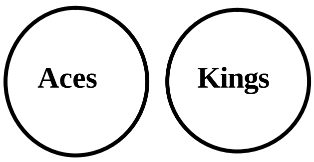
Laws of Probability

Summary Sheet

A or B

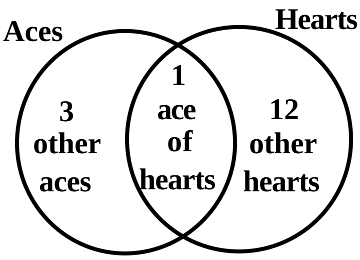
Mutually exclusive means that A and B cannot both happen at the same time.

Venn Diagram showing mutually exclusive events:



The events 'draw an Ace' and 'draw a Heart' are **not *mutually exclusive*** as the Ace of Hearts means both events happen together.

Venn Diagram showing non-mutually exclusive events:



When events A and B are ***mutually exclusive***:

$$\mathbf{P(A\ or\ B) = P(A) + P(B)}$$

For example, if a card is drawn at random from a pack of 52

$$\text{P(Ace)} = \frac{4}{52} = \frac{1}{13} \qquad \text{P(King)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{P(Ace } \textbf{or} \text{ King)} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

This can also be calculated directly using the fact that 8 out of the 52 cards are aces or kings, giving the probability $\frac{8}{52} = \frac{2}{13}$

When events A and B are ***not mutually exclusive***, you cannot just add the probabilities.

For example, if a card is drawn at random from a pack of 52

$\text{P(Ace)} = \frac{4}{52}$ and $\text{P(Heart)} = \frac{13}{52}$ Adding these gives $\frac{17}{52}$

But this is ***not*** the probability of an Ace or a Heart. Since 13 cards are hearts and there are another 3 aces, there are just 16 cards out of 52 cards that are either hearts or aces so:

$$\text{P(Ace } \textbf{or} \text{ Heart)} = \frac{16}{52} \text{ not } \frac{17}{52}$$

In this case adding gives a value that is too high because the Ace of Hearts is included twice.

A and B

Independent means that A has no effect on B and vice versa.

When events are ***not independent***, it is necessary to use ***conditional probabilities***. This is not required for the FSMQ 'Hypothesis Testing'.

When combined events A and B are ***independent***:

$$\mathbf{P(A\ and\ B) = P(A) \times P(B)}$$

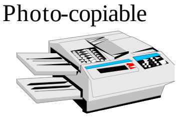
For example, if a coin is tossed and a card is taken at random from a pack of 52

$$\text{P(Head)} = \frac{1}{2} \qquad \text{P(King)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{P(Head } \textbf{and} \text{ King)} = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

For independence, if 2 cards are taken from the pack the first must be replaced before the second is taken. In this case

$$\text{P(2 Kings)} = \frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$$



Probability Tree Diagrams

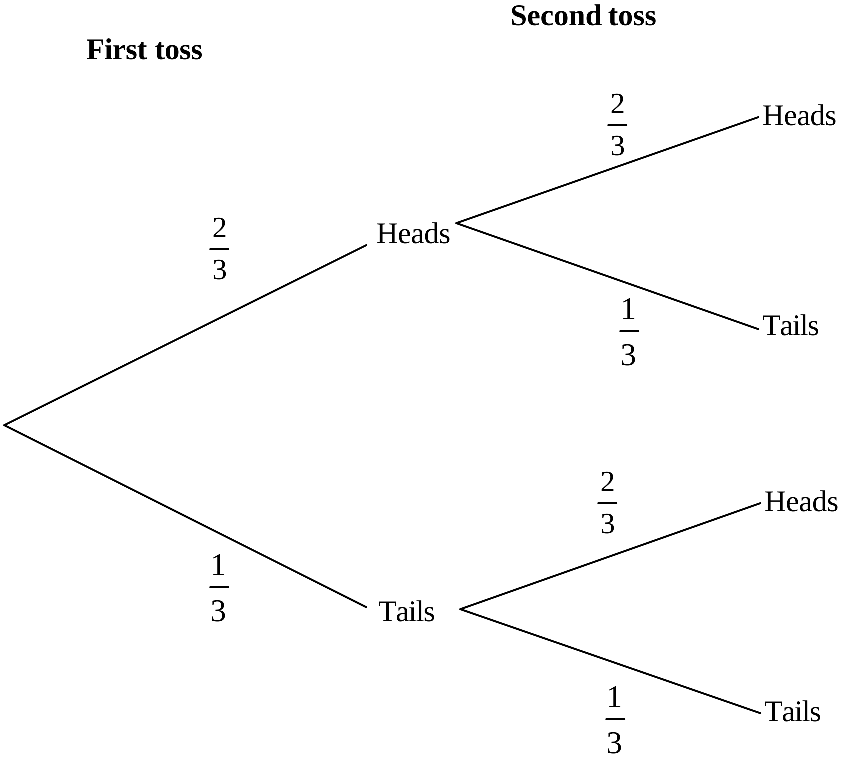
These show all the possibilities for combined events together with their probabilities.

