

Towards universal participation in post-16 mathematics: lessons from high-performing countries

Country profile: Germany (Rhineland-Palatinate)

Population (end of 2011):	81,800,800 ¹
Population aged 5-19 (2010):	11,672,475 ²
Population of aged 15-19 (2010):	4,140,394 ²
Registered school students:	11,424,948 ³
Number of schools: Primary & secondary:	34,486 ⁴
	8,876
	Secondary:

The Federal System

In Germany, state education is free. Full-time education is compulsory between the ages of 6 and 15 or 16 (depending on the region), and part-time education is compulsory until the age of 18 for those who do not attend a full-time school. Germany is a federal republic and the ministers and senators of the federal states (the Länder) are responsible for education, higher education and research as well as cultural affairs. School policy is in the responsibility of the federal states.⁵ School-systems and curricula differ between the federal states dependent in part on the political parties. Responsibility for education lies primarily with the Länder.⁶

A particular issue in German mathematics education is referred to as the PISA crisis. The PISA 2000 survey results led to a considerable focus both at Länder and Federal level on increasing attainment (despite differences between the PISA focus on problem solving and

¹ https://www.destatis.de/DE/PresseService/Presse/Pressemitteilungen/2012/01/PD12_014_12411.html

² Calculated from: <https://www.destatis.de/DE/ZahlenFakten/GesellschaftStaat/Bevoelkerung/Bevoelkerungsstand/Tabellen/> with support from country policy expert

³ <https://www.destatis.de/DE/ZahlenFakten/GesellschaftStaat/BildungForschungKultur/Kennzahlen.html?nn=50760>, see also: http://www.kmk.org/fileadmin/pdf/Statistik/Dok_195.pdf which provides a slightly higher estimate of 11,485,000

⁴ <http://www.datenportal.bmbf.de/portal/K23.gus?rid=T2.3.1#T2.3.1>

⁵ <http://www.bmbf.de/en/1263.php>

⁶ <http://www.kmk.org/information-in-english/standing-conference-of-the-ministers-of-education-and-cultural-affairs-of-the-laender-in-the-federal-republic-of-germany/organization-and-proceedings.html>

Germany's traditionally more formal and academic approach to mathematics education). The increased Federal focus has had some centralising effects although there remain considerable differences between the Länder.⁷

School structure

At a federal level, schooling structure in Germany is currently undergoing structural reform and change, resulting in different systems in different Länder. Following primary school (*Grundschule*) pupils progress to one of three types of secondary school; secondary education in Germany is exceptional due to its stratified approach. Primary pupils are recommended a place in either the *Gymnasium* which prepares students for the *Abitur* exam for entrance to university, the *Realschule* providing an intermediate education or the *Hauptschule* focussing on vocational training.

The three phases of compulsory education are shown in the table below.⁸

Phases	Age range	Grades
Primary education (<i>Grundschule</i>)	6 to 10 years (6 to 12 years in the Länder of Berlin and Brandenburg)	Grades 1 to 4 (or 1 to 6) of compulsory education
Lower secondary education (Secondary 1) (<i>Gymnasium, Realschule, Hauptschule or Gesamtschule</i>) ⁹	10 to 15/16 years (or 12 to 15/16 years)	Grades 5 to 9 (or 7 to 9/10) of compulsory education
Upper secondary education (Secondary 2)	15/16+ to 18/19 years	Grades 10/11 to 12/13 of compulsory education, either in general education schools or vocational education schools in the dual system.

