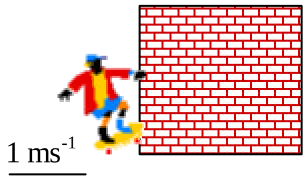
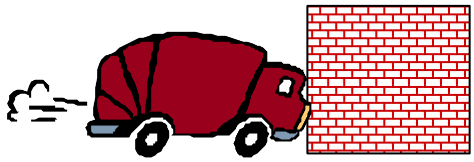
Collisions

Imagine that a wall of your house is hit on one occasion by a boy on a skateboard and on another by a large juggernaut.



Suppose both had a speed of 1 ms^{-1} at impact.

Which collision damages your house less?



Your answer to this question shows that when you model a collision, you must consider more than just the velocities of the bodies involved. Their masses are also significant.

Momentum

When an object of mass *m* kilograms moves with velocity **v** metres per second, its momentum is *m***v** (measured in kg ms^{-1} or Ns).

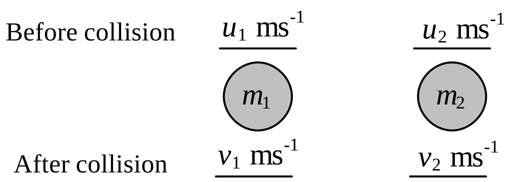
Momentum, like velocity, is a vector having both magnitude and dimension.

Whenever two objects are in contact, they exert equal and opposite forces on each other. In the case of a collision in which both objects can move, the contact produces equal and opposite changes in momentum. Provided that no external force acts on either object, the total momentum of the two objects remains constant. For the Dynamics FSMQ, you will only need to use this property for objects moving in one dimension.

The Principle of Conservation of Linear Momentum

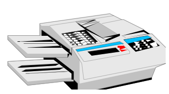
When a collision occurs between two objects, the total momentum after collision is equal to the total momentum before the collision (assuming that there are no external forces).

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$



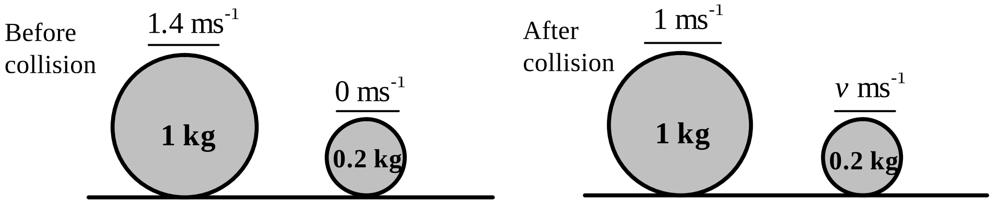
In some collisions the two colliding objects separate after the collision, whereas in others they join together. In the first case the objects have different velocities after the collision whereas in the second case they move together with the same velocity after the impact.

When solving a problem involving a collision, draw a clear diagram showing the velocities before and after the impact. In cases where the objects move in opposite directions, take care to define one direction as the 'positive' direction – an object moving in the opposite direction will have negative velocity. Some examples in real contexts follow.



Bowls

In a game of bowls a wood of mass 1 kg hits the jack, of mass 0.2 kg, which is stationary. The wood has a speed of 1.4 ms^{-1} before the collision and a speed of 1 ms^{-1} in the same direction after the collision. What is the speed of the jack after the collision?



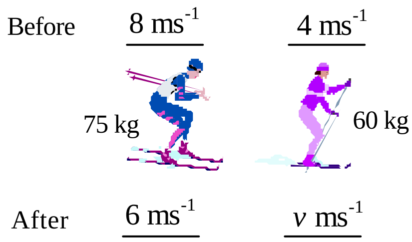
By the principle of conservation of linear momentum

$$1 \times 1 + 0.2 \times v = 1 \times 1.4 + 0.2 \times 0$$
$$0.2v = 1.4 - 1 = 0.4$$
$$v = 2$$

The jack has speed 2 ms^{-1} after the collision.

Skiers

A skier of mass 75 kg who is moving at 8 ms^{-1} collides with a skier of mass 60 kg moving at 4 ms^{-1} in the same direction. The faster skier's speed is reduced to 6 ms^{-1} after the collision. What happens to the speed of the other skier?



