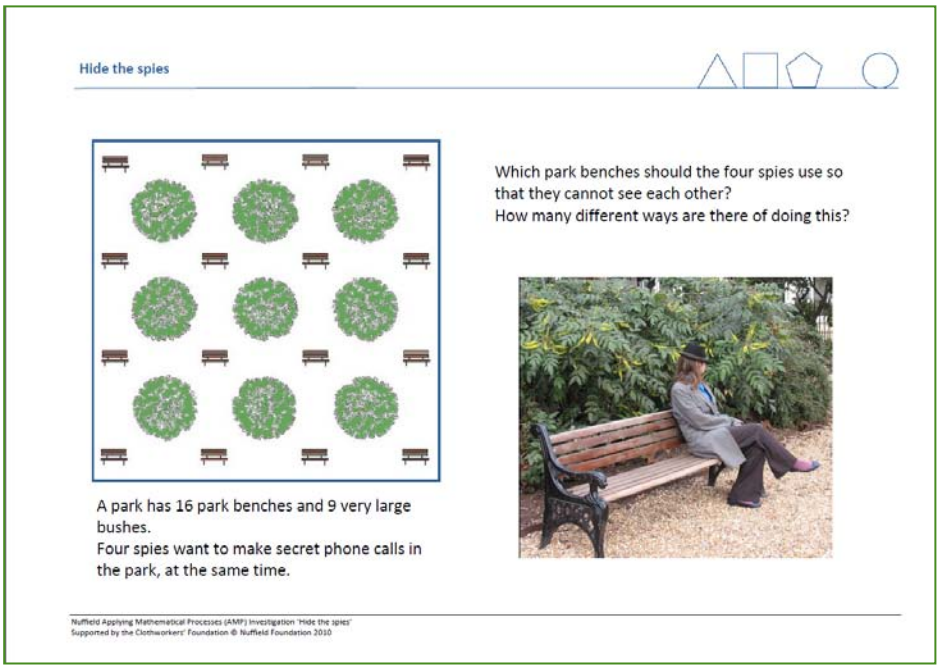
Activity description

Pupils investigate where four spies should sit in a square park so that they cannot see each other, and how many such arrangements are possible.



Suitability

Pupils working as individuals or pairs

Time

Up to 1 hour

AMP resources

Pupil stimulus
Print worksheets
PDF Interactive

Equipment

Counters, cubes, cups, etc. to simulate spies and bushes

Key mathematical language

lines of sight, arrangement/configuration, symmetry, reflection, rotation

Key processes

Representing Choosing a mathematical approach to finding arrangements.

Analysing Working systematically, generating arrangements, using symmetry.

Interpreting and evaluating Identifying patterns and making generalisations, considering findings to form convincing arguments.

Communicating and reflecting Explaining the approach taken and the outcomes achieved at each stage of the work.

Teacher guidance

Explain the context of the problem and emphasise that no spy should be able to see another spy. Preliminary discussion may be needed so that pupils understand how the very large bushes obscure vision. It might be worth investigating lines of sight within the school to help support understanding.

The interactive PDF provided can be used to introduce the task, or you may prefer to model the problem physically, perhaps using pupils and desks, or using counters and the worksheet provided.

Starting with one spy sitting on a bench, ask pupils to find all the benches where another spy could sit so that the spies cannot see each other. Invite one or two pupils to present their findings.

Pupils can use the interactive PDF, or printed worksheets, to investigate the problem. Pupils could also record solutions on squared paper.

During the activity

Encourage pupils to explore their ideas, generating their own systems for finding results, but highlight the need to work systematically. This could be done as a group discussion at some stage in the investigation, asking pupils to share how they have been organising their work.

Showing each spy in a different colour can help pupils to be more systematic, but it can lead to an unwieldy number of results if permutations are taken into account. Stress that the task is about which park benches the spies are sitting on, rather than the individual spies.

Probing questions and feedback

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- Pupils who have two diagrams that are equivalent, say via a rotation, can be engaged in a discussion on 'equating' congruent/symmetric arrangements.
- How are you deciding what result to find next?
- Have you found all the different ways? How can you be certain?
- Could you have 5 spies for a '4x4' array of benches? Why not?

