers than the AS papers, with one exception.

The percentage of question parts with mathematical content.

A level	A	B	C	D	E
AS unit 1	50	57	61	100	86
AS unit 2	50	35	47	6	89
AS average	50	46	54	53	88
A2 unit 3	60	8	64	69	96
A2 unit 4	54	47	47	61	93
A2 average	57	28	56	65	95
A level average	54	37	55	59	91

Note: For multiple choice questions, if only one option had mathematical content and it was not the correct option, then the question was judged to have no mathematical content.

Extent – number of marks for mathematical content

On a number of question papers, students have a choice of questions. We calculated percentages by taking into account all options and giving an overall indication of the extent of mathematics. The actual percentage for a particular choice of questions would be higher or lower depending on the questions the student chooses to answer.

The percentage of marks requiring mathematics ranged from 10% to 46%. The percentage of marks that could be gained using a mathematical route was between 24% and 50%. The variation between marks that required mathematics and marks that could be gained using mathematics was much smaller in Psychology than in Business Studies, Economics and Sociology.

The variation in mathematical content of the examples and approaches students use in the their answers, coupled with their ability to choose questions, means there can be a marked difference between the mathematics used by two students achieving the same or comparable grades. Depending on individual choices, students may use mathematics in anything from one seventh to three quarters of their responses.

Measure 1: Percentage of marks that required mathematics

A level	A	B	C	D	E
AS unit 1	22	32	27	88	32
AS unit 2	6	11	20	47	33
AS average	14	22	24	67	33
A2 unit 3	3	04	18	17	37
A2 unit 4	10	26	12	31	18
A2 average	6	15	15	24	28
A level average	10	18	20	46	30

Measure 2: Percentage of marks that could be gained using mathematics

	A	B	C	D	E
AS unit 1	38	47	42	88	32
AS unit 2	26	20	26	56	50
AS average	32	34	34	72	41
A2 unit 3	19	04	24	22	49
A2 unit 4	22	26	20	33	22
A2 average	21	15	22	27	35
A level average	26	24	28	50	38

Difficulty – number of steps

The vast majority of the calculations required only a single step (62% to 80%), while 19% to 38% were multi-step. A levels B and E did not use any extended calculations, and extended calculations represented only 2% of the total for calculations for A levels A, C and D.

The number of single (S), multi-step (M) and extended (E) calculations for each unit, and the overall A level percentage of each type.

A level	A			B			C			D			E		
	S	M	E	S	M	E	S	M	E	S	M	E	S	M	E
AS unit 1	60	40	0	100	0	0	79	21	0	50	50	0	100	0	0
AS unit 2	78	22	0	91	9	0	64	29	7	58	42	0	31	69	0
AS average	69	31	0	96	5	0	72	25	4	54	46	0	66	35	0
A2 unit 3	100	0	0	33	66	0	89	11	0	82	18	0	85	15	0
A2 unit 4	60	33	7	22	78	0	88	13	0	64	27	9	100	0	0
A2 average	80	17	4	28	72	0	89	12	0	73	23	5	93	8	0
A level average	75	24	2	62	38	0	80	19	2	64	34	2	79	21	0

Difficulty – Complexity of task

The vast majority of calculations were categorised as levels one or two for complexity. Awarding Organisation A had the highest proportion of level one calculations, but it also had the highest number of level three and four calculations. A levels C and E did not have any level three or four calculations.

The number of calculations by level of complexity and the overall A level percentage of each level.

A level	A				B				C				D				E			
complexity	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
AS units 1	3	5	1	1	5	8	0	0	10	4	0	0	4	10	0	0	2	4	0	0
AS unit 2	5	4	0	0	6	5	0	0	4	10	0	0	21	5	0	0	2	14	0	0
AS percentage	42	47	5	5	46	54	0	0	50	50	0	0	63	38	0	0	18	82	0	0
A2 units 3	9	0	0	0	2	1	0	0	11	7	0	0	13	9	0	0	21	5	0	0
A2 unit 4	9	4	2	0	2	6	1	0	2	6	0	0	5	5	1	0	0	13	0	0
A2 percentage	75	17	8	0	33	58	8	0	50	50	0	0	55	42	3	0	54	46	0	0
A level total percentage	60	30	7	2	42	56	3	0	50	50	0	0	59	40	1	0	41	59	0	0

Levels of complexity

1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of context

Between 54% and 81% of the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). Between 5% and 46% of the question parts used mathematics where the context had novel aspects (category two). The papers for A level E were the only ones to use mathematics set in a context that the students were unlikely to have met before (category three).