The diagram on the following page gives the general structure of the German education system.¹⁰

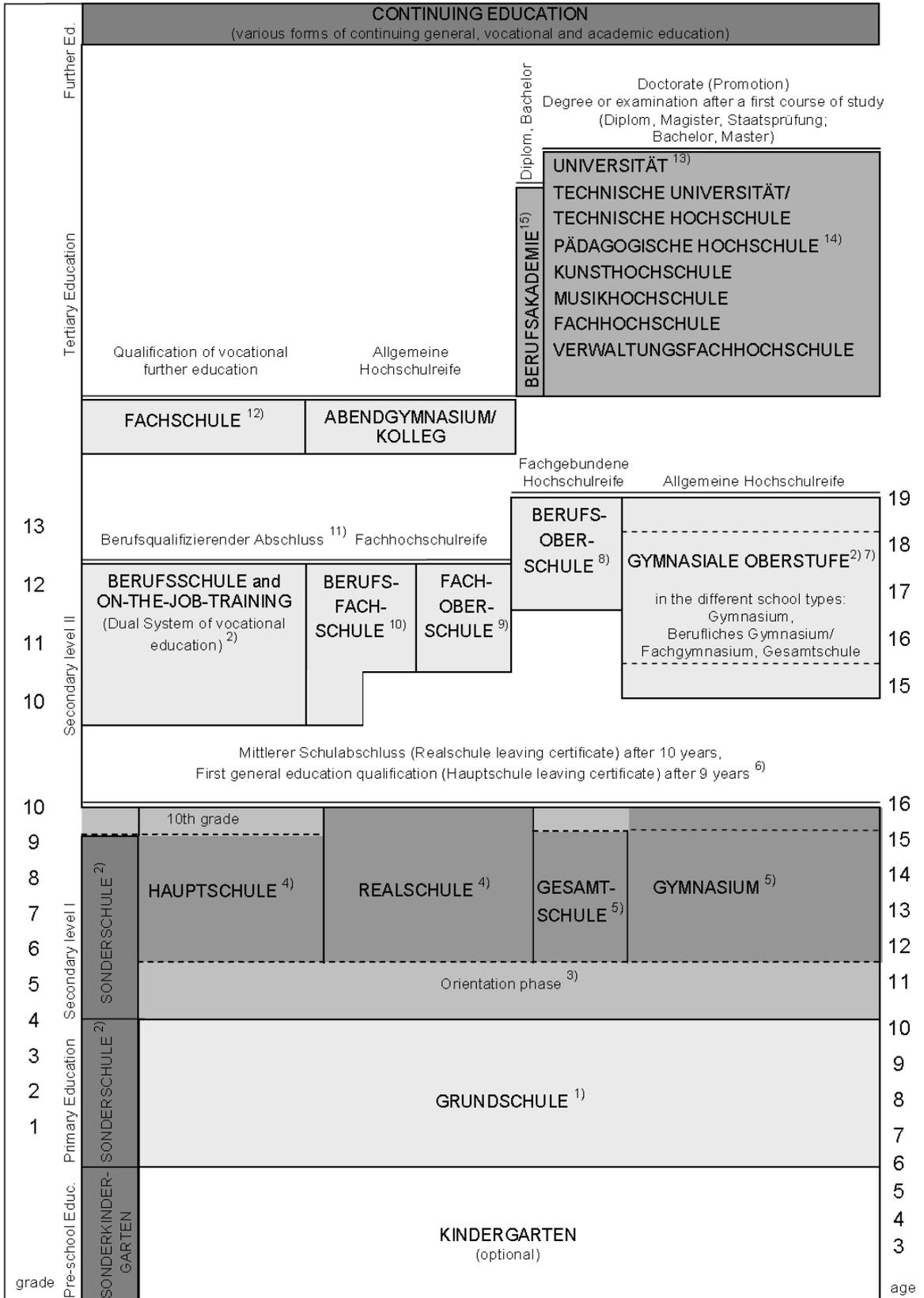
⁷ Grek, S. (2009). Governing by numbers: the PISA 'effect' in Europe. *Journal of Education Policy*, 24(1), 23-37.