Extensions

- Exploring connections between Hide the spies and Sudoku.
- Using different sizes of rectangular parks, and/or different numbers of spies.
- Numbering the park benches as shown on the right, and adding together the numbers of the park benches that the spies are sitting on: What do pupils notice? Ask them to investigate and explain any patterns they find.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

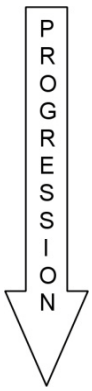
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

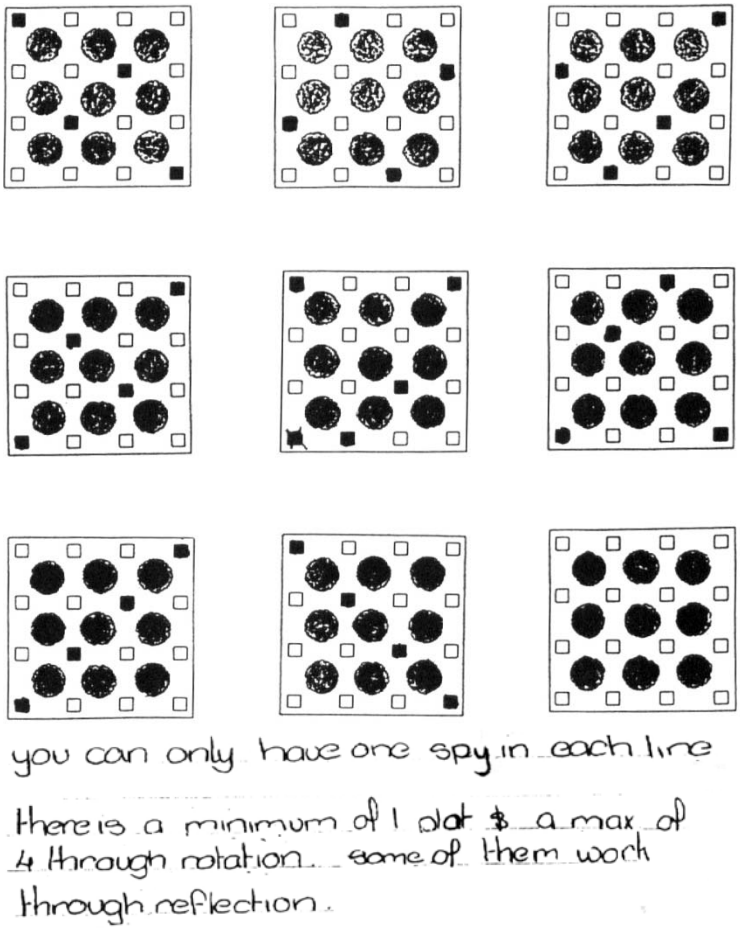
Representing *Choosing a mathematical approach*	**Analysing** *Logical ordering of the outcomes and accuracy of results*	**Interpreting and evaluating** *Identifying patterns and exceptions, and generalising*	**Communicating and reflecting** *Explaining the approach taken and the outcomes achieved at each stage*
Shows understanding of the task by identifying benches where spies can or cannot sit	Produces arrangements that are correct Pupils A, B	Makes simple observations on arrangements found	Arrangements shown clearly
Recognises that there are mathematical principles that can be used to generate families of results Pupils A, B, C	Identifies patterns and uses them to find further arrangements and seeks all arrangements Pupil C	Makes observations about relationships between arrangements and how these can be used to produce further results Pupils A, C	Presents arrangements and describes some common features and differences, and/or gives some indication of approach Pupils A, B, C
Develops a mathematical approach to find arrangements Pupil D	Uses a mathematical approach accurately to generate all arrangements Pupil D	Seeks general principles within arrangements for a given size and/or across varying sizes Pupil D	Presents results systematically Communicates the approach taken and how conclusions were arrived at Pupil D
Chooses an effective and efficient strategy to explore all possibilities	Uses a well-defined mathematical approach systematically and accurately to find all arrangements Finds the number of possible arrangements	Makes insightful observations about arrangements and strategies found and relates these to the original problem, and possibly to extensions	Describes clearly and succinctly the systematic approach taken and how conclusions were reached Reflects on how approaches can be generalised to solve related problems



Sample responses

Pupil A

Pupil A has drawn 8 different results, and recognises the role of reflection and rotation in generating further results.

