Activity description

In this activity students explore the idea that the area under speed-time graphs can be used to find the distance travelled. They also use triangles and trapezia to estimate areas under curves.

Suitability

Level 2 (Intermediate/Higher)

Time

Approximately 3 hours (or 2 × 1½ hours)

Resources

Student sheets
Optional: slideshow

Equipment

Optional: graph paper
or graphics calculator
or a computer and spreadsheet

Key mathematical language

Graph, speed–time graph, triangle, trapezium, area, approximation, acceleration, deceleration, estimate, function

Notes on the activity

You may wish to divide the task into two parts:
students could undertake A and B after work on straight-line graphs,
and the remainder (C and D) after work on functions and curves.

During the activity

Answers to the questions in the optional slideshow are given on the student sheets. So it is a good idea not to hand out the student sheets until after slideshow if you want students to discuss these questions before seeing the answers. This also allows class discussion on 'Think about' points without students being able to see what comes next.

Points for discussion

The 'Think about' points in **part A** introduce distance travelled. Students should recognise the value of 140 when it occurs again as the shaded area, and make the link between the two concepts.

Students need to appreciate that distance travelled is only equivalent to the area under the graph when the time units on the two axes are the same. Students may need additional practice in converting speeds from kilometres per hour to metres per second (perhaps using 18, 36, 72, 90 and 108 kph, which give convenient results).

The first 'Think about' question in **part B** discusses the fact that in a straight-line graph (constant acceleration) the average speed is halfway between the minimum and the maximum. This is confirmed when the 'Think about' is discussed at the end of Part B. The first 'Reflect on your work' question at the end of this section requires a reverse calculation. You could suggest that students sketch a graph to help them work out the answer
($\frac{1}{2} \times 30 \times \text{time} = 180$; time = 12 seconds).

The first 'Think about' question in **part C** should prompt students to recognise that the curve is symmetrical, so it is only necessary to work out the area of three strips, add them, and then double the answer.

At the end of **part C** discuss how the method used gives an under-estimate, and that the actual distance will be greater than 70 metres. A better estimate could be achieved by splitting the area into more strips. Also discuss the way in which the graph models the motion, and how the actual changes in speed are unlikely to be as smooth.

The **'At the end of the activity'** questions (in slideshow as well as student sheets) encourage students to think again about the use of triangles and trapezia to measure the area under a curve, and how this method can only give an approximation.

Discuss how it gives an under-estimate when the curve is convex, and an over-estimate when the curve is concave.

Also emphasise how using more strips can improve the estimate.

Perhaps most important of all is to discuss how graphs and functions provide simple models of the real situation, but that in practice the motion of a car will be much more complex.

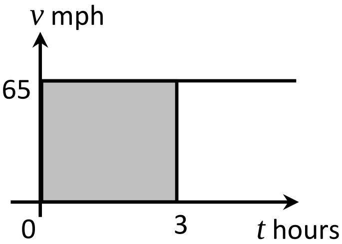
Extensions

Study other cases where the area under a graph gives useful information. This is usually the case when a rate is plotted against time. For example, if the rate of flow of water from a tap (or other source) in litres per second is plotted against time, the area under the graph will give the volume of water.

(Note that this activity could provide a useful introduction to the concept of integration for students going on to advanced work.)

