In this activity you will use the laws of probability to solve problems involving mutually exclusive and independent events. You will also use probability tree diagrams to help you to calculate probabilities.

Information sheet

Given events A and B, the probability of the combined events (A or B) or (A and B) are often of interest. In certain situations these can be calculated from the probability of A and the probability of B. Two of these situations are explained below.

Mutually exclusive events

When events A and B cannot both happen at the same time, they are called ***mutually exclusive*** events.

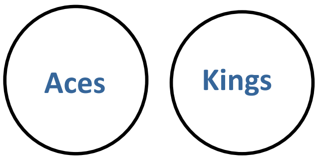
When a card is drawn at random from a pack of 52 playing cards it can be an Ace or a King, but it can't be both. Drawing an Ace and drawing a King are mutually exclusive events.

For a card drawn at random from the pack,

P(Ace) = $\frac{4}{52} = \frac{1}{13}$ and P(King) = $\frac{4}{52} = \frac{1}{13}$

P(Ace *or* King) = $\frac{8}{52} = \frac{2}{13}$ = P(Ace) + P(King)

Venn diagram showing Aces and Kings:



When events A and B are ***mutually exclusive***

P(A or B) = P(A) + P(B)

The events 'draw an Ace' and 'draw a Heart' are **not *mutually exclusive*** because both events happen together if you draw the Ace of Hearts.

When events A and B are ***not mutually exclusive***, you cannot just add the probabilities.

If a card is drawn at random from a pack of 52

P(Ace) = $\frac{4}{52}$ and P(Heart) $= \frac{13}{52}$

Think about

What is P(Ace or Heart)?

Since 13 cards are hearts and there are another 3 aces, there are just 16 cards out of 52 cards which are either hearts or aces so:

$$\text{P(Ace or Heart)} = \frac{16}{52} \; \textit{not} \; \frac{17}{52}$$

In this case adding P(Ace) and P(Heart) gives a value which is too high because the Ace of Hearts is included twice.

Think about

What does the Venn diagram for Aces and Hearts look like?

Independent events

If a fair coin is tossed and a card is taken at random from a pack of 52 playing cards,

$\text{P(Head)} = \frac{1}{2}$ and $\text{P(King)} = \frac{1}{13}$.

Each of the events 'throw a Head' or 'draw a King' has no effect on the probability of the other event occurring. These events are said to be **independent**.

For independent events, the probability of both events occurring can be calculated by multiplying the probabilities of the events together.

$$\text{P(Head } \textit{and} \text{ King)} = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26}$$

When combined events A and B are **independent**

P(A and B) = P(A) × P(B)

Probability tree diagrams

These show all the possibilities for combined events together with their probabilities.

Example

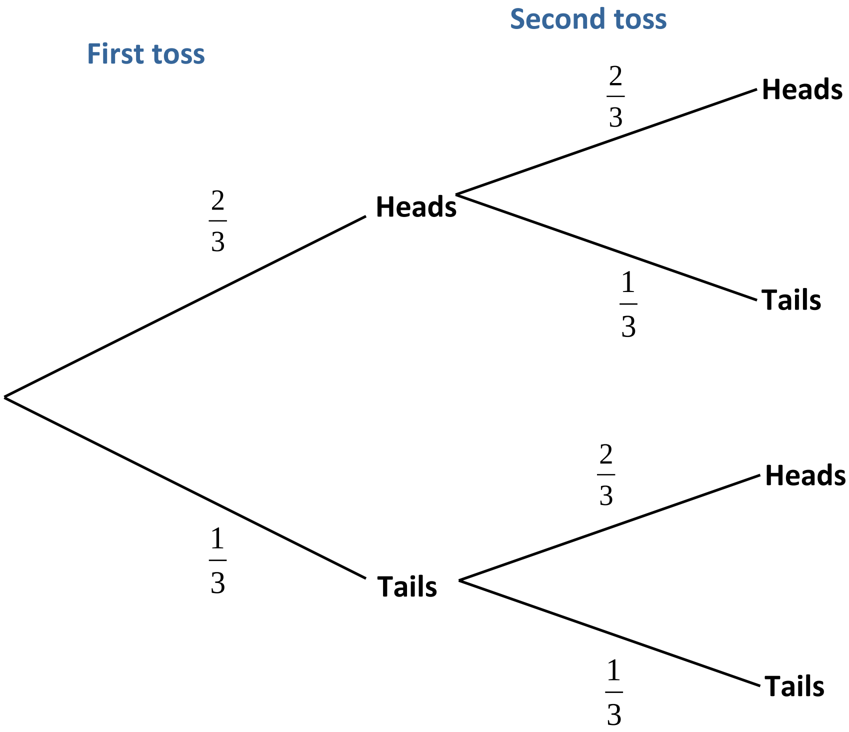
A coin is biased so that it is twice as likely to give heads as it is to give tails.