The number of calculations by familiarity of context and the overall A level percentage for each category.

A level	A			B			C			D			E		
Context	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
AS total	17	2	0	16	11	0	16	12	0	23	17	0	12	0	10
A2 total	18	6	0	5	7	0	26	0	0	26	7	0	36	3	0
A level total percentages	81	19	0	54	46	0	78	22	0	67	33	0	79	5	16

Familiarity of context: 1 = typical, 2 = some novel aspects, 3 = unlikely to have been met before

Mathematical skills or subject-specific comprehension

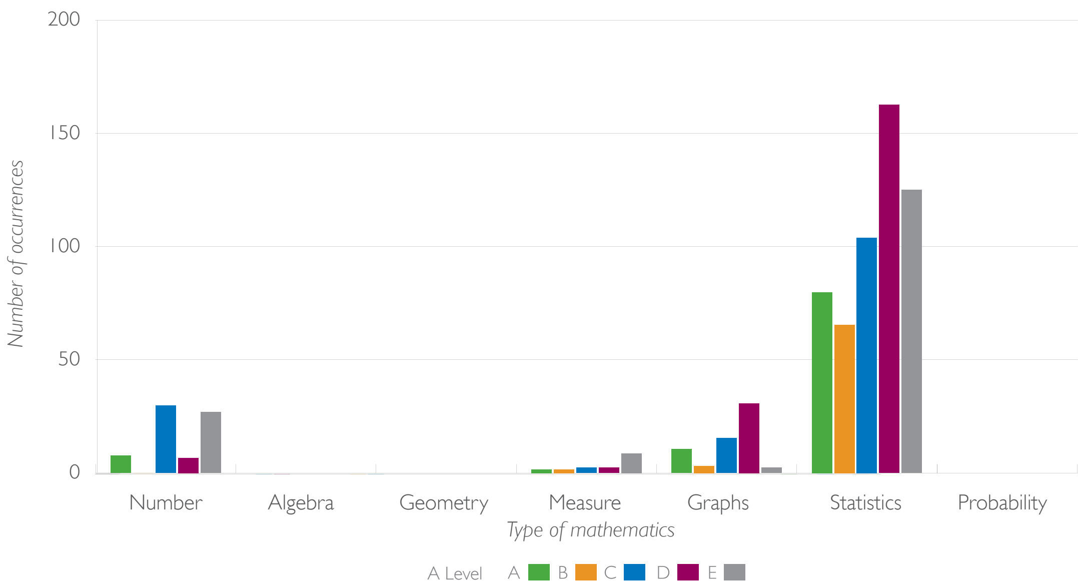
With the exception of one A level (A), most question parts with mathematical content required both subject-specific comprehension and mathematical skills. A levels B, C, D and E all awarded some marks (between 3% and 13%) purely for mathematical skills, but for A, this was over 60%. Although it should be noted that A level A had the least total value of marks that required mathematics (10%).

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

	A			B			C			D			E		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
AS total	7	4	8	0	0	24	4	0	24	1	0	39	0	0	22
A2 total	20	0	4	1	4	7	3	0	23	2	0	31	7	11	21
A level total as a percentage	63	9	28	3	11	86	13	0	87	4	0	96	11	18	70

Type of mathematics

The number of occurrences of each type of mathematics in A level Psychology assessments (summer 2010)



Statistics

Statistics was the most commonly found type of mathematics: research methods; design and presentation; and analysis and interpretation. Due to the statistical content in the Psychology papers, we completed two separate analyses. Firstly, we analysed the papers using statistics in the generic list of mathematical requirements (5a – 5g). In this analysis, the comparison of qualitative and quantitative methods was not considered to be mathematics. Our second analysis was completed using a framework of requirements with detailed content on statistical elements, especially for research methods (8a – 8r). Part 8j-8r of the mathematical requirements in this second analysis captured the comparison of quantitative and qualitative methods.

