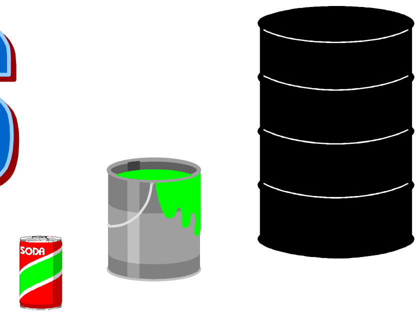
Containers



Assignment

One of the things manufacturers consider when designing a container is the amount of material it will take to make it. The greater the surface area of the container, the greater its cost.

Many commodities are sold in cylindrical containers, for example fizzy drinks, baked beans, paint and oil. Investigate how the surface area of a cylindrical can containing a fixed volume depends on its radius, and how manufacturer can keep the cost of the material needed to make the can to a minimum.

As part of the investigation you should:

- decide on a fixed volume for a cylindrical can containing a particular commodity eg 300 ml of cola, 2.5 litres of paint, 5 litres of oil;
- find a formula for the total surface area of the can, A, in terms of its radius, r;
- use a numerical method to find the radius for which the area is a minimum;
- use differentiation to find the radius for which the area is a minimum;
- find also the corresponding height of the can and the area of material required;
- illustrate your work using graphs of the function and its first and second derivatives;
- describe clearly how gradients and changing gradients relate to the situation;
- compare the effectiveness of the numerical method with the method of differentiation.

If you have time, investigate other container volumes and/or shapes.

Useful formulae:

Area of curved surface of cylinder = $2\pi rh$

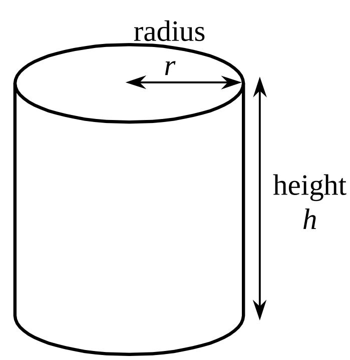
Area of each end of cylinder = πr^2

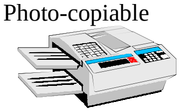
Volume of cylinder $V = \pi r^2 h$

Conversion factors

1 litre = 1000 ml = 1000 cm^3

1 m^3 = 1000 litres





Teacher Notes

Unit Advanced Level, Modelling with calculus

Skills required in this assignment:
rearranging formulae and substituting one formula into another
solving equations
differentiation – including numerical methods
sketching and interpreting graphs of functions and first and second derivatives

Notes

Students will need to be aware that they should use a consistent system of units for the volume, radius, height and area. They may find it difficult to find a formula relating the total surface area and the radius. They could be advised to have this checked before proceeding with the rest of the assignment.

The general formula for surface area in terms of radius is:

$A = 2\pi r^2 + \frac{2V}{r}$ or $A = 2\pi r^2 + 2Vr^{-1}$

This differentiates to give $\frac{dA}{dr} = 4\pi r - 2Vr^{-2}$

then $\frac{d^2A}{dr^2} = 4\pi + 4Vr^{-3}$

The shapes of the graphs of the formula and the first and second derivatives are as shown in the sketches.

The value of r that gives the minimum surface area is $r = \sqrt[3]{\frac{V}{2\pi}}$

The corresponding height $h = 2r = 2\sqrt[3]{\frac{V}{2\pi}}$

The minimum surface area $A = 3\sqrt[3]{2\pi V^2}$

Students who consider other container volumes and shapes will have the opportunity of working independently.