Example

A coin is biased so that it is twice as likely to give heads than tails.

This means that whenever the coin is tossed P(H) = $\frac{2}{3}$ and P(T) = $\frac{1}{3}$.

The tree diagram below shows the possible outcomes when this coin is tossed twice.



$P(H, H) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

$P(H, T) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

$P(T, H) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

$P(T, T) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Notes

- the first set of branches shows the possibilities for the first toss of the coin.
- the second sets of branches show the possibilities for the second toss of the coin
- the **probabilities on each set of branches add up to 1**
- the probability of any combination is found by **multiplying** the probabilities on the path along the branches
- the **sum of the resulting probabilities is 1** i.e. $\frac{4}{9}+\frac{2}{9}+\frac{2}{9}+\frac{1}{9}=\frac{9}{9}=1$

  This provides a good check.

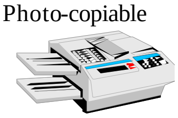
- you can also **add the resulting probabilities** to find the probabilities of other events
  For example, the probability that both tosses give the same result is:

  $P(H, H) + P(T, T) = \frac{4}{9}+\frac{1}{9}=\frac{5}{9}$

  The probability that the tosses give different results is:

  $P(H, T) + P(T, H) = \frac{2}{9}+\frac{2}{9}=\frac{4}{9}$

N.B. Note that $\frac{5}{9}+\frac{4}{9}=\frac{9}{9}=1$ as these cover all possibilities.



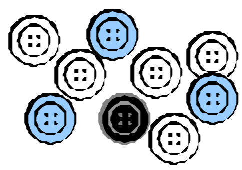
Some to try:

Mutually Exclusive Events

Each row in the table gives a pair of events.
In each case show whether the events are mutually exclusive or not.

	Mutually exclusive?	
	Yes	**No**
Angela goes for her train to work: Event A: she catches the train Event B: she misses the train		
Rory throws a dice: Event A: he gets an odd number Event B: he gets less than 4		
Rory throws a dice: Event A: he gets more than 3 Event B: he gets less than 3		
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a club		
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a queen		

Buttons



A box contains 1 black button, 3 blue buttons and 5 white buttons.
If a button is taken out of the box at random, what is the probability that it is

(a) black …………….. (b) blue ………… (c) white …………

(d) black or blue ………… (e) blue or white ………… (f) black or white…………

Independent Events

A coin is tossed and a dice is thrown. Assuming independence, find the probability of

(a) heads and a six ……………..……………………………………………………..

(b) heads and an even number ……………..……………………………………………

(c) tails and more than 4 ……………..……………………………………………………

Deliveries

A delivery firm delivers 75% of packages the next day.
Jack posts 2 packages.

(a) Complete the tree diagram.
(Assume deliveries are independent.)

(b) Write down the probability that

(i) both packages are delivered the next day ………..

(ii) neither package is delivered the next day ………..