⁸ <http://www.inca.org.uk/germany-system-mainstream.html>

⁹ There are also new school types. The *Hauptschule* and *Realschule* courses of education are also offered at schools with several courses of education, for which the names differ from one Land to another. The following types of school bring the courses of education of *Hauptschule* and *Realschule* under one educational and organisational umbrella: *Mittelschule* (Saxony), *Regelschule* (Thuringia), *Sekundarschule* (Bremen, Saxony-Anhalt), *Erweiterte Realschule* (Saarland), *Verbundene Haupt- und Realschule* (Hesse), *Haupt- und Realschule* (Hamburg), *Regionale Schule* (Mecklenburg-Vorpommern), *Realschule plus* (Rhinland-Palatinate), *Regionalschule* (Schleswig-Holstein), *Oberschule* (Brandenburg), *Mittelstufenschule* (Hesse). Information supplied by country policy expert.

¹⁰ <http://www.kmk.org/information-in-english/the-education-system-in-the-federal-republic-of-germany.html>

Basic Structure of the Educational System in the Federal Republic of Germany



Issues specific to Rhineland-Palatinate:

Course system: In Rhineland-Palatinate students at the Gymnasium, in the two years prior to the Abitur (the final examination/qualification at the end of the Gymnasium – taken at an equivalent time to A-Levels in England) – choose 3 advanced courses (5 lessons a week) instead of 2, and 7 or more basic courses (2 or 3 weekly lessons, including sports and religion or ethics). To calculate the points for the Abitur two subjects at the advanced level are needed. Points acquired in the basic courses also count towards the Abitur.¹¹

Centralized Abitur: The Abitur-exam in Rhineland-Palatinate is not administered centrally. The teachers devise examination questions which are sent to the Ministry of Education for final task selection. Rhineland-Palatinate is the only Land with this procedure. The Länder inform each other about the topics selected for the written Abitur-exams (e.g. calculus versus statistics).¹²

Examinations at the end of lower-secondary education (after Grade 9 and 10): In Germany there are no end of year examinations during secondary education. The Abitur is the only examination German students take in school (taken by students in the Gymnasium), although some Länder have introduced examinations for the end of other types of secondary school.

School structure: In Rhineland-Palatinate, some 8-year Gymnasium have been introduced, so parents can choose, in theory, to send their children there if they want to. However, this choice is only readily available in the city and does not exist outside of city areas. In 2012 there are 19 in the Land all of which are full-time schools.¹³

1. What is the national policy for, and structure of, mathematics education provision for 16-18/19 year-old (pre-university level) learners?

- Is upper secondary education compulsory or optional?
- What is the structure of upper secondary programmes?
- Is any mathematics compulsory in the upper secondary age group?
- What, if any, are the mathematics options in upper secondary education?

In all Länder mathematics is compulsory. In most Länder there is a choice between basic and advanced mathematics courses. In some (Bavaria, Baden-Württemberg) pupils have to choose mathematics as a core subject at a higher level. In others they can choose the level, but the basic level is compulsory. 4 courses in mathematics are the minimum and the marks gained here also count towards the final mark in the Abitur examination.¹⁴

From the age of 16 students will be enrolled either in a general upper secondary school or in vocational education. These may be full time, or, in the case of *Duales System* – *Berufsschule + Betrieb* (Dual System: part-time vocational school and part-time on-the-job

¹¹ Information provided by country policy expert, see also: http://gymnasium.bildung-rp.de/fileadmin/user_upload/gymnasium.bildung-rp.de/downloads/mss_2015_web.pdf

¹² <http://www.standardsicherung.schulministerium.nrw.de/abitur-gost/fach.php?fach=2> See also <http://bildung-rp.de/unterricht/materialien/sekundarstufe-ii/mathematik.html> (chapter 5, pp.30-64 for examples for mathematics)

¹³ <http://gymnasium.bildung-rp.de/info-zu-g8gts.html>

¹⁴ http://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/1989/1989_12_01-EPA-Mathe.pdf and <http://www.kmk.org/bildung-schule/allgemeine-bildung/schulpflicht.html>

training) a mixture of schooling and job-based training. Compulsory curricula/syllabuses exist for all subjects.¹⁵