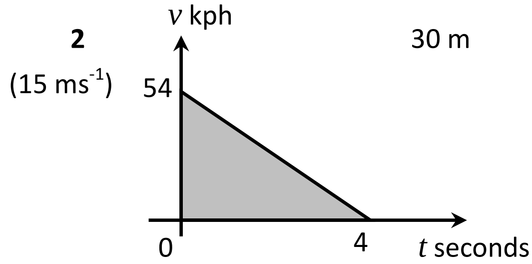
Answers part B

1



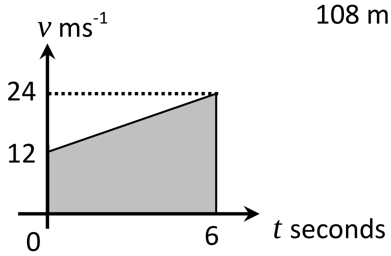
195 miles

2



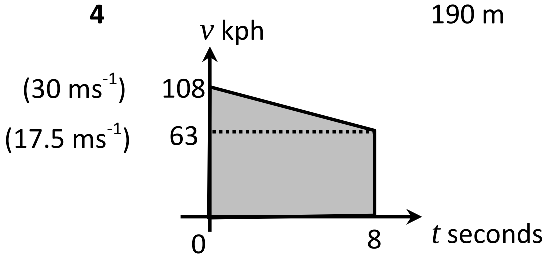
30 m

3



108 m

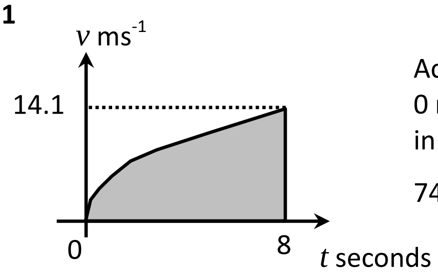
4



190 m

Answers part D

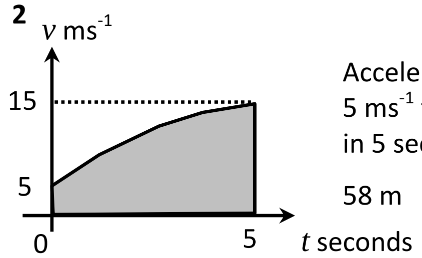
1



Accelerates from
0 ms^{-1} to 14.1 ms^{-1}
in 8 seconds

74 m

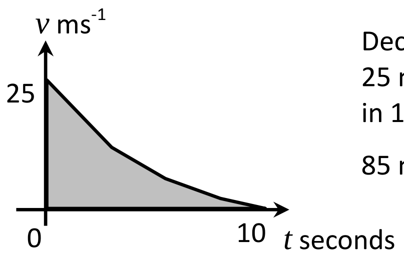
2



Accelerates from
5 ms^{-1} to 15 ms^{-1}
in 5 seconds

58 m

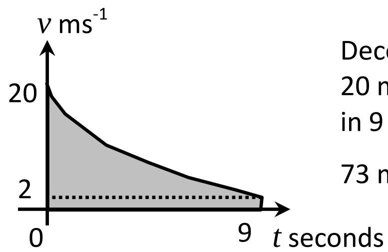
3



Decelerates from
25 ms^{-1} to 0 ms^{-1}
in 10 seconds

85 m

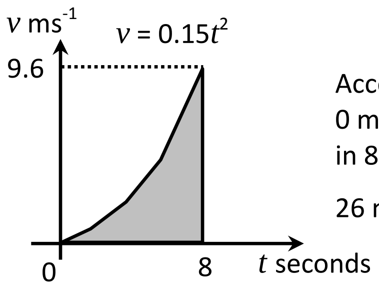
4



Decelerates from
20 ms^{-1} to 2 ms^{-1}
in 9 seconds

73 m

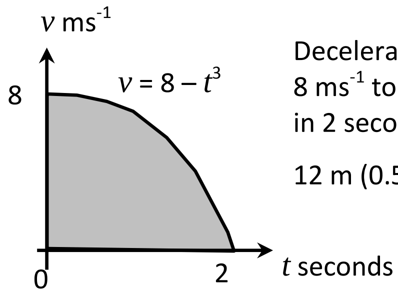
5



Accelerates from
0 ms^{-1} to 9.6 ms^{-1}
in 8 seconds

26 m (1s intervals)

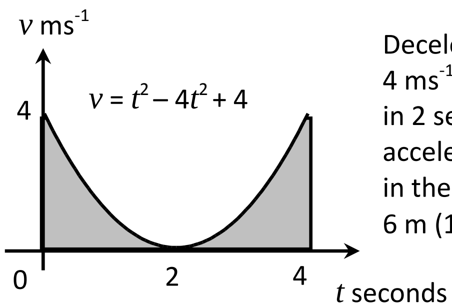
6



Decelerates from
8 ms^{-1} to 0 ms^{-1}
in 2 seconds

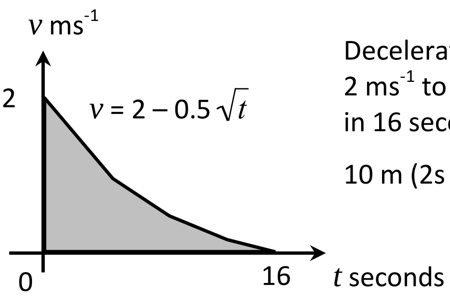
12 m (0.5s intervals)

7



Decelerates from
4 ms^{-1} to 0 ms^{-1}
in 2 seconds, then
accelerates to 4 ms^{-1}
in the next 2 seconds
6 m (1s intervals)

8



Decelerates from
2 ms^{-1} to 0 ms^{-1}
in 16 seconds

10 m (2s intervals)