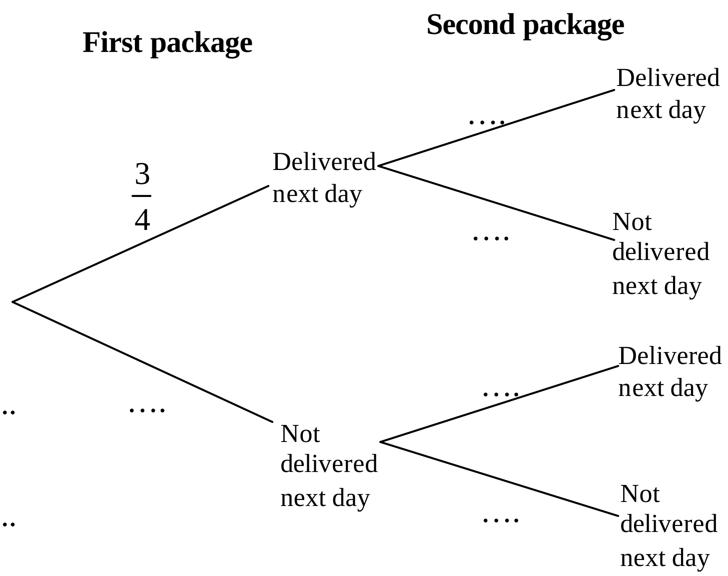
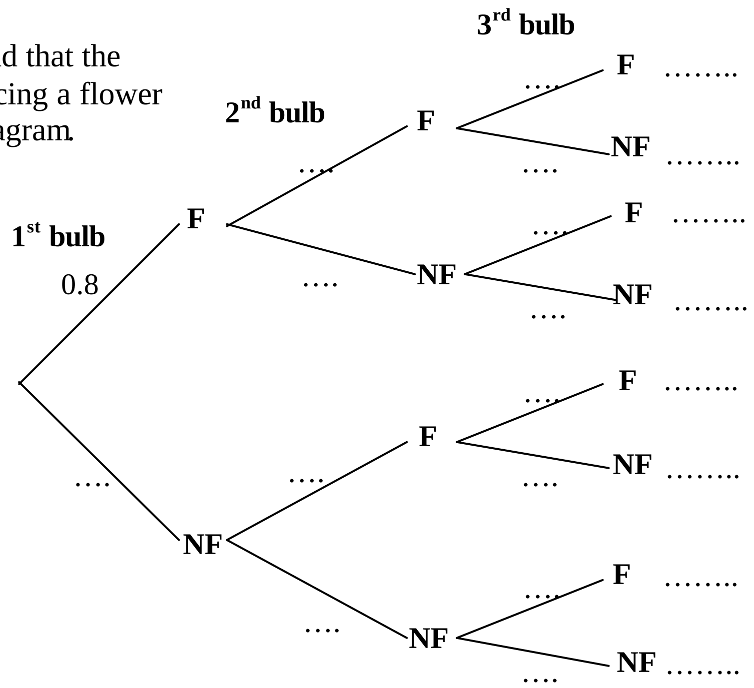


Photo-copiable

Potting Bulbs
Neil plants three bulbs in a pot.

(a) Assuming independence and that the probability of a bulb producing a flower is 0.8, complete the tree diagram.

Key
F a flower is produced
NF no flower is produced



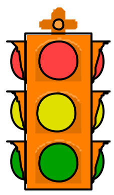
(b) What is the probability that

(i) all 3 bulbs produce a flower? ……………………………………………

(ii) none of the bulbs produce a flower? ……………………………………………..

(iii) two or more bulbs produce a flower? ……………………………………………

Traffic Lights
Kate passes two sets of traffic lights on her way to work.



The probability that she has to stop for the first set of traffic lights is $\frac{1}{3}$.

The probability that she has to stop for the second set of traffic lights is $\frac{2}{5}$.

(a) On a separate piece of paper draw a tree diagram to show the probabilities of her stopping for these traffic lights.

(b) What is the probability that on one journey to work Kate will:

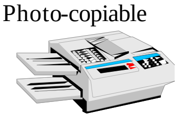
(i) have to stop for both sets of traffic lights ………………………………….

(ii) not have to stop for either set of traffic lights …………………………………..

(c) Kate travels to work on 220 days each year. On how many of these days would you expect her to have to stop for at least one set of traffic lights?

……………………………………………………………………………………………

Photo-copiable



Teacher Notes

Unit Advanced level, Hypothesis Testing

Notes

The examination for this FSMQ will only include probabilities of mutually exclusive and independent events, so the examples included in this resource concentrate on contexts where this can be assumed. Pages 1 and 2 give a summary of the main points. The PowerPoint presentation includes the same examples and can be used when this topic is introduced and/or for revision later. Pages 3 and 4 give some examples for learners to try.

Answers

Mutually Exclusive Events

	Mutually exclusive?	
	Yes	**No**
Angela goes for her train to work: Event A: she catches the train Event B: she misses the train	√	
Rory throws a dice: Event A: he gets an odd number Event B: he gets less than 4		√
Rory throws a dice: Event A: he gets more than 3 Event B: he gets less than 3	√	
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a club	√	
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a queen		√