Statistics 1 (5a – 5g): We found many question parts involving understanding and using the principles of sampling and interpreting statistical data. There were some parts using statistical tests, calculating averages and framing null or alternate hypotheses. The greatest spread of statistical requirements was found in papers from A level C and the least in those from A level B.

Statistics 2 (8a – 8r): We found examples of all the areas of statistics listed in the requirements from 8a to 8r. Knowledge and understanding of statistical research methods had a very high incidence. We also found many instances of identifying strengths and weaknesses of research methods; commenting on reliability and validity; and evaluating the strengths and weaknesses of any aspect of investigations, including designs and samples.

Number

Many question parts involved using percentages, and we also found a significant amount of work on the four basic operations and the use of proportions and ratios. The papers for A level B did not have any number content. A level C had the greatest spread of number content, incorporating all of the requirements listed under number (other than using binary) being found across the A level.

Graphs

Most of the graph work found in the A level papers concerned recognising correlations and lines of best fit, and interpreting and presenting tables. We found a handful of occurrences of scatter graphs and histograms, but no evidence of bar charts, line graphs, pie charts or log scales.

Measure

We found a few question parts across the papers that included interpreting scales on a range of measurements. The papers from A level E had the highest incidence of these.

Probability

Students are required to complete non-assessed coursework involving elements of probability. They need to have completed this work in order to answer the theory paper questions, but we did not capture this in our analysis of the assessment papers.

Algebra

We did not find any examples of algebra.

Sociology

Three of the five Awarding Organisations in our study offer an A level in Sociology (AQA, OCR and WJEC), although AQA has at least 80% of the market share. There are four externally assessed units: two AS and two A2. We have anonymised the three Awarding Organisations, using letters A, B and C.

Extent – proportion of question parts with mathematical content

We found considerable variation between Awarding Organisations on the overall percentage of question parts with mathematical content, ranging from 43% to 85%. Although these percentages look substantial, the mathematical content was limited and they are a measure of the spread of the mathematics rather than the volume (see next measure of extent).

The percentage of question parts with mathematical content.

Awarding Organisation	A	B	C
AS unit 1	86	67	25
AS unit 2	100	44	61
AS average	93	55	55
A2 unit 3	67	38	67
A2 unit 4	90	18	100
A2 average	79	30	78
A level average	85	43	65

Extent – number of marks for mathematical content

On a number of question papers, students had a choice of questions. We calculated percentages by taking into account all options and giving an overall indication of the extent of mathematics. The actual percentage for a particular choice of questions would be higher or lower depending on the questions the student chose to answer.

Sociological studies referenced by students in their answers ranged from those that were highly statistical in nature, to those that were purely theoretical. Similarly, both qualitative and quantitative research was used. Both these factors affected the level of mathematical content in students' responses, and it was possible to achieve a high mark using references to studies with high mathematical content **or** to achieve the same mark using references to studies with *no* mathematical content. In other words, the number of marks that *required* the student to use mathematics was very low (1-3%).

The percentage of marks that could be gained using a mathematical route ranged between 12% and 19%. Again, some papers contained multiple options, some of which provided minimal mathematical opportunities.

Measure 1: Percentage of marks that required mathematics

Awarding Organisation	A	B	C
AS unit 1	4	2	2
AS unit 2	4	1	4
AS average	4	1	3
A2 unit 3	1	1	1
A2 unit 4	3	0	2
A2 average	2	0	1
A level average	3	1	2

Measure 2: Percentage of marks that could be gained using mathematics

Awarding Organisation	A	B	C
AS unit 1	22	24	2
AS unit 2	20	23	10
AS average	21	24	8
A2 unit 3	14	9	13
A2 unit 4	23	2	21
A2 average	18	6	15
A level average	19	14	12

Difficulty – Number of steps

The majority of calculations were single or multi-step, with Awarding Organisations A and B having predominantly multi-step calculations (100% and 77% respectively). Awarding Organisation C had the highest percentage of single-step calculations (62%), but was the only Organisation to use extended calculations (15%).

The length and multi-mark nature of Sociology questions means that on certain question parts students may discuss, for example, percentage changes as well as money and averages. However, as these separate domains are not used when discussing the same point, we considered these calculations to be multi-step rather than extended.

The number of single (S), multi-step (M) and extended (E) calculations for each unit, and the overall A level percentage of each type.

