



### Activity description

In this activity students use logarithms to find a relationship between magnitude and frequency of earthquakes. By plotting a straight line log graph, they are able to estimate parameters of the non-linear relationship.

### Suitability and Time

Level 3 (Advanced)

Up to 1 hour, or longer if activity is used in a more open-ended way.

### Resources

Student information sheet

Optional: slideshow

### Equipment

Calculators (spreadsheet facility is optional), graph paper

### Key mathematical language

Frequency, model, exponential, ratio, logarithm, linear, intercept, gradient, average

### Notes on the activity

This activity uses the context of earthquakes to introduce modelling of data by plotting a log graph and using this to determine the exponential relationship between magnitude of earthquakes and average annual frequency. The student sheets include an information sheet (page 1), questions based on it (page 3), and an optional sheet (page 2) that guides students through the modelling process.

The optional sheet uses approximately constant ratios to justify modelling the data with an exponential function, and helps clarify how taking logarithms leads to plotting a straight line graph. You may, however, prefer not to give this information to the students at the outset and instead start the activity by plotting the log graph and allow the students to explore how they can use this to determine the relationship.

The slideshow can be used to introduce the context. Some questions are included to aid class discussion. The graph on the information sheet is on a slide and could be used to discuss which functions could help model the data.

### During the activity

Students might need help computing the values of  $\log_{10} N$ , especially if they have only worked with natural logarithms. It might be worth exploring how they could use properties of logs to help them estimate or check values.

## Points for discussion

How do we know to model data with a particular function?

Why is it useful to have a straight line graph?

How does the choice of base 10 affect the model?

Does a straight line log graph necessarily mean that there is an exponential relationship?

The notes in the information sheet show that constant ratios of successive terms suggest an exponential relationship, but students should understand that they could simply start by plotting a log graph.

Although it is suggested that students work with  $\log_{10} N$  rather than  $\ln N$ , students might be curious about whether the base of the logarithm will affect the model they produce. You could ask one or two students to use natural logarithms for comparison, or use this as an extension activity. At the end of the session, use the reflection section at the end of the slideshow or student sheets to help students reflect on their work.

## Extensions

1. The US Geological Survey has an Earthquake Archive search facility <http://earthquake.usgs.gov/earthquakes/search/> that allows one to extract (up-to-date) bespoke data sets, using which one could model the data for particular geographical regions.

See also, the Gutenberg – Richter Law.

[http://en.wikipedia.org/wiki/Gutenberg%E2%80%93Richter\\_law](http://en.wikipedia.org/wiki/Gutenberg%E2%80%93Richter_law)

2. Students could use work related to their other studies and interests to suggest other situations that could be modelled by an exponential function and/or analysed using log graphs. Possible situations include:

- speed and braking distance
- population and time
- compound interest
- mass of a radioactive substance and time
- temperature of a cup of tea and time
- the number of times a ball has bounced and the height of a bounce.