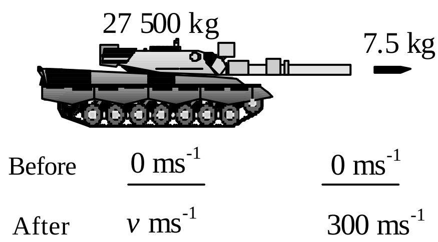
By the principle of conservation of linear momentum

$$75 \times 6 + 60v = 75 \times 8 + 60 \times 4$$
$$60v = 600 + 240 - 450 = 390 \quad \Rightarrow \quad v = 6.5$$

The skier's speed is increased to 6.5 ms^{-1}

Tank

A tank of mass 27.5 tonnes fires a shell of mass 7.5 kg with a speed of 300 ms^{-1} . Find the initial speed of recoil of the tank.



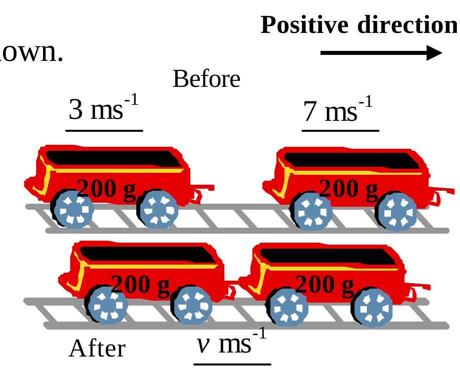
By the principle of conservation of linear momentum

$$27\,500v + 7.5 \times 300 = 0$$
$$27\,500v = -2250$$
$$v = -0.0818$$

The tank recoils with an initial speed of 8.18 cms^{-1}

Toy trucks

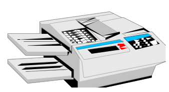
Two identical trucks, each of mass 200 g collide on a track. Before the collision they are moving towards each other as shown. After the collision they move together along the track at a speed of v ms^{-1} . Find v.



By the principle of conservation of linear momentum

$$0.4v = 0.2 \times 3 - 0.2 \times 7 = -0.8$$
$$v = -2$$

The trucks move at 2 ms^{-1} in the direction of the faster truck.

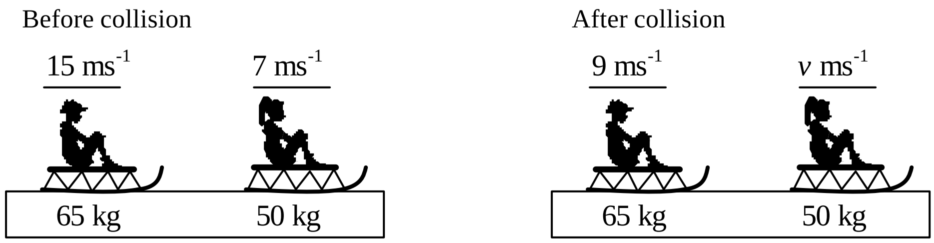


Collisions

Worksheet

1 A boy and his toboggan have a combined mass of 65 kg. They slide at a speed of 15 ms^{-1} along a horizontal snow surface and then collide into a girl on her toboggan travelling in the same direction at a speed of 7 ms^{-1}. The girl and her toboggan have a total mass of 50 kg.

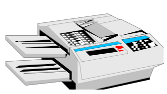
After collision, the speed of the boy and his toboggan is reduced to 9 ms^{-1}.
The situations before and after the collision are shown below.
Find the new speed, v ms^{-1}, of the girl and her toboggan after the collision.



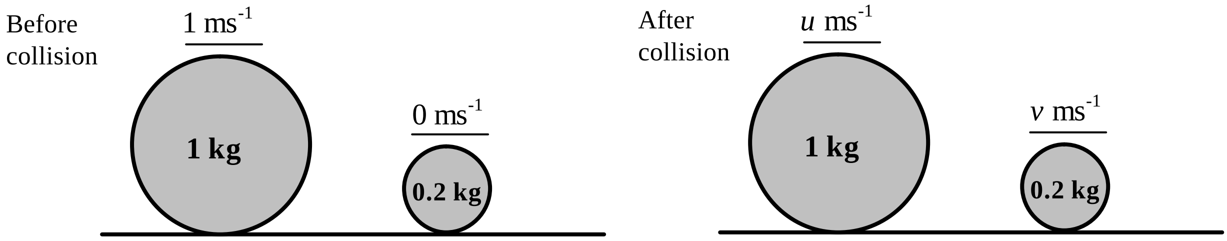
2 A student experiments with some small trolleys which can move freely along a plastic track.

a) The first collision which she investigates is arranged so that a trolley moves with initial speed 1 ms^{-1} and collides with another which is stationary, as shown below. Both trolleys are identical and have mass m kg. The trolleys stick together at collision and move away together with speed v ms^{-1}. Use the principle of conservation of momentum to find v.

b) In the second experiment she investigates how the final speed, v ms^{-1}, of the combined trolleys after the collision varies with the initial speed u ms^{-1} of the moving trolley before impact as shown below. Find an expression for v in terms of u.