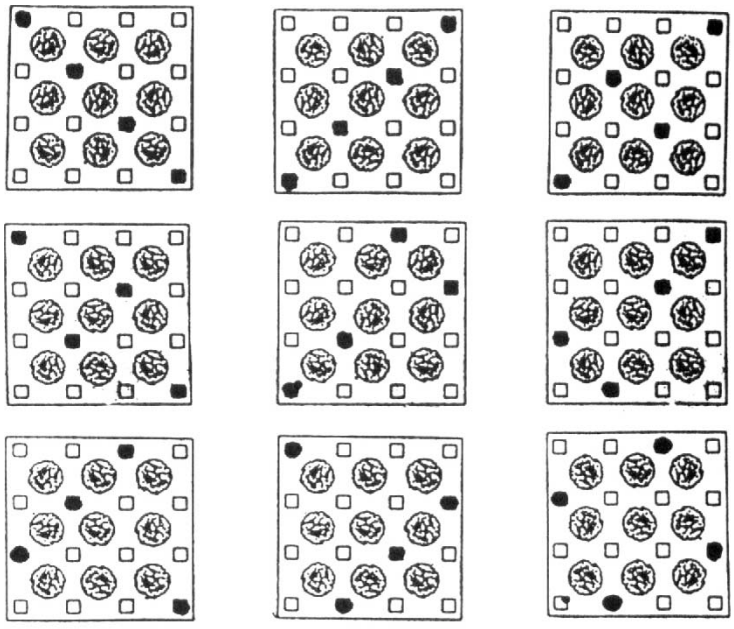


Probing questions

- What approach have you been using to generate new results?
- How can symmetry help you to find and classify other results?
- How will you determine that you have found all possible arrangements?

Pupil B

Pupil B used symmetries to generate results and explained '... means you don't miss out ones by mistake and you can concentrate on thinking where the next one is going to go'. But there is no evidence that the order of symmetry has been recognised for different arrangements.



Probing questions

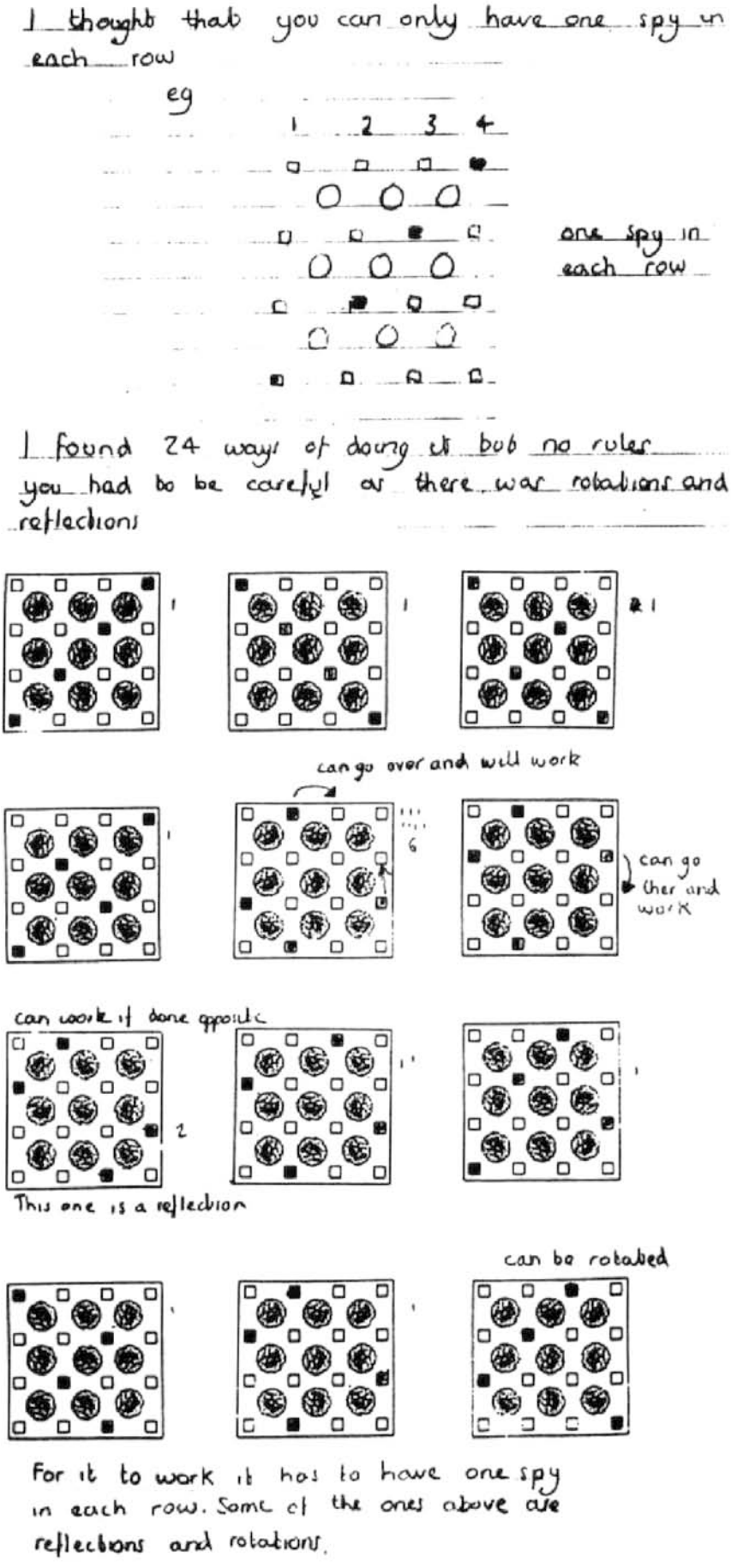
- What did you do to diagram 3 (top right) to get diagram 4 (middle left)? Could you use these to get further arrangements?
- Can you tell how many more arrangements you can get from one particular arrangement by using symmetry?