## Answers

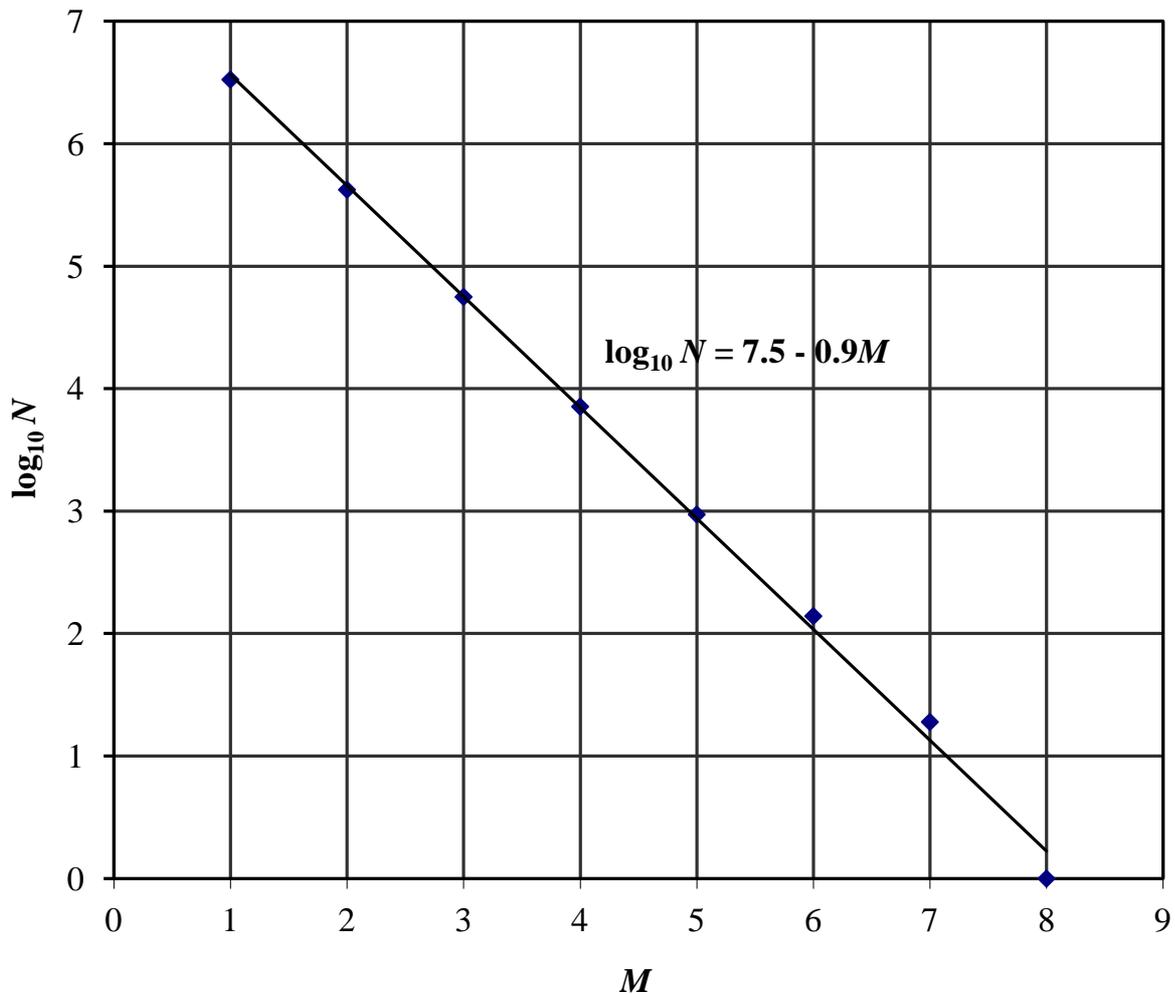
### Question 1.

M	1	2	3	4	5	6	7	8
N	3341000	421000	56000	7100	939	139	19	1
$\log_{10} N$	6.52	5.62	4.75	3.85	2.97	2.14	1.28	0

### Questions 2 and 3.

A graph of  $\log_{10} N$  against  $M$ , produced using an Excel spreadsheet, is given below.  
The equation of the line of best fit is  $\log_{10} N = 7.5 - 0.9M$  (to 1 dp).

**Graph of  $\log_{10} N$  against  $M$**



The relationship  $\log_{10} N = c - bM$ , between the magnitude ( $M$ ) of earthquakes and the average annual number ( $N$ ) of earthquakes of magnitude at least  $M$ , is a particular case of the **Gutenberg-Richter Law**.

[http://en.wikipedia.org/wiki/Gutenberg%E2%80%93Richter\\_law](http://en.wikipedia.org/wiki/Gutenberg%E2%80%93Richter_law)

This relationship has been found to apply to particular regions as well as to the world as a whole, and across different intervals of time. The constant  $c$  reflects the level of seismicity of the region and the constant  $b$  has been found to be consistently close to the value 1 in seismically active regions.

For the region of the UK from  $10^\circ W$  to  $2^\circ E$  and from  $49^\circ N$  to  $59^\circ N$  the Gutenberg-Richter formula has been found to be  $\log_{10} N = 3.82 - 1.03M$ .

#### Question 4.

Some students may draw up a table of ratios, as in the information sheet, while others might start by plotting a log graph.

Magnitude	Frequency $F$	$\log_{10} F$
1.5	2 920 000	6.47
2.5	365 000	5.56
3.5	49 000	4.69
4.5	6 200	3.79
5.5	800	2.90
6.5	120	2.08
7.5	18	1.26
8.5	1	0

The equation of the line of best fit is  $\log_{10} F = 7.8 - 0.9M$  (to 1dp).