This means that whenever the coin is tossed $\text{P(H)} = \frac{2}{3}$ and $\text{P(T)} = \frac{1}{3}$.

The tree diagram below shows the possible outcomes when this coin is tossed twice.

The first set of branches shows the possibilities for the first toss of the coin.

The second sets of branches show the possibilities for the second toss of the coin.



P(H, H) = $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

P(H, T) = $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

P(T, H) = $\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

P(T, T) = $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

Think about

What is the sum of the probabilities on each set of branches? Why?

Are the events Heads on the first toss and Tails on the second toss independent?

Are the events (H,H), (H,T), (T,H), (T,T) mutually exclusive?

The result of the first toss of the coin has no effect on the second toss, so these events are independent. So, the probability of any combination is found by **multiplying** the probabilities on the branches along the path that leads to the combination.

Think about

What is the sum of the probabilities of all the combinations?

You can **add the resulting probabilities** to find the probabilities of other events

For example, the probability that both tosses give the same result is:

P(H, H) + P(T, T) = $\frac{4}{9} + \frac{1}{9} = \frac{5}{9}$

Think about

What is the probability that the tosses give different results?

Try these

1 Each row in the table gives a pair of events.
In each case decide whether the events are mutually exclusive or not.

	Mutually exclusive?	
	Yes	No
Angela goes for her train to work: Event A: she catches the train Event B: she misses the train		
Rory throws a dice: Event A: he gets an odd number Event B: he gets less than 4		
Rory throws a dice: Event A: he gets more than 3 Event B: he gets less than 3		
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a club		
Sue takes a card at random from a pack of 52: Event A: she gets a spade Event B: she gets a queen		

2 A box contains 1 black button, 3 blue buttons and 5 white buttons.

If a button is taken out of the box at random, what is the probability that it is:

a black ………… **b** blue ………… **c** white …………

d black or blue ………… **e** blue or white ………… **f** black or white …………

3 A fair coin is tossed and a fair dice is thrown. Assuming independence, find the probability of:

a heads and a six ……………………………………………………………………….

b heads and an even number …………………………………………………………….

c tails and more than 4 ……………………………………………………………….

4 A delivery firm delivers 75% of packages the next day.

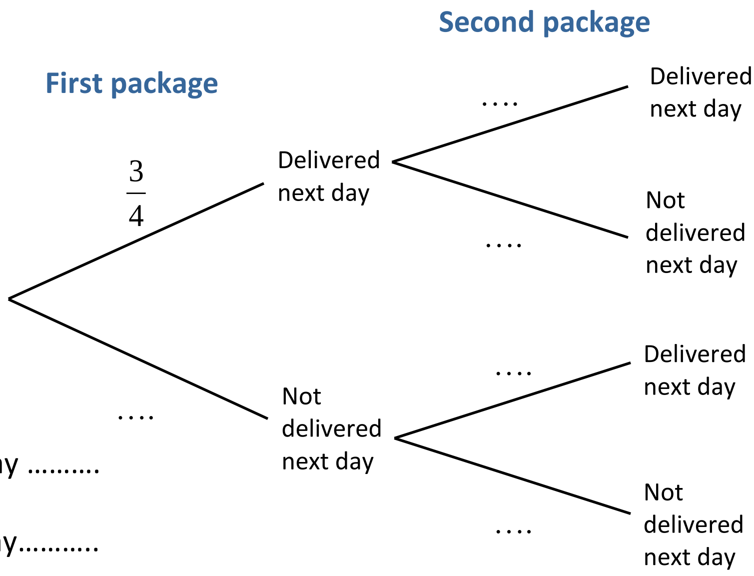
Jack posts two packages.

a Complete the tree diagram.
(Assume deliveries are independent.)

b Write down the probability that

i both packages are delivered the next day ……….

ii neither package is delivered the next day……….