Awarding Organisation	A			B			C		
Calculation steps	S	M	E	S	M	E	S	M	E
AS unit 1	0	6	0	0	10	0	1	0	0
AS unit 2	0	8	0	2	6	0	7	4	0
AS total	0	14	0	2	16	0	8	4	0
A2 unit 3	0	9	0	3	3	0	3	1	4
A2 unit 4	0	9	0	1	1	0	5	1	0
A2 total	0	18	0	4	4	0	8	2	4
A level percentage	0	100	0	23	77	0	62	23	15

Difficulty – Complexity of task

While none of the mathematical tasks were categorised as level four in complexity , the majority of tasks used by Awarding Organisation A were categorised as level three (69%). Tasks from Awarding Organisations B and C were almost all categorised as level one or two in complexity, with a healthy percentage of level two ranging from 22% to 35%.

The number of calculations by level of complexity and the overall A level percentage of each level.

Awarding Organisation	A				B				C			
Complexity	1	2	3	4	1	2	3	4	1	2	3	4
AS unit 1	0	0	6	0	4	5	1	0	1	0	0	0
AS unit 2	0	3	5	0	6	2	0	0	10	1	0	0
AS total	0	3	11	0	10	7	1	0	11	1	0	0
A2 unit 3	0	2	7	0	4	2	0	0	2	6	0	0
A2 unit 4	3	2	4	0	2	0	0	0	5	1	0	0
A2 total	3	4	11	0	6	2	0	0	7	7	0	0
A level percentage	9	22	69	0	62	35	4	0	69	31	0	0

Levels of complexity
1. Straightforward/routine – requires recall of procedures and relatively straightforward application.
2. Requires application and understanding of maths within one domain.
3. Requires understanding/use of maths across domains and necessitates a decision about the direction in which to proceed.
4. Complex activity requiring synthesis across a number of domains with structuring/decision-making being necessary.

Difficulty – Familiarity of context

All the mathematical content was set in familiar contexts, meaning it would have been covered in the student learning programme (category one). While the studies presented in the written assessments would likely be unfamiliar to students (other than in cases of pre-release material), we felt the mathematics within them was presented in a familiar way.

The number of calculations by familiarity of context and the overall A level percentage for each category.

Awarding Organisation	A			B			C		
Familiarity of Context	1	2	3	1	2	3	1	2	3
AS unit 1	6	0	0	10	0	0	1	0	0
AS unit 2	8	0	0	8	0	0	11	0	0
AS total	14	0	0	18	0	0	12	0	0
A2 unit 3	9	0	0	6	0	0	8	0	0
A2 unit 4	9	0	0	2	0	0	6	0	0
A2 total	18	0	0	8	0	0	14	0	0
A level percentage	100	0	0	100	0	0	100	0	0

Familiarity of context: 1 = typical, 2 = some novel aspects, 3 = unlikely to have been met before

Mathematical skills or subject-specific comprehension

For all three Awarding Organisations, all question parts with mathematical content required both subject-specific comprehension and mathematical skills.

The number of question parts where the mathematical content required mathematical skills (MS), subject-specific comprehension (SSC), or both (BOTH), and the overall A level percentage for each category.

