



This resource can be used as a classroom activity or an assignment. It involves students setting up their own network as the basis for a sightseeing tour.

In identifying their route, students will need to make use of the Chinese Postman Algorithm and/or the Travelling Salesman Algorithm, depending on whether their route involves visiting the edges or the vertices of their network.

### Suitability and Time

Level 3 (Advanced); 3–4 hours

### Resources and equipment

Student information sheet, internet access

*Optional:* slideshow

### Key mathematical language

Algorithm, edge, vertex, traversable graph, Eulerian trail, tour, Hamiltonian cycle, least upper bound, greatest lower bound

### Notes on the activity

The students should have already studied the Chinese Postman and Travelling Salesman problems. Initially students will need to:

- select places of interest – this may depend on the visitors and their interests
- research possible routes between places, estimating distances and times, using maps or the internet
- draw a network to represent the roads that might be used – the arcs could be weighted with distances or times, depending on which of these the students think is most important.

If students aim to visit each place once and only once, using a tour that returns to the starting point and covers as little distance as possible, this is a 'Travelling Salesman' problem.

In this case, students should appreciate the connection with the classical problem of finding a Hamiltonian cycle. They can find an upper bound by using the Nearest Neighbour algorithm. Students can also determine a lower bound by using Kruskal's or Prim's algorithm to find the length of a minimum spanning tree for a network formed by deleting a given node, and then adding the two shortest distances to the given node.

The students should appreciate when a solution is sufficiently good, realise that a solution they find is not necessarily the best, and comment on how appropriate it is in this context.

Where there are a number of possible routes between two places of interest, students may decide to use Dijkstra's Algorithm to find the shortest or quickest route.

If your local area is one of outstanding beauty, it may be more appropriate to plan a tour that includes all the most scenic roads.

In trying to find a route that will travel along each of these roads exactly once before returning to the starting point, students will be attempting to solve a 'Chinese Postman' problem.

In this case, students should appreciate the connection with the classical problem of finding an Eulerian trail. They will need to understand the significance of any odd vertices in their network.

If there is no Eulerian cycle, then some edges (roads) will have to be used more than once unless students extend the network to include other roads. If they decide not to include more roads, the Chinese Postman algorithm could be used to find which repeat edges will add the least distance.

Again the appropriateness of the possible solutions should be considered in terms of the real situation.

### During the activity

Students could work individually, in pairs or small groups on this activity.

They will initially need internet access or maps to collect relevant information to form the basis of their tour and set up their network.

If this is used as a group activity, each group could be asked in turn to present their findings.

### Points for discussion

Students should be encouraged to discuss:

- what places of interest they should select for their tour
- is it the *route* that is of interest for the tour or the *places* they are travelling to
- what possible routes are there between places
- how they should weight the edges for their network, either in distance or time.

### Extensions

Students could be asked to compare the different routes found by each group, and discuss their effectiveness and limitations.