Buttons

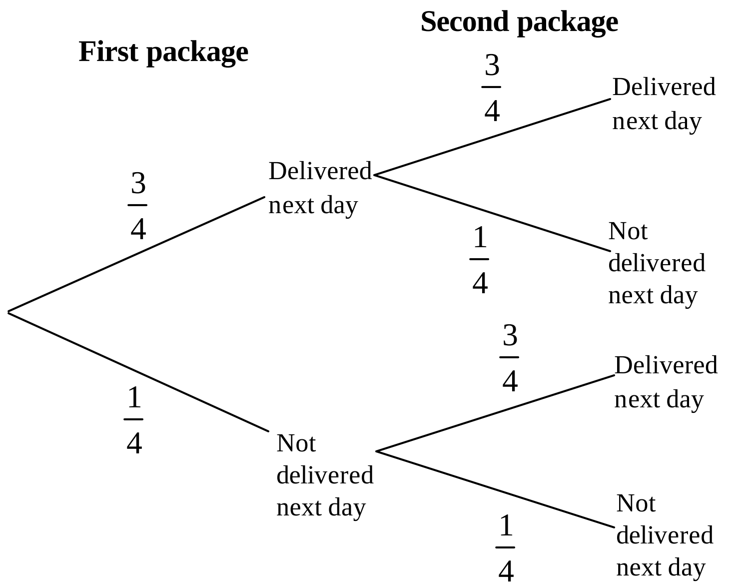
(a) $\frac{1}{9}$ (b) $\frac{3}{9} = \frac{1}{3}$ (c) $\frac{5}{9}$ (d) $\frac{4}{9}$ (e) $\frac{8}{9}$ (f) $\frac{6}{9} = \frac{2}{3}$

Independent Events

(a) $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ (b) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ (c) $\frac{1}{2} \times \frac{2}{6} = \frac{1}{6}$

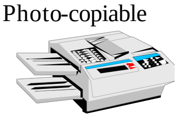
Deliveries

(a)



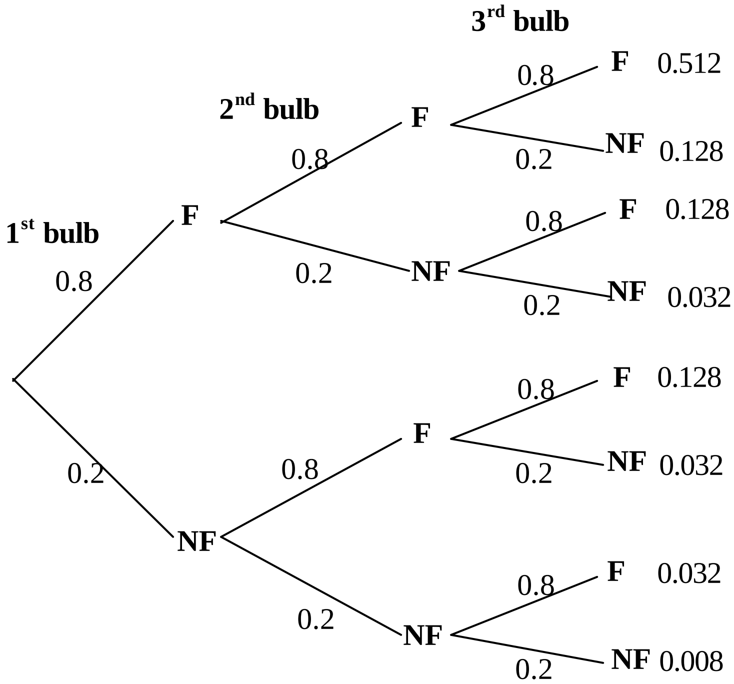
(b) (i) $\frac{9}{16}$

(ii) $\frac{1}{16}$



Potting Bulbs

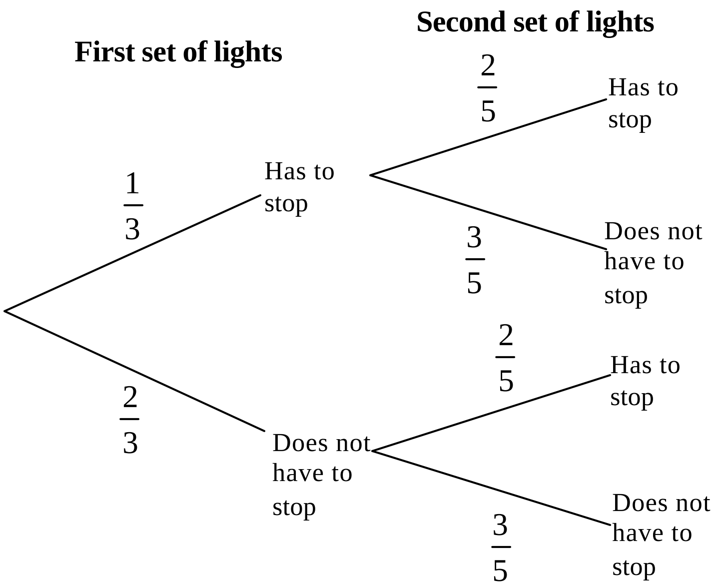
(a)



(b) (i) 0.512
(ii) 0.008
(iii) 0.896

Traffic Lights

(a)



(b) (i) $\frac{2}{15}$ (ii) $\frac{2}{5}$ (c) 132