In this activity you will use a spreadsheet to calculate the gradients of functions of the form x^n and find a general rule for finding gradients for functions of this type.

Information sheet

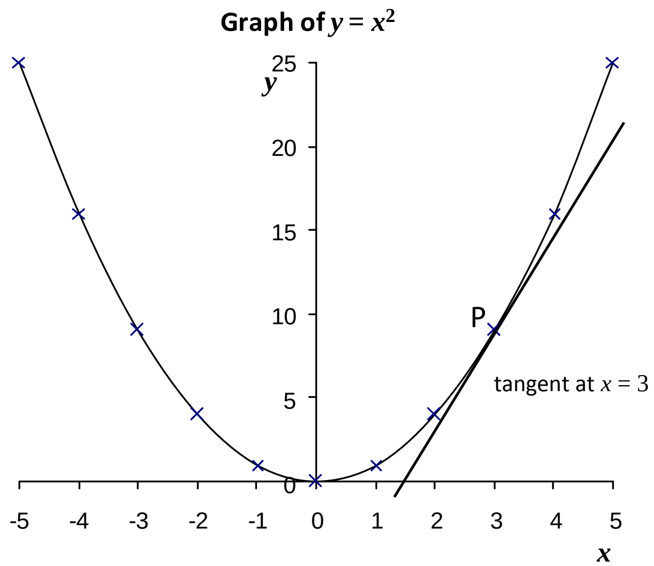
The gradient of a curve has different values at different points.

The sketch shows the graph of $y = x^2$.

When x is negative the gradient is negative, because y is decreasing as x increases.

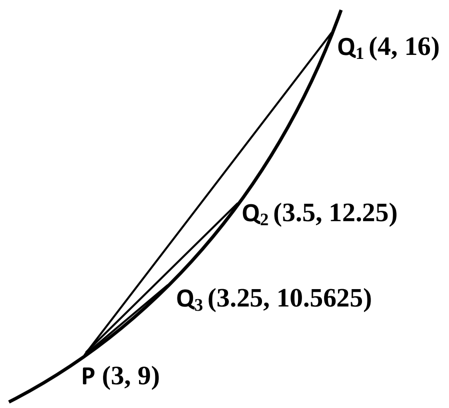
When x is positive the gradient is positive and increases as x increases.

This means the curve becomes steeper.



A tangent has been drawn at the point, P, on the curve where $x = 3$.
The gradient of the curve at this point is the gradient of this tangent.

The gradient can be found from a hand-drawn graph, but this method gives an estimate rather than an accurate result.



Look at the second sketch.

It shows the section of the curve between the points P(3, 9) and Q_1(4, 16).

The gradient of the straight line PQ_1 is given by:

$$\text{gradient of } PQ_1 = \frac{\text{difference in } y \text{ values}}{\text{difference in } x \text{ values}}$$

$$= \frac{16 - 9}{4 - 3} = 7$$

Now suppose another point, nearer to P, is used rather than Q_1

At the point Q_2, the x coordinate is 3.5, and the y coordinate is $3.5^2 = 12.25$

$$\text{Gradient of } PQ_2 = \frac{12.25 - 9}{3.5 - 3} = \frac{3.25}{0.5} = 6.5$$

Repeating this at the point Q_3, where the x coordinate is 3.25, and the y coordinate is $3.25^2 = 10.5625$

$$\text{Gradient of } PQ_3 = \frac{10.5625 - 9}{3.25 - 3} = \frac{1.5625}{0.25} = 6.25$$

As the point Q moves nearer to P, the gradient of PQ becomes nearer to the gradient of the tangent at P. Carrying out this process a number of times is time-consuming because of the arithmetic involved. A spreadsheet can be used to perform the calculations and give results more quickly.

Try this … Use the spreadsheet to find gradients as follows.

A Gradient of $y = x^2$

The x **squared** worksheet has been set up to show the gradients calculated on page 1.

The spreadsheet formulae that have been used are given below. Look at them carefully.

	A	B	C	D
1	**Point**	x	$y = x^2$	**Gradient of PQ**
2	P	3	9	
3	Q_1	4	16	7
4	Q_2	3.5	12.25	6.5
5	Q_3	3.25	10.5625	6.25

	A	B	C	D
1	**Point**	x	$y = x^2$	**Gradient of PQ**
2	P	3	= B2^2	
3	Q_1	= B2 + 1	= B3^2	= (C3 –-C2)/(B3 – B2)
4	Q_2	= (B2 + B3)/2	= B4^2	= (C4 –-C2)/(B4 – B2)
5	Q_3	= (B2 + B4)/2	= B5^2	= (C5 –-C2)/(B5 – B2)

gradient is difference in y values divided by difference in x values. Absolute references for cells C2 and B2 mean the coordinates of P are used each time.