School	Age	Leaving certificate
General upper secondary school (<i>Gymnasiale Oberstufe</i>) (Also in berufliches Gymnasium, Fachgymnasium, they count towards vocational education)	15/16-18/19	<i>Abitur</i>
Full time vocational education (<i>Berufsfachschule</i> or <i>Fachoberschule</i>)	15/16-18	<i>Zeugnis der Fachgebundenen Hochschulreife</i> or <i>Fachhochschulreife</i>
Part-time vocational school and part-time on-the-job training (<i>Duales System – Berufsschule + Betrieb</i>)	15/16-18/19	<i>Zeugnis der Fachgebundenen Hochschulreife</i>

Students attending the *Gymnasium* must study subjects from 3 areas:

- languages, literature and the arts
- social sciences, religion or ethics
- mathematics, natural sciences and technology
- (plus sports)

They choose from basic and advanced courses, with a minimum of two subjects at advanced level.¹⁶ Typically a student may take 2 advanced and 5 basic courses. One advanced course must be either German, continuation of a foreign language, mathematics, or a natural science. Two of the three subjects - German, mathematics and foreign language – must be included into the *Abitur* examination. This is a new formulation from 2012.¹⁷

For students attending vocational school, the compulsory curriculum varies between courses. Students attending a *Berufsfachschule* (which introduces students to one or several occupations) will have 60 per cent of their classes focused on the skills needed to do their job and the remainder will cover general education subjects: German, social studies, economics, religion, and sport. In Rhineland-Palatinate they also have mathematics.¹⁸

In *Fachoberschulen* (general and specialised theoretical and practical knowledge and skills), instruction is given in the following subjects: German, foreign language, mathematics, natural sciences, economics and society, and a field-specific subject.

2. What are the overall participation rates in mathematics study for 16-18 year-olds both as proportions of students and proportions of the age cohort?

- What are current levels of participation in mathematics overall amongst the upper secondary cohort and age group?
- What are the current levels by gender?
- How have these participation rates changed over time?

¹⁵ <http://www.inca.org.uk/1430.html>

¹⁶ http://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/1972/1972_07_07-Vereinbarung-Gestaltung-Sek2.pdf

¹⁷ Information provided by country policy expert, see also:

http://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/1972/1972_07_07-Vereinbarung-Gestaltung-Sek2.pdf

¹⁸ <http://berufsbildendeschule.bildung-rp.de/schulformen/berufsfachschule/berufsfachschule-i.html>