Pupil C

Pupil C has found all 24 results using a 'careful' analysis, but claims to have found 'no rules'. Not all the arrangements shown are those that work, and the method is unclear.

Probing questions

- Why doesn't each result have the same number of rotations?
- How would you explain your approach in greater detail? And how has it led you to 24 arrangements?



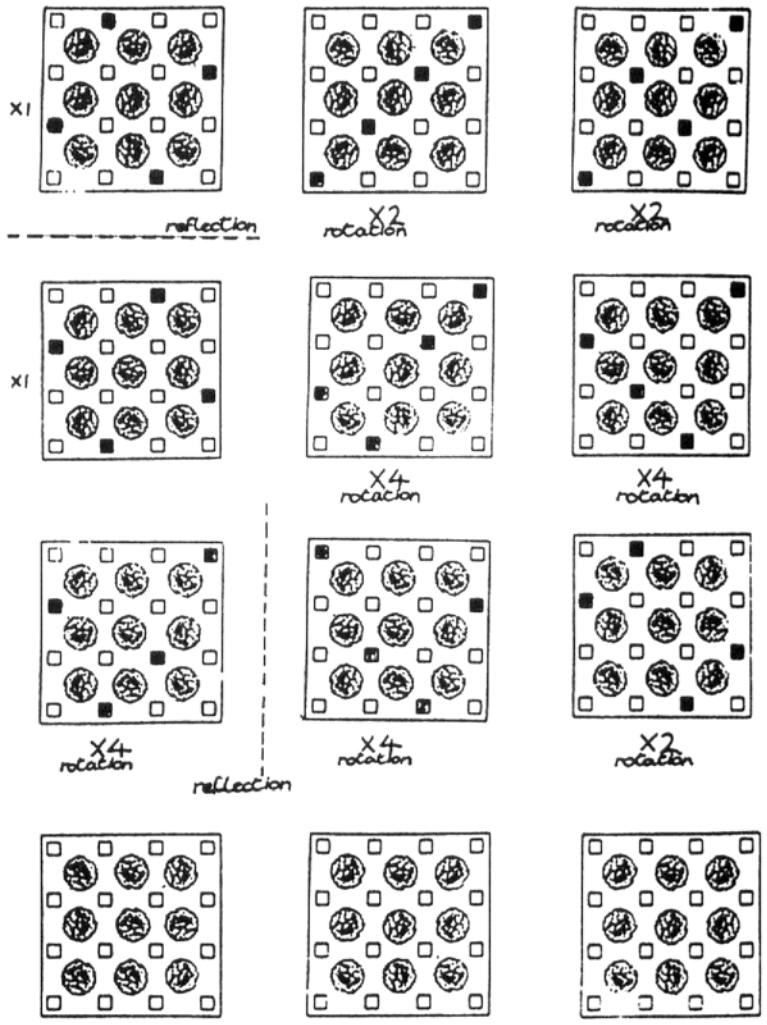
Pupil D

Pupil D shows all 24 results, some of which were found by controlling the positions of spies.

Different sized square parks have been investigated, but the result for a 5 by 5 park is incorrect.

Probing questions

- How can you be sure that you have all the results for the 5 by 5 park?
- What do you notice about the results for different sizes of park? Can you explain your observations?



Only one box can be used in each row and each column.

Results:

1x1 with 1 row = 1 way
2x2 with 2 rows = 2 ways
3x3 with 3 rows = 6 ways
4x4 with 4 rows = 24 ways
5x5 with 5 rows = 36 ways

From these results I cannot see any obvious rules (apart from number one)