



In this activity you will use some real data to estimate the probability of an event happening. You will also use a variety of methods to work out theoretical probabilities. Theoretical and experimental probabilities can be used to predict the number of trials in which an event will occur.



Information sheet

Relative frequency

Suppose you want to find the probability of a drawing pin landing point up when you toss it up in the air.

To estimate the probability you could carry out an experiment to find in what fraction of the trials the drawing pin lands point up. This fraction, called the relative frequency, gives an estimate of the probability.

$$\text{Relative frequency of E} = \frac{\text{Number of times E happened}}{\text{Total number of trials}}$$

For example, if the drawing pin was tossed 200 times and it landed point up on 140 of these trials,

$$P(\text{drawing pin lands point up}) \approx \frac{140}{200} = \frac{7}{10} = 0.7$$

The greater the amount of data used, the more reliable the relative frequency is as an estimate of the probability. For example, doing the experiment 2000 times would give a better estimate of the probability than doing it 200 times.

In other situations you might use a survey or data that has been collected in the past. For example, if you wanted to estimate the probability of a train being late you could look at the train operator's past record on punctuality for that journey. Again, the more data used, the better the estimate.

Think about

Can you think of other situations where data can help you to estimate probabilities?

Theoretical probability

When something can happen in a number of *equally likely* ways, the probability of event, E, can be calculated using:

$$P(E) = \frac{\text{Number of ways E can happen}}{\text{Total number of possibilities}}$$

For example, if a fair dice is thrown, there are 6 possibilities ...



$P(6) = \frac{1}{6}$ This means the probability of getting a six is $\frac{1}{6}$

$P(\text{even number}) = P(2 \text{ or } 4 \text{ or } 6) = \frac{3}{6} = \frac{1}{2}$

$P(\text{more than } 2) = P(3 \text{ or } 4 \text{ or } 5 \text{ or } 6) = \frac{4}{6} = \frac{2}{3}$



Think about

How do these examples use the fact that the dice is fair?

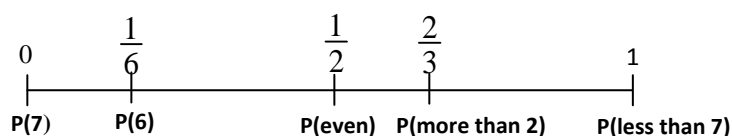
The probability of something that is **impossible** is **0**.

For example, on the dice $P(7) = 0$.

The probability of something that is **certain** is **1**.

For example, on the dice $P(\text{score} < 7) = 1$.

These results can all be shown on a **probability line**:



E and not E

As it is certain that an event E either occurs or does not occur

$P(\text{E doesn't happen}) + P(\text{E happens}) = 1$.

Using the notation $P(\bar{E})$ for the probability that E doesn't happen,

$P(\bar{E}) = 1 - P(E)$.

Think about

Use this result and the probabilities given above to answer these questions:

- What is the probability of the drawing pin from the first example not landing point up?
- For a fair dice, what is the probability of not getting a 6?
- What is the probability of getting at most 2 when a fair dice is thrown?

Expectation

Probabilities can be used to make predictions. For example, if a fair dice is thrown 300 times, the expected number of sixes is one sixth of 300, which is 50.

In general:

Expected number of trials in which E occurs = $P(E) \times$ total number of trials

Using the probabilities given earlier,

- expected number of even scores = $\frac{1}{2} \times 300 = 150$
- expected number of scores above 2 = $\frac{2}{3} \times 300 = 200$

Combined events

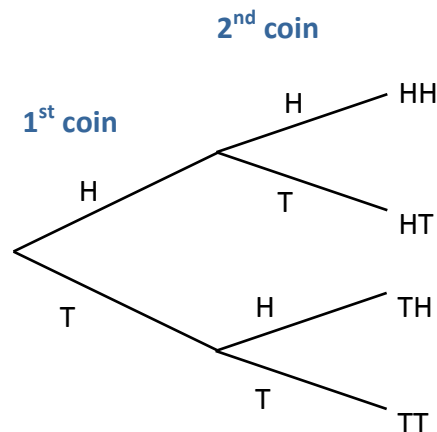
If two fair coins are tossed the possibilities are:

HH HT TH TT



These can be systematically listed, as above, or shown in a table, or in a tree diagram.

		2 nd coin	
		H	T
1 st coin	H	HH	HT
	T	TH	TT



Each of these methods gives all four possible outcomes, and allows probabilities to be found.

