



The area under a graph often gives useful information.

In this activity you will explore one example of this, the area under a speed–time graph.

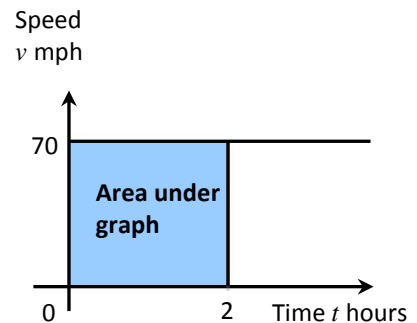
Student information sheet

A: Speed-time graphs with constant speed

The sketch shows the speed–time graph for a car that is travelling along a motorway at a steady 70 mph.

Think about

If the car is travelling at 70 mph, how far will it travel in 2 hours?



The area under this graph is rectangular in shape.
The shaded area = $2 \times 70 = 140$

Area under a speed–time graph = distance travelled

B: Speed–time graphs with constant acceleration

This graph shows the speed of a car that accelerates steadily from 0 to 72 kilometres per hour (kph) in 10 seconds.

$$72 \text{ kph} = \frac{72 \times 1000}{60 \times 60} = 20 \text{ metres per second (ms}^{-1}\text{)}$$

(1 km = 1000 m)

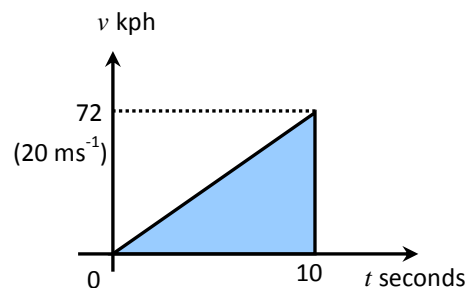
The distance travelled is given by the area under the graph:

$$\text{Distance travelled} = \frac{10 \times 20}{2} = 100 \text{ metres}$$

Think about

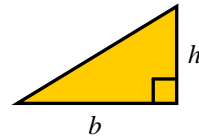
If the car has travelled 100 metres in 10 seconds, what is its average speed?

What is the connection with the values on the graph?

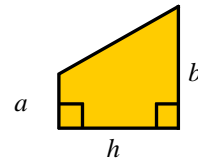


Formulae

$$\text{Area of a triangle} = \frac{\text{base} \times \text{height}}{2} = \frac{bh}{2}$$



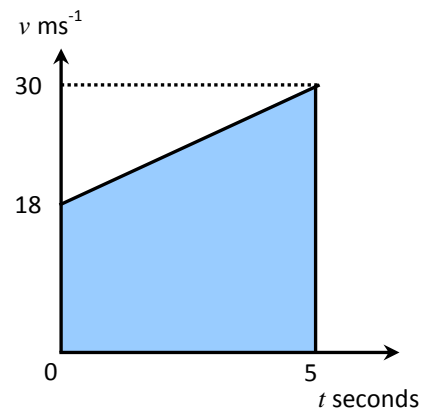
$$\begin{aligned} \text{Area of a trapezium} &= \frac{\text{sum of parallel sides} \times \text{distance between}}{2} \\ &= \frac{(a+b)h}{2} \end{aligned}$$



This graph represents a car accelerating steadily from 18 to 30 ms⁻¹ in 5 seconds.

The shaded area is a trapezium with area = $\frac{(18+30) \times 5}{2}$

Distance travelled = 120 metres



Think about

If the car has travelled 120 metres in 5 seconds, what is its average speed?

What is the connection with the values on the graph?

Student worksheet

Try these

In each case sketch a speed-time graph and find the distance travelled:

- 1 Car travels at a steady speed of 65 mph for 3 hours.
- 2 Car decelerates steadily from a speed of 54 kph until it stops 4 seconds later.
- 3 Car accelerates steadily from 12 ms⁻¹ to 24 ms⁻¹ over a period of 6 seconds.
- 4 Car decelerates steadily from 108 kph to 63 kph over a period of 8 seconds.

Reflect on your work

Can you use what you have learnt to solve the following problem:

A car accelerates from 0 to 30 ms⁻¹ over a distance of 180 metres. How long did this take?

How realistic is it to represent changes in the speed of cars as straight-line graphs?

