Are there significant differences between body measurements taken from male and female children?

Do differences emerge at particular ages?

In this activity you will use anthropometric data to carry out significance tests to answer such questions.

Information sheet A Anthropometric data

Manufacturers of children's clothing need to consider the body measurements of boys and girls. Toy manufacturers may also need to consider their weights. These body measurements will depend on age and sometimes gender. In some cases boys and girls of the same age may be similar in size, but in other cases they may be different.

The spreadsheet contains anthropometric data collected for a 1977 study carried out in the USA by the Consumer Product Safety Commission (CPSC). The data includes:

- weight (newtons)
- stature (mm)
- head, chest and waist circumferences (mm)
- shoulder breadth (mm)
- hand length and breadth (mm)
- foot length and breadth (mm)
- age (months)

The 'Male' and 'Female' worksheets give data for males and females sorted by age and stature.

Mean

The mean of a sample can be found using $\bar{x} = \frac{\sum x}{n}$

or using the spreadsheet formula AVERAGE.

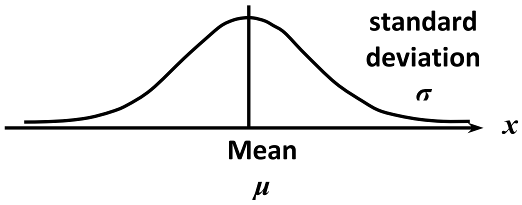
Standard deviation

The formula for the best estimate of the population standard deviation from a sample is:

$$\sigma_{n-1} = \sqrt{\frac{n}{n-1}\left(\frac{\sum x^2}{n} - \bar{x}^2\right)}$$

The corresponding spreadsheet formula is STDEV.

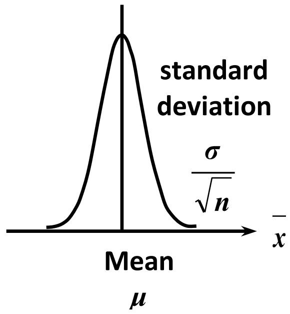
Information sheet B Testing a mean



Distribution of a sample mean

When samples of size n are taken from a normal population with mean μ and standard deviation σ, then the sample mean, $\overline{X}$, follows a normal distribution with mean μ and standard deviation $\dfrac{\sigma}{\sqrt{n}}$.

Distribution of Sample Mean



Think about …

Why is the standard deviation of the sample mean smaller?

Summary of method for testing a mean

To test whether the mean of a population has a value μ:

Null hypothesis H_0 : population mean, μ = value suggested

Alternative hypothesis H_1 : $\mu \neq$ value suggested (two-tail test)
or $\mu <$ value suggested or $\mu >$ value suggested (one-tail test)

Test statistic $z = \dfrac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

> ### Think about …
> Explain the formula for the test statistic.

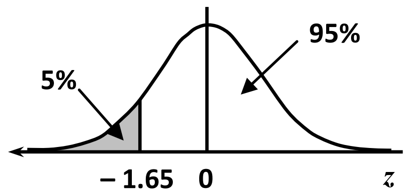
where $\overline{x}$ is the sample mean, σ is the standard deviation and n is the size of the sample.

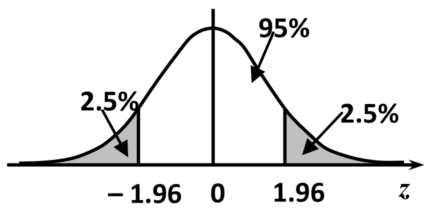
If the standard deviation of the population is not known, then the best estimate from the sample, σ_{n-1} is used instead (see page 1).

Compare the test statistic with critical values of z.

Critical values

Test type	Significance level	Critical values
one-tail test	5%	1.65 or −1.65
	1%	2.33 or −2.33
two-tail test	5%	±1.96
	1%	±2.58





If the test statistic is **in the critical region** (that is, a tail of the distribution), **reject the null hypothesis in favour of the alternative.**
If the test statistic is **not in the critical region**, **accept the null hypothesis**.

Testing a mean: T-shirt example

A clothing manufacturer designs boys' t-shirts for a chest circumference of 540 mm. Is the mean for 4-year-old boys larger than this?

Using data from the spreadsheet to test this mean:

H_0 : population mean, μ = 540 mm

H_1 : μ > 540 mm (one-tail test)

Think about ...

Why is a one-tail test used here rather than a two-tail test?

Test statistic $$z = \frac{\bar{x} - \mu}{\frac{\sigma_{n-1}}{\sqrt{n}}}$$ with μ = 540

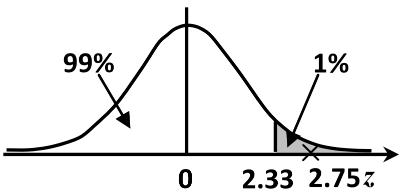
From the data, $\bar{x}$ = 546.94355 and n = 124

So $$z = \frac{546.94355 - 540}{\frac{28.07432}{\sqrt{124}}} = 2.75$$

For a one-tail 1% significance test, the critical value is 2.33.