5 Neil plants three bulbs in a pot.

a Assuming independence and that the probability of a bulb producing a flower is 0.8, complete the tree diagram.

b What is the probability that:

i all three bulbs produce a flower?

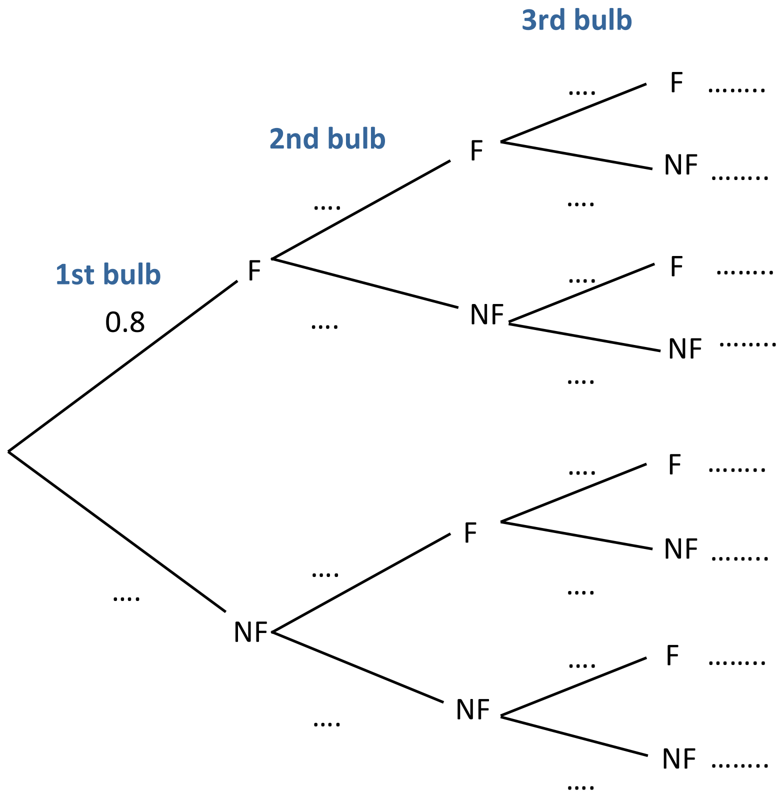
…………………………………………………

ii none of the bulbs produce a flower?

…………………………………………………

iii two or more bulbs produce a flower?

…………………………………………………



Key
F a flower is produced
NF no flower is produced

6 Kate passes two sets of traffic lights on her way to work.

The probability that she has to stop for the first set of traffic lights is $\frac{1}{3}$.

The probability that she has to stop for the second set of traffic lights is $\frac{2}{5}$.

a On a separate piece of paper draw a tree diagram to show the probabilities of her stopping for these traffic lights.

b What is the probability that on one journey to work Kate will:

i have to stop for both sets of traffic lights ………………………………

ii not have to stop for either set of traffic lights

………………………………

c Kate travels to work on 220 days each year. On how many of these days would you expect her to have to stop for at least one set of traffic lights?

……………………………………………………………………………………………………

Extension

A biased coin is tossed twice. The probability of getting a Tail and then a Head is $\frac{6}{25}$ and the probability of getting two Tails is $\frac{4}{25}$.

What is the probability of getting a Head if the coin is tossed once?

If the coin is tossed three times, what is the probability of getting at least one Head?

Reflect on your work

Explain what is meant by the term 'mutually exclusive' and give an example. What is the law of probability that can be applied to mutually exclusive events?

What does the Venn diagram of two mutually exclusive events look like? What does it look like if the events are not mutually exclusive?

Explain what is meant by the term 'independent' and give an example. What is the law of probability that can be applied to independent events?

Explain how the laws of probability are applied in a tree diagram.