Student information sheet

C: Speed–time graphs with variable speed and acceleration

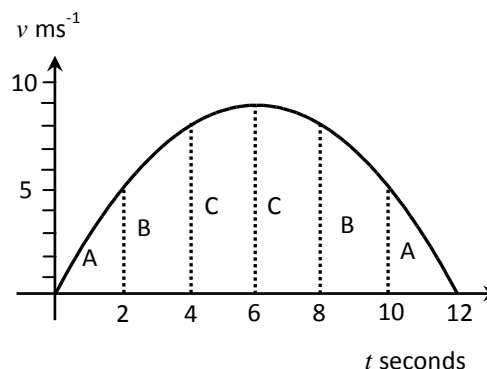
The table and graph give the speed of a car as it travels between two sets of traffic lights.

t (s)	0	2	4	6	8	10	12
v (ms^{-1})	0	5	8	9	8	5	0

An **estimate** for the area under this graph can be found by splitting it into strips as shown.

Think about

Why are the shapes labelled **A**, **B** and **C**?
How does this reduce the work needed?



The strips at each end are approximately triangular in shape, and each strip between them is approximately in the shape of a trapezium.

$$\text{Area of A: } \frac{2 \times 5}{2} = 5$$

$$\text{Area of B: } \frac{(5+8)2}{2} = 13$$

$$\text{Area of C: } \frac{(8+9)2}{2} = 17$$

$$\begin{aligned} \text{Total area: } & 5 + 13 + 17 + 17 + 13 + 5 \\ & = 70 \text{ metres} \end{aligned}$$

The distance travelled by the car between the traffic lights is approximately 70 metres.

Think about

The graph is a curve, but the shapes used to estimate the area have straight edges. Do you think the car actually travels further than 70 m or not as far?

The estimated distance was 70 m. How could you improve this estimate?

Is a real car likely to speed up and then slow down in the way shown by the graph?

Student information sheet

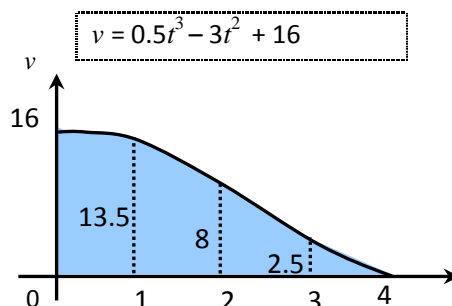
D: Speed–time graphs with speed modelled by a function

Suppose the speed of a car, $v \text{ ms}^{-1}$ is modelled by the function

$$v = 0.5t^3 - 3t^2 + 16 \text{ for } 0 \leq t \leq 4$$

Working out a table of values allows a graph to be drawn by hand (or you could use a graphic calculator or spreadsheet).

$t \text{ (s)}$	0	1	2	3	4
$v \text{ (ms}^{-1}\text{)}$	16	13.5	8	2.5	0



Think about

Can you describe what the car did in the time shown on the graph?

The distance travelled by the car can be found from the area under the graph:

$$\begin{aligned} \text{Area under graph} &= \frac{(16 + 13.5)1}{2} + \frac{(13.5 + 8)1}{2} + \frac{(8 + 2.5)1}{2} + \frac{1 \times 2.5}{2} \\ &= 14.75 + 10.75 + 5.25 + 1.25 \\ &= 32 \end{aligned}$$

Distance travelled is 32 metres

Think about

How could you find a more accurate value for the area under the graph?

Student worksheet

Try these

In each of the following:

- draw a speed–time graph on graph paper, a graphic calculator or a spreadsheet
- describe what happens during the given time interval
- estimate the distance travelled to the nearest metre.

1.

t (s)	0	1	2	3	4	5	6	7	8
v (ms^{-1})	0	5.0	7.1	8.7	10.0	11.2	12.2	13.2	14.1

2.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
v (ms^{-1})	5.0	6.9	8.6	10.1	11.4	12.5	13.4	14.1	14.6	14.9	15.0

3.

t (s)	0	2	4	6	8	10
v (ms^{-1})	25	16	9	4	1	0

4.

t (s)	0	1	2	3	4	5	6	7	8	9
v (ms^{-1})	20.0	14.0	11.5	9.6	8.0	6.6	5.3	4.1	3.0	2.0

5. Speed of a car, v ms^{-1} modelled by $v = 0.15t^2$ for $0 \leq t \leq 8$
6. Speed of a car, v ms^{-1} modelled by $v = 8 - t^3$ for $0 \leq t \leq 2$
7. Speed of a car, v ms^{-1} modelled by $v = t^2 - 4t + 4$ for $0 \leq t \leq 4$
8. Speed of a car, v ms^{-1} modelled by $v = 2 - 0.5\sqrt{t}$ for $0 \leq t \leq 16$

At the end of the activity

- Explain why using triangles and trapezia can only give an estimate of the area under a curve.
- When is the estimate smaller than the actual value? When is it larger? How can you improve the estimate?
- How well do you think the functions you have studied model the actual speed of real cars?
- In what way would graphs showing actual speeds differ from those used in this activity?