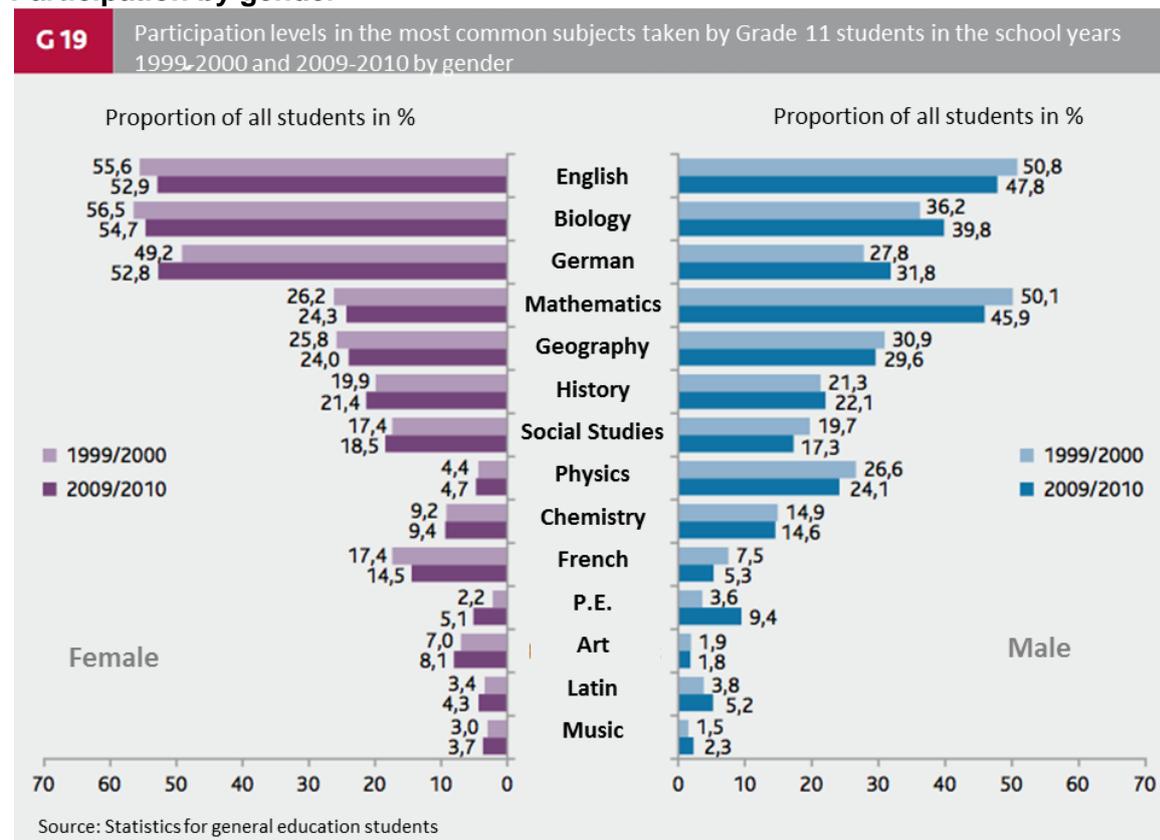
In all Länder, some mathematics is compulsory in upper secondary education in the Gymnasium. Participation levels will be different depending upon whether students choose mathematics as one of their two advanced subjects (out of three core options).¹⁹ For Rhineland-Palatinate 33.8% of students chose mathematics as one of their core advanced subjects.²⁰

We estimate that between 2.5% and 10% of upper secondary students in vocational education are studying advanced mathematics. This was based on judgement of participation in courses available.²¹

On this basis we estimated the participation in upper secondary overall (including both general and vocational education) to be between 8% and 14%. The wide interval is due to the wide estimate interval for vocational education.

The diagram below shows levels of participation in different subjects (as a percentage of all students) for Grade 11 male and female students in the academic years 1999/2000 and 2009/2010 in general education. This shows that for 2009/2010, mathematics has the second highest level of participation for male students (45.9%) and the fourth highest for female students (24.3%). This demonstrates a large gender difference in participation in mathematics at Grade 11. For both male and female students, participation in mathematics has fallen slightly over the 10-year period in this diagram.

Participation by gender²²



¹⁹ http://eacea.ec.europa.eu/education/eurydice/documents/thematic_reports/132EN.pdf (p.75)

²⁰ http://www.statistik.rlp.de/fileadmin/dokumente/berichte/B1083_201100_1j_K.pdf

²¹ <http://www.bildungsserver.de>

²² Translated and adapted from a diagram available in the original (German) at: http://www.bmbf.de/pub/band_dreissig_bildungsforschung.pdf

4. What are the patterns of participation in terms of following different routes involving mathematics?

→ What are current levels of participation in different mathematics options amongst the upper secondary cohort and age group?

Information not available.

5. What is the content and level of the different kinds of provision?

→ What is the structure and content of the mathematics options?
 → How is teacher education organised in order to offer the mathematics options?

State regulations dictate the syllabi and textbooks (with an approved list for schools to choose from) but the way teachers teach and when they teach particular topics is up to the school / individual teacher (state dependent).²³ All Länder have common standards in core subjects. Standards for the Abitur are under construction.