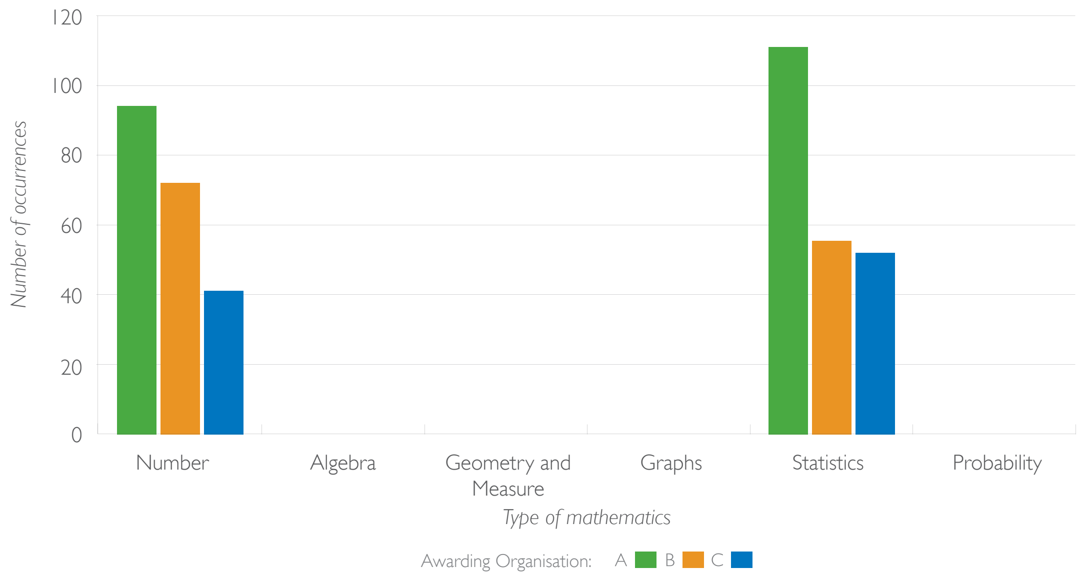
Awarding Organisation	A			B			C		
	MS	SSC	BOTH	MS	SSC	BOTH	MS	SSC	BOTH
Unit 1	0	0	6	0	0	10	0	0	1
Unit 2	0	0	8	0	0	8	0	0	11
AS total	0	0	14	0	0	18	0	0	12
Unit 3	0	0	9	0	0	6	0	0	8
Unit 4	0	0	9	0	0	2	0	0	6
A2 total	0	0	18	0	0	8	0	0	14
A level percentage	0	0	100	0	0	100	0	0	100

Type of mathematics

When analysing the mathematical content of the assessments, a detailed list of statistical requirements (arising from the Psychology specifications) were considered alongside the more basic list of statistical requirements (criteria 5 in the list of requirements). Depending on the studies that the students chose to reference in their responses, their answers may have included many of the statistical requirements listed, none of the statistical requirements listed, or anywhere in between. As a result, we decided not to use the more detailed statistical criteria as no meaningful conclusions could be drawn from it.

The predominant mathematics in Sociology assessments, across all Awarding Bodies, involved statistics and number.

The number of occurrences of each type of mathematics in A level Sociology assessments (summer 2010)



Statistics

The statistics required was primarily understanding and using the principles of sampling (5b); and interpreting statistical data (5c). We found roughly half as many instances of using the mean, median, mode (5d); and using the range, quartiles and interquartile range (5f).

Number

The number work was related to the statistical computations and analysis in questions, involving basic arithmetic operations (1a); using percentages (1e); and using proportions and ratios (1f).

Annex 2: Research methodology

Research design

We used papers from the summer 2010 assessments, as these were the most recent set of papers publicly available at the time of the SCORE research. For its study, SCORE selected the Awarding Organisations that offered single science A levels in the UK: AQA, CCEA, Edexcel, OCR and WJEC. For reasons of comparability, we included the same organisations in our study.

We used the SCORE framework, with a variation that allowed us to account for separate measures relating to the extent of mathematical content (mathematics required and mathematics that could be used). Prior to its use, the SCORE framework was piloted to test its validity and the effectiveness of the analysis process.

We established groups of subject experts (nine to twelve members) to analyse the assessments. These groups included A level teachers, former teachers employed in research and development, and people employed by Awarding Organisations as markers, question writers or examiners.

At the start of a group meeting, subgroups of two to four analysts looked at sample A level questions, along with instructions for analysing the questions. They then undertook a standardisation exercise, which was followed by a discussion to reach agreement on the meaning of the measures. The measures were then judged by another subgroup of two to four members. The groups were given the A level papers and mark schemes, the specifications, data sheets, and all other materials from the Awarding Organisations necessary to fully comprehend the demand and scope of the complete A level.

The composition of the groups was changed throughout the process in order to verify the reliability of judgements. Any new issues arising were addressed and clarified as a whole group in order to reach agreement in line with existing measures and to ensure consistency of approach. At the end of each analysis session, groups checked their final judgements against those made at the start of the day. While analysts were familiar with most of the assessment materials, some judgements regarding the Welsh and Irish papers were subjective, but informed.

Using a printed framework (see page 56), the following data was gathered:

1. **Question number and part**: this allowed for the number of questions parts with mathematical content to be worked out as a proportion of the total number of questions parts, giving a measure of the extent of the mathematical content.