The x coordinate used for Q is halfway between the x coordinate of P and the previous x coordinate.

squaring the x value gives the y value

Use 'fill-down' to extend the table as far as Q_{10}.

	A	B	C	D
1	**Point**	x	$y = x^2$	**Gradient of PQ**
2	P	3	9	
3	Q_1	4	16	7
4	Q_2	3.5	12.25	6.5
5	Q_3	3.25	10.5625	6.25
6	Q_4	3.125	9.765625	6.125
7	Q_5	3.0625	9.37890625	6.0625
8	Q_6	3.03125	9.188476563	6.03125
9	Q_7	3.015625	9.093994141	6.015625
10	Q_8	3.0078125	9.046936035	6.0078125
11	Q_9	3.00390625	9.023452759	6.00390625
12	Q_{10}	3.001953125	9.011722565	6.001953125

As Q approaches P the gradient of PQ approaches the value 6.

This is the gradient of the tangent to the curve at P.

The gradient of $y = x^2$ at the point (3, 9) is 6.

On this worksheet, the numerical values in the cells giving x, y and gradient values are all determined from the value entered in cell B2. This means that the gradient at any other point on $y = x^2$ can be found by simply replacing the value in B2 by the x coordinate of the new point.

Enter the value 2 in cell B2.

You should find that the values in the other cells change to give you the gradient of $y = x^2$ at the point (2,4).

The gradient is 4.

The values of the gradients found so far are given in the table.

Use the spreadsheet to find the gradients needed to complete this table.

Point	x coordinate	Gradient
(–4, 16)	–4	
(–3, 9)	–3	
(–2, 4)	–2	
(–1, 1)	–1	
(0, 0)	0	
(1, 1)	1	
(2, 4)	2	4
(3, 9)	3	6
(4, 16)	4	

Think about …

What is the relationship between the gradient and the x coordinate?

Gradient = ……………………………………

Gradient functions

For the curve $y = x^2$ the gradient function is $2x$

B Gradient of $y = x^3$

Open the x cubed worksheet.

This has been set up to find the gradient of the curve $y = x^3$ at the point (1, 1)

Look carefully at the spreadsheet functions used to start the table.

Compare them with those that were used for $y = x^2$ (given on page 2).

Use 'fill-down' to extend the table as far as Q_{10}.

This will show that the gradient of the curve $y = x^3$ at the point (1, 1) is 3.

Use the spreadsheet to find the gradients needed to complete this table.

Point	x coordinate	Gradient
(–4, 64)	–4	
(–3, 27)	–3	
(–2, 8)	–2	
(–1, –1)	–1	
(0, 0)	0	
(1, 1)	1	3
(2, 8)	2	
(3, 27)	3	
(4, 64)	4	

Think about …

What is the gradient function for $y = x^3$?

Gradient = ……………………………………

C Gradients of other functions of the form $y = x^n$

Set up a new worksheet to find the gradient of the curve $y = x^4$ at the point (1, 1)

Use your worksheet to complete the table.

Point	x coordinate	Gradient
(–4, 256)	–4	
(–3, 81)	–3	
(–2, 16)	–2	
(–1, 1)	–1	
(0, 0)	0	
(1, 1)	1	
(2, 16)	2	
(3, 81)	3	
(4, 256)	4	

Think about …

What is the gradient function for $y = x^4$?

Gradient = ………………………………….

Complete the second and third rows of this table:

Equation of curve	Gradient function
$y = x^2$	$2x$
$y = x^3$	
$y = x^4$	
$y = x^5$	

Think about …

What do you think is the gradient function for $y = x^5$?

Put your answer into the table above.

Use the spreadsheet to check your answer.

What do you think is the gradient function for $y = x^6$?

Can you write down a general rule for the gradient function of $y = x^n$?

Reflect on your work

Describe the way in which the gradient of a curve can be found using a spreadsheet.

What advantages does this have on drawing a tangent to a hand-drawn graph?

What is the gradient function of $y = x^n$?