The curriculum is the responsibility of the Länder. Standards do not relate content to grades, although there are now general mathematical competencies which have to be reached.²⁴

The standards in mathematics to be reached by the end of Grade 4 are:²⁵

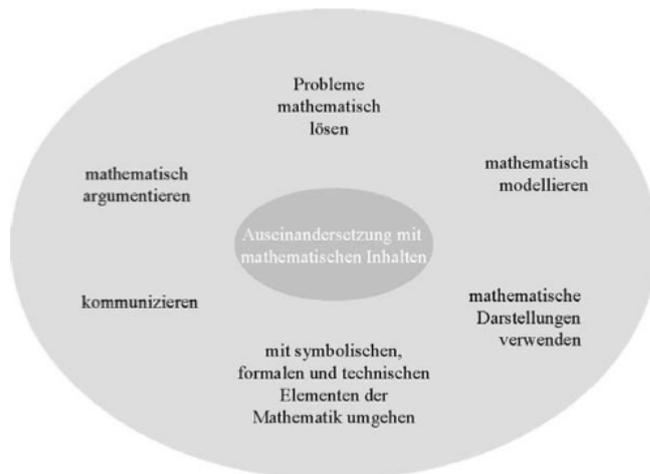
Mathematikunterricht in der Grundschule		
Allgemeine mathematische Kompetenzen		
Problemlösen		
Argumentieren	Inhaltsbezogene mathematische Kompetenzen	Kommunizieren
Darstellen von Mathematik		Modellieren

²³ <http://www.didaktik.mathematik.uni-wuerzburg.de/history/meg/weidiga2.html>

²⁴ Information provided by country policy expert

²⁵ http://www.kmk.org/fileadmin/veroeffentlichungen_beschluesse/2004/2004_10_15-Bildungsstandards-Mathe-Primar.pdf

The standards in mathematics to be reached by the end of Grade 9 are:



In addition to the Standards, there are content-specific competencies for Grade 4 and Grades 9/10.

Students in the upper-secondary years of the Gymnasium take mathematics at either a general or advanced level, depending on their chosen subjects. The general course is aimed at providing students with a broad understanding of mathematics, while the advanced course is intended to serve as an in-depth introduction to academic study. A general course will contain around 3 hours of teaching each week, while an advanced course will have between 4 and 5 hours.

Teacher Training

Teacher training courses are provided in universities and other higher education establishments and are usually divided into two stages: a course of higher education, followed by a period of practical teacher training.

The curricular content of courses depends on the level and type of education for which the trainee teacher is preparing. Teachers are trained specifically for the type of school they will be teaching in:²⁶

- Type 1 : Teaching careers at the primary level
- Type 2 : General teaching careers at primary level and lower secondary level schools
- Type 3 : Teaching careers at lower secondary level schools
- Type 4 : Teaching careers for the general education subjects at upper secondary level or for the Gymnasium (Normally they teach two subjects).
- Type 5 : Teaching careers in vocational subjects at upper secondary level or at vocational schools
- Type 6 : Teaching careers in special education

In 2008, common content requirements for subject-related studies and subject-related didactics in teacher training were adopted. These ensure that in terms of subject specific training, trainee teachers have:

²⁶ http://eacea.ec.europa.eu/education/eurydice/documents/eurybase/eurybase_full_reports/DE_EN.pdf

- compatible subject-related knowledge
- subject-related cognitive and working methods
- compatible subject-related teaching methods

Upper-secondary training involves 4 years of university study in two subjects (with two 5-week practicum phases between semesters) followed by 2 years teacher-training involving time in school (40%) and educational seminars. Teachers who fail the specialist upper-secondary programme (type 4) may move down to a lower-secondary course (type 3) or sideways to a different subject in upper-secondary.²⁷

In Germany there is a project providing in-service training for lower secondary mathematics teachers called SINUS, in which teachers work together on prepared tasks but there is no comparable initiative for the upper secondary level.²⁸

6. How are the different mathematics options assessed?

- How and when are students assessed for summative purposes?
- Are any alternative assessment pathways available?