For example:

HH is one of the four equally likely outcomes, so the probability of getting 2 heads is $\frac{1}{4}$.

Similarly, the probability of getting a head and tail in any order = $\frac{2}{4} = \frac{1}{2}$

The probability of getting the same on each coin = $\frac{2}{4} = \frac{1}{2}$

The probability of getting at least one tail = $\frac{3}{4}$

Think about

Can you use $P(\bar{E}) = 1 - P(E)$ to relate these results?

Try these

1 Lost luggage

The table gives the number of bags checked in for British Airways and bmi flights in 2007 and the number of these bags that were delayed or lost.

	Number of bags	
	checked in	delayed/lost
British Airways	43 064 346	1 141 205
bmi	5 229 520	88 902

a Use this data to estimate the probability that:

i a bag checked in for a British Airways flight will be delayed or lost

.....

ii a bag checked in for a bmi flight will be delayed or lost

.....

b If British Airways do not deliver a bag at the end of a flight, the probability that it will be returned within 48 hours is 0.85.

i What is the probability that a delayed bag will not be returned within 48 hours?

.....

ii 450 bags are checked in for a British Airways flight.
How many of these bags do you expect to be delayed?

.....

iii How many of the bags do you expect to be delayed more than 48 hours?

.....

2 Dice differences

a The table shows the possible differences between the scores of two fair dice when they are thrown. One row of the table has been filled in.

Complete the other rows.

		2 nd dice					
		1	2	3	4	5	6
1 st dice	1						
	2	1	0	1	2	3	4
	3						
	4						
	5						
	6						

b Complete the table to give the probability of each of the possible differences.

Difference	0	1	2	3	4	5
Probability						

c What is the probability that the difference is:

- i greater than 3?
- ii less than or equal to 2?

3 Three coins

a Complete the diagram to show all the possibilities when 3 fair coins are tossed.

b What is the probability that

i all 3 coins show tails?

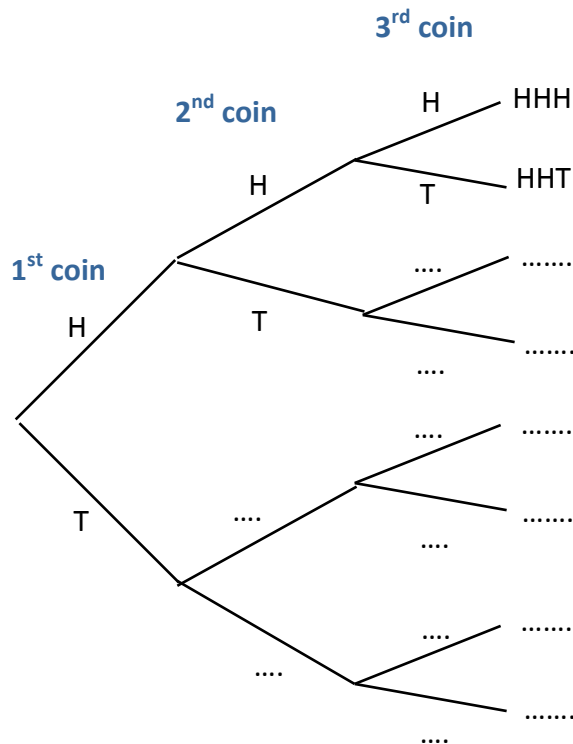
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ii all 3 coins show the same?

.....

iii there are 2 heads and 1 tail?

.....



4 Children

a Each child could be a boy or girl. List all the possibilities for 3 children.

.....

.....

b Assuming that any child is equally likely to be a boy or girl, find the probability that if a couple have 3 children:

i all the children will have the same gender

ii two of the children will be boys

iii there will be more boys than girls

c List all the possibilities for 4 children.

.....

.....

.....

.....

d Assuming that any child is equally likely to be a boy or girl, find the probability that a couple who have 4 children have an equal number of boys and girls.

.....

Extension

What connections are there between probabilities for the three coin tosses in Question 3 and those for the three children in Question 4?

Why?

Question 2 is about the differences in the scores when two fair dice are thrown. Make up and solve a probability question when the scores on two dice are combined in some other way, such as adding or multiplying, or taking the larger score.

Reflect on your work

Explain how relative frequency is used to estimate probabilities.

Give some examples.

Write down a formula for the theoretical probability of an event E.

Why is it important to assume that dice and coins are fair when you are calculating theoretical probabilities?

Describe three ways of finding probabilities of combined events.