The test statistic is in the critical region (greater than the critical value).

The result is **significant** at the 1% level.



So reject the null hypothesis and accept the alternative.

The test has provided strong evidence that the mean chest circumference of 4-year-old boys is more than 540 mm.

Think about...

Explain the reasoning behind this conclusion.

Information sheet C Testing the difference between means

Summary of method for testing the difference between means

To test the difference between means:

Null hypothesis, $H_0 : \mu_A = \mu_B$ $(\mu_A - \mu_B = 0)$

Alternative hypothesis, $H_1 : \mu_A \neq \mu_B$ (two-tail test)

$\mu_A < \mu_B$ (one-tail test)

$\mu_A > \mu_B$ (one-tail test)

The test statistic is $z = \dfrac{\bar{x}_A - \bar{x}_B}{\sqrt{\left(\dfrac{\sigma_A{}^2}{n_A} + \dfrac{\sigma_B{}^2}{n_B}\right)}}$

where $\bar{x}_A$ and $\bar{x}_B$ are the means from samples of size n_A and n_B.

σ_A and σ_B are standard deviations of the populations. If these are not known, use the best estimates that can be found from the samples.

Think about…

Why are the variances added in the formula for the test statistic?

Compare the test statistic with critical values of z.

Critical values:

Test type	**Significance level**	**Critical values**
one-tail test	5%	1.65 or −1.65
	1%	2.33 or −2.33
two-tail test	5%	±1.96
	1%	±2.58

If the test statistic is **in the critical region,**
reject the null hypothesis in favour of the alternative.

If the test statistic is **not in the critical region**, **accept the null hypothesis**.

Testing the difference between means: Hand length example

Using data from the spreadsheet to test whether the hand lengths of 2-year-old boys are significantly different from those of 2-year-old girls:

$H_0 : \mu_M = \mu_F \quad (\mu_M - \mu_F = 0)$
$H_1 : \mu_M \neq \mu_F$ (two-tail test)

The test statistic is $z = \dfrac{\bar{x}_M - \bar{x}_F}{\sqrt{\left(\dfrac{\sigma_{(n-1)M}^2}{n_M} + \dfrac{\sigma_{(n-1)F}^2}{n_F}\right)}}$

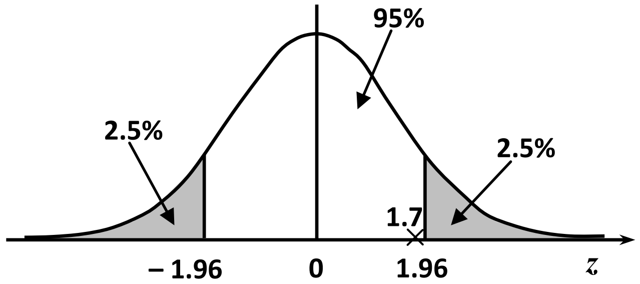
The table below gives the values calculated from the data on the spreadsheet:

Gender	**Sample mean,** $\bar{x}$	**Standard deviation,** σ_{n-1}	**Sample size,** n
Male	103.25 mm	7.9332 mm	52
Female	101 mm	5.1478 mm	49

Using these values, the test statistic is:

$$z = \frac{103.25 - 101}{\sqrt{\left(\dfrac{7.9332^2}{52} + \dfrac{5.1478^2}{49}\right)}} = 1.70$$

For a two-tail test at the 5% level, the critical values of z are ± 1.96, so this value of z is not significant at the 5% level.



There is no significant difference between the hand lengths of 2-year-old boys and girls.

Think about…

Explain the reasoning behind this conclusion.

Try these

1 Chest circumference

a A manufacturer assumes that the mean chest circumference of 6-year-old boys and the mean chest circumference of 6-year-old girls are both 580 mm.

Use the data in the spreadsheet to carry out hypothesis tests to decide whether the true means are greater than 580 mm.

b Repeat part **a** to test whether the mean chest circumference of 8-year-old boys and the mean chest circumference of 8-year-old girls are both 640 mm or greater than this.

2 Head circumference

a Use data from the spreadsheet to test whether there is any difference between the mean head circumferences of 2-year-old boys and 2-year-old girls.

b Repeat part **a** for one other age group chosen from:
4-year-olds, 8-year-olds, 10-year-olds, 16-year-olds

3 Difference between means

a Choose a body measurement and age group where you think there *will be* a significant difference between boys and girls. Use the data on the spreadsheet to test the difference between means.

b Choose a body measurement and age group where you think there will *not* be a significant difference between boys and girls. Use the data on the spreadsheet to test the difference between means.

Reflect on your work

- What are the mean and standard deviation of the distribution of a sample mean?
- Describe the steps in a significance test for a population mean.
- Describe the steps in a significance test for the difference between means.
- When should you use a one-tail test and when a two-tail test?
- Would you be more confident in a significant result from a 5% significance test or a 1% significance test? Explain why.