While Germany does not have a tradition of end of year examinations, proficiency tests have been introduced in core subjects at the end of Grade 3 and Grade 8 in nearly all Länder. During Grades 5 and 6 teachers make recommendations about suitable secondary pathways.

At primary and secondary level, students are assessed by their subject teachers based on their overall work. Grades are awarded and reported twice a year, with each student given a grade from Mark 1 (very good) to Mark 6 (very poor/fail). In many subjects, students require Mark 4 or above to progress to the next year. Students receive a leaving certificate at the end of lower secondary. In all Länder except Rhineland-Palatinate students are required to sit an examination.

The main academic qualification on completion of upper secondary education in the Gymnasium is the Abitur, which allows successful students to progress to higher education.²⁹ Marks obtained in the last two years of upper secondary education, alongside the Abitur grades, count towards the students' final grading.

At the upper secondary level in vocational education, students take an intermediate examination halfway through their course. Successful completion of many vocational courses at upper secondary level enables students to enter higher education or to practise a particular profession.³⁰

²⁷ Information supplied by country expert

²⁸ Information provided by country policy expert

²⁹ <http://www.inca.org.uk/germany-system-mainstream.html>

³⁰ <http://www.inca.org.uk/germany-system-mainstream.html>

7. What information is available on students' learning outcomes in secondary education?

→ What research or policy evidence is available on students' expectations, attitudes and attainment in relation to mathematics in lower and upper secondary education?

In the 2009 PISA assessment, Germany's mean score was 513. This is statistically significantly above the OECD average.³¹ OECD student attitude measures in 2000 showed that 25% of students did not enjoy going to school and 49% found it boring.³² In spite of projects aimed at showing students the relevance of mathematics to their lives, students in Germany have been found to believe the subject has no use in their lives, with mathematics viewed as traditional and formula-driven.³³

8. What vocational education options are available at upper secondary level?

→ What is the structure and content of the vocational courses available?
→ What status do vocational courses have in comparison to other options?
→ What are the participation levels in these courses?
→ How much mathematics is included in vocational education courses and at what levels?

Bundesinstitut für Berufsbildung (BIBB) is the federal institute responsible for Vocational Education and Training (<http://www.bibb.de>). They produced a VET policy report for Germany in 2011.³⁴

Vocational education is notably strong in Germany. Education at post-compulsory level / upper secondary is characterised by the predominance of vocational education. Mathematics is compulsory in all vocational courses. There has recently been an increase in the number of pupils from vocational schools who go on to University or Fachhochschule. A clear link is made in Germany between continuing general education, and vocational education through the framework provided by the dual system of combined school and workplace education.³⁵

- 80% of students in upper secondary education follow a route involving at least some vocational education (vocational schools with combined general education).
- 20% of students follow the general education courses (Gymnasium).³⁶
- All students in 'technical' vocational and 90% of other vocational students are required to take mathematics. Some students of vocational education, such as engineering, also study advanced mathematics.³⁷

³¹ http://www.oecd.org/document/61/0,3746,en_32252351_32235731_46567613_1_1_1_1,00.html

³² <http://www.nationmaster.com/country/gm-germany/edu-education>

³³ http://www.erzwiss.uni-hamburg.de/personal/gkaiser/pdf-publist/festbandhenn_2006.pdf : G.Kaiser, K.Maaß: Vorstellungen über Mathematik und ihre Bedeutung für die Behandlung von Realitätsbezügen

³⁴ [http://www.refernet.de/images_content/VET_in_Europe_2011-Country_Report_2._Auflage\(3\).pdf](http://www.refernet.de/images_content/VET_in_Europe_2011-Country_Report_2._Auflage(3).pdf)

³⁵ <http://www.inca.org.uk/1418.html>

³⁶ <http://www.inca.org.uk/1418.html>

³⁷ Hodgen, J., Pepper, D., Sturman, L., & Ruddock, G. (2010). Is the UK an outlier? An international comparison of upper secondary mathematics education. England: Nuffield Foundation.