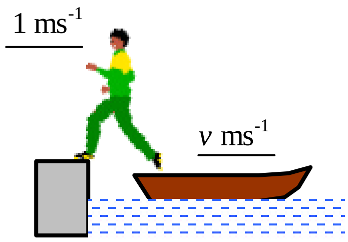
3 In a game of bowls a wood of mass 1 kg hits the jack, of mass 0.2 kg, which is stationary. The wood has a speed of 1 ms^{-1} before the collision and a speed of u ms^{-1} after the collision. The jack has a speed of v ms^{-1} after the collision as shown below.



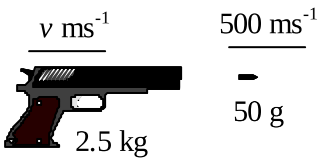
a) Find an expression for v in terms of u.

b) Draw a graph showing how v varies with u. Explain what would happen to the wood and jack at each of the points where your graph cuts the axes.

4 A man stands up in a small rowing boat. He steps out of the rowing boat onto the river bank. Before he leaves the boat both the man and boat are stationary. The man has a mass 80 kg and the boat has a mass of 250 kg. When the man leaves the boat he has a speed of 1 ms^{-1}. The boat travels in the opposite direction with a speed of v ms^{-1} as shown. Find v.

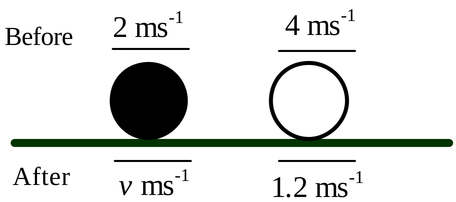


5 A bullet is fired from a gun. The mass of the gun is 2.5 kg; that of the bullet is 50 g. The bullet leaves the gun with speed 500 ms^{-1}. Find the initial speed of recoil of the gun.



6 A girl, of mass 60 kg, jumps onto a stationary skateboard of mass 3 kg. Just before she hits the skateboard, which is on horizontal ground, the girl is travelling in a horizontal direction with speed 4 ms^{-1}. Find the speed of the girl on the skateboard just after the collision.

7 Two snooker balls, both of mass 150 g collide directly. Just before the collision they are travelling with the speeds shown in the diagram. After the collision the white snooker ball rebounds with a speed of 1.2 ms^{-1}. What happens to the black ball?



8 A 10 tonne train engine, which is rolling at a speed of 0.5 ms^{-1}, collides with a 2 tonne truck which is rolling in the opposite direction with a speed of 0.1 ms^{-1}. After the collision the train and carriage move together. Find the speed and the direction in which they move after the collision.

Teacher Notes

Unit Advanced Level, Dynamics

Notes on Activity

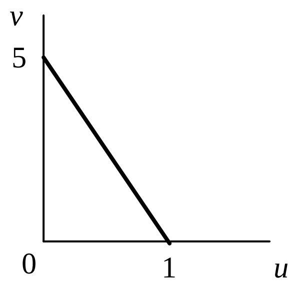
Pages 1 and 2 introduce the principle of conservation of momentum and show how this can be used to solve problems involving collisions. The worksheets on pages 3 and 4 give practice in solving similar problems. Some of these originally appeared in the book ***Mechanics 1*** which was funded by the Nuffield Foundation and published by Longman in 1994 (ISBN 0-582-09979-X).

Answers

1 14.8 ms^{-1}

2 a) $v = \frac{1}{2}\ \text{ms}^{-1}$ b) $v = \frac{1}{2}u$

3 a) $v = 5(1 - u)$ b)



Point (0, 5)
The wood is reduced to rest and the jack moves off with velocity 5 ms^{-1}

Point (1, 0)
The jack is at rest and the wood moves at 1 ms^{-1} (the initial situation)

4 0.32 ms^{-1}

5 10 ms^{-1}

6 3.81 ms^{-1} (to 3 sf)

7 Black ball rebounds with velocity 3.2 ms^{-1}

8 0.4 ms^{-1} in the initial direction of the engine's motion.