2. **The number of marks that required mathematics**: the total number of marks that required mathematics were measured. Whole marks were counted rather than parts of marks. The number of marks that required mathematics was then worked out as a proportion of the marks overall, giving a second measure of the extent of the mathematical content.

3. **The number of marks that could be gained using mathematics**: the maximum number of marks that could be achieved using mathematical content/approaches was measured. Whole marks were counted rather than parts of marks. The number of marks that could be gained using mathematics was then worked out as a proportion of the marks overall, giving a second measure of the extent of the mathematical content.

For measures 2 and 3, we included all options, even when students could choose only one. Consequently, the resulting proportion does not represent any one student's trajectory, but rather gives the extent for all options considered collectively in the absence of any choices. We believe this is the best way to offer a meaningful average given the desire to map what is available.

4. **The type of mathematics**: Lists of mathematical requirements (see page 53) building on the A level specifications were used for this purpose. All of the requirements needed for each question part were recorded.

5. **The number of steps in a calculation**: this measure of difficulty discriminated between questions where only one step was needed to gain the solution to a problem (single step), questions where more than one step was needed in one calculation to gain the solution to a problem (multi-step), and questions where a value, for example x, had to be found and that value, x, used in a subsequent calculation in order to find the solution to the problem, y (extended calculation). It could be argued that questions with multiple steps may require students to use higher order skills, and extended calculations could be said to require extended reasoning.

6. **The complexity of the task**: this measure of difficulty established the complexity of the task as defined by Geoff Wake's[8] four descriptions of increasing difficulty. Category one is defined as straightforward or routine, requiring recall of procedures and relatively straightforward application. Category two requires application and understanding of mathematics in one domain. Category three requires understanding and use of mathematics across domains, and necessitates a decision about the direction in which to proceed. Category four involves complex activity requiring synthesis and application across a number of domains, with structuring or decision-making being necessary.

7. **Familiarity of context**: if a context is more familiar it is generally accepted that applying mathematics is easier than if the context is unfamiliar. Three categories were used to judge the familiarity of the task in relation to recognising which mathematics to apply, as defined by Geoff Wake's work referenced above: (1) typically met through the learning programme, (2) has some novel aspects, and (3) situation unlikely to have been met before.

8. **Mathematics skills or scientific comprehension**: This measure considers the balance between comprehension of the subject and mathematical skills. The three categories correspond to whether the marks were awarded purely for mathematical skills, purely for subject-specific understanding, or both.

8 Drake P, Wake G. and Noyes A (in press) 'Assessing `functionality´ in school mathematics examinations: what does being human have to do with it?' Research in Mathematics Education

Mathematics requirements

The following list of mathematical requirements was built around the A level specifications, taking into account the content of GCSE mathematics. We found that Psychology and some of the social sciences drilled deeper into statistical methods, especially on design and evaluation of statistical experiments. As a result, we have supplemented the original data handling section (section 5), with a more detailed listing of statistical research methods (section 8). We also made some additions and changes to the list during our analysis in order to capture information regarding the mathematical content from the A level papers. These changes are <u>underlined</u>.

Quantitative Methods in A levels Mathematical requirements

1 Number

a. add, subtract, multiply and divide whole numbers
b. recognise and use expressions in decimal form
c. recognise and use expressions in standard form
d. use fractions
e. use percentages, including changes as percentages
f. use proportions and ratios
g. use <u>and understand</u> binary <u>and other</u> number systems (amended for Computing)
h. handle calculations involving appropriate numbers of significant figures
i. handle calculations involving rounding errors
j. Recognise number type, including Boolean (inserted for Computing)

2 Algebra

a. understand and use the symbols =, <, <<, >>, >, ~, μ, <u>and the notion of inequalities</u> (amended for Economics)
b. solve simple algebraic equations
c. develop and use algorithms
d. use formula
e. generate terms of a sequence using term to term definitions
f. use linear expressions to describe the nth term of an arithmetic sequence
g. use and understand functions (inserted for Computing)

3 Geometry

a. identify area of standard shapes (inserted for Economics)
a. measure and appreciate angles in 2-D and 3-D shapes (inserted for Geography)
b. visualise and interpret two-dimensional representations of 3-D objects (inserted for Geography)
c. calculate areas of triangles, circumferences and areas of circles, surface areas and volumes of rectangular blocks, cylinders and spheres (inserted for Geography)
d. use Pythagoras' theorem, similarity of triangles and the angle sum of a triangle (inserted for Geography) use sines, cosines and tangents in physical problems (inserted for Geography)