The statistics on participation for 2010 show that:³⁸

All pupils in upper secondary level	General education in upper secondary	Vocational education in upper secondary		
		Full Time	Part Time	Total
3,780,320	1,092,300 (28.9%)	934,050	1,753,970	2,688,020

Vocational courses involving mathematics are tailored towards the individual requirements of the associated profession, and as such will range from everyday uses of mathematics, to advanced mathematics, for subjects such as engineering. Students undertaking courses leading to the Abitur examination will cover: calculus, vector space, analytic geometry and probability and statistics. Where possible the mathematics should be connected to the vocational course.³⁹

9. What drives the pattern of take-up? How is it linked to the needs of HE, employers and national policy objectives?

- What are the official criteria, if any, for acceptance to the mathematics options?
- Are there any unofficial / local criteria for acceptance to the mathematics options?
- What information, advice or guidance is there about the mathematics options?
- Are any mathematics recruitment policies targeted to specific groups or types of students?
- Which subjects and options are students expected or required students to take?
- Please note the views of, for example, further/higher education institutions, employers, parents or the public more generally
- Are there 'unofficial' expectations to have particular mathematics qualifications for entry to particular HE courses?

In all Länder except Rhineland-Palatinate, students obtain a leaving certificate after completing Grade 10 and passing the final examination. The leaving certificate allows entry to upper secondary education including vocational training. In order to enter upper secondary education in the Gymnasium students must meet achievement standards at the end of Grade 9 or 10. Students not in the Gymnasium in lower-secondary may take an entrance qualification to transfer there for their upper-secondary education.⁴⁰

Take-up of mathematics for students in general education is driven largely by university admission requirements. Prior to 2004, students completing the Abitur were granted access to any course of study. However, a decision by the Standing Conference of the Ministers of Education and Cultural Affairs of all the Länder allowed universities to select up to half their intake. This allows them to require grades in individual subjects, and to hold entrance exams and/or interviews.

For students in vocational education, take-up will be governed largely by the compulsory content of their course. Current policy initiatives aim to increase the numbers of students taking MINT (Mathematics, Informatik [computer science], Natural Science and Technical Subjects) with schools getting additional money and awards.⁴¹ Some initiatives are targeted

³⁸ http://www.kmk.org/fileadmin/pdf/Statistik/Dok_195.pdf

³⁹ Information provided by country policy expert

⁴⁰ http://eacea.ec.europa.eu/education/eurydice/documents/eurybase/eurybase_full_reports/DE_EN.pdf

⁴¹ <http://www.kmk.org/presse-und-aktuelles/meldung/kultusministerkonferenz-uebernimmt-schirmherrschaft-fuer-die-ausszeichnung-mint-freundlicher-schulen.html>

specifically at female students, including those from universities⁴² and national/international competitions.⁴³

10. What policies and practices are there for transition and retention?

- What policies or practices are there to support students' transition from lower secondary to upper secondary mathematics options?
- More generally, what policies or practices are directed at students struggling with upper secondary mathematics?

Teachers give advice to pupils at grade 10 as to whether they are capable of doing mathematics at advanced level.

During the first term of upper-secondary education in the Gymnasium, it is possible for students to change subjects.⁴⁴

In some Länder, "catch-up" mathematics courses are offered. However, this depends on the schools' budgets and teachers' availability.⁴⁵

11. What information is available on (other) factors affecting recruitment and retention?

- What factors would you attribute to the upper secondary mathematics recruitment levels in your education system?
- Please give details of any supporting information or sources

No further information available.

⁴² <http://www.ada-lovelace.com/english/>

⁴³ E.g. "Jugend forscht" (<https://www.jugend-forscht.de/>), "Kangaroo" (<http://www.mathe-kaenguru.de>), Mathematical Olympiade (<http://www.mathematik-olympiaden.de/>)

⁴⁴ Information provided by country policy expert

⁴⁵ Information provided by country expert