4 Measure

a. interpret scales on a range of measuring instruments and in a range of situations (amended for Geography)
b. use and interpret maps and scale drawings
c. convert measurements from one unit to another
d. use and interpret 4 and 6 figure coordinates (inserted for Geography)

5 Statistics

a. select, use and understand simple statistical tests
b. understand and use the principles of sampling
c. interpret statistical data
d. calculate/interpret the mean, median, mode (amended for Psychology)
e. calculate and use standard deviation
f. calculate/interpret the range, quartiles and interquartile range (amended for Psychology)
g. frame null hypothesis

6 Probability

7 Graphs

a. translate information between text, graphical, numerical and algebraic forms (amended for Computing)
b. interpret and present flowcharts
c. interpret and present scatter graphs
d. recognise correlation and draw and / or use lines of best fit by eye, understanding what these represent
e. interpret and present bar charts
f. interpret and present histograms
g. interpret and present line graphs
h. interpret and present pie charts
i. interpret and present tables
j. use and interpret log scales
k. sketch diagrams or models (inserted for Economics)
k. use and interpret triangular graphs (inserted for Geography)

Quantitative methods in A levels Statistical requirements

8 Statistical research methods

a. Know and understand the following research methods:
   i. Experiments, including laboratory, field and natural experiments
   ii. Observations, including participant and structured observation, time sampling and event sampling
   iii. Self reporting techniques, including questionnaire and interview, rating scales and open and closed questions
   iv. Studies using correlation

Design

b. Identify strengths and weaknesses of research methods, in general terms and in relation to source material
c. Identify variables, including distinguishing between independent and dependent variables

d. Suggest how variables may be operationalised and measured
e. Control for extraneous variables
f. Be familiar with features of experimental design, including independent groups, repeated measures and matched pairs
g. Frame hypotheses – null and alternate, one- and two-tailed
h. Comment on reliability and validity of measurements
i. Be familiar with the following features of sampling:
    i. Selection of participants and sampling techniques, including random, opportunity and volunteer/self-selected sampling
    ii. Identify strengths and weaknesses of sampling techniques, including for source material

Presentation, analysis and interpretation

j. Distinguish between, and identify strengths and weaknesses of, qualitative and quantitative data
k. Sketch/present appropriate summary graphs, scatter plots, tables
l. Use measures of central tendency, including mean, median, mode
m. Use measures of dispersion, including range and standard deviation
n. Analyse and interpret correlation, including using correlation coefficients
o. Draw conclusions from summary data and graphs
p. Use inferential analysis/non-parametric tests, including sign test, Chi-square, Wilcoxon, Mann-Whitney, Spearman
q. Know and use levels of significance, including interpretation of Type1/Type 2 errors and probability
r. Evaluate strengths and weaknesses of any aspect of the investigation, including knowledge and experience of possible future research and alternative designs and samples

Additional subject-specific requirements

9 Business Studies-related mathematics

e. understand, construct and use critical path analysis
f. use of bracket notation
g. use of negatives to denote inverse relationships
h. construct and interpret decision trees

9 Computing-related mathematics

a. understand and use the set and Boolean operators AND, OR, NOT, $\cup$, $\cap$, IF, THEN, ELSE
b. understand and manipulate arrays
c. understand and use graph theory

9 Economics-related mathematics

d. use and interpret indices
e. understand and use the relationship between variables over time

Analysis framework

Framework for analysis of A level assessments

Subject:				Awarding Organisation:		Unit:							
Q number and part	Number of steps			Type of maths	Complexity of task	Number of marks that require maths	Number of marks that could be gained with maths	Are the associated marks purely for maths skills or do they require subject-specific comprehension?			Familiarity of context		
	Single	Multi	Extended calculation		1 to 4			Maths skills	Subject-specific comp	both	1	2	3

Published by the Nuffield Foundation, 28 Bedford Square, London WC1B 3JS
Telephone +44 (0)20 7631 0566

Available in print and electronic formats.

www.nuffieldfoundation.org

ISBN 978-0-904956-85-6