



In this activity you will use the **Chinese postman algorithm**, (also called the Route Inspection Problem), to solve practical problems. It is called the Chinese postman algorithm because it was studied in 1962 by a Chinese mathematician called **Kwan Mei-Ko**, who was interested in minimising the total distance walked by a postman delivering mail. It is based on **Euler's** findings for **traversable** graphs.

Information

A graph is made up of a collection of vertices called **nodes**, joined by edges called **arcs**. A **traversable** graph is one that can be drawn without taking a pen from the paper and without retracing the same edge.

Try this ...

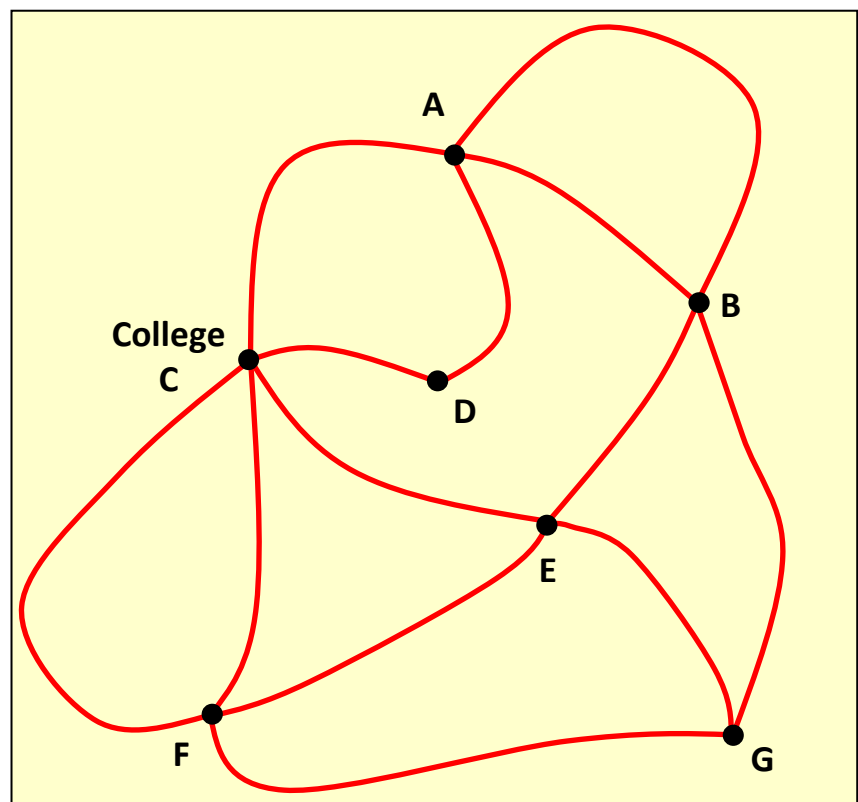
College Open Day

A college is planning an Open Day.

The college car park is small, and visitors are likely to park on streets near the college. The map shows those streets likely to be affected.

The Principal wants to deliver a leaflet to the houses on these streets to tell residents about the open day, and to apologise for any inconvenience caused.

Your task is to find the most efficient way for one person to deliver these leaflets.



Think about...

Do you think it is possible to deliver leaflets to the houses on these streets without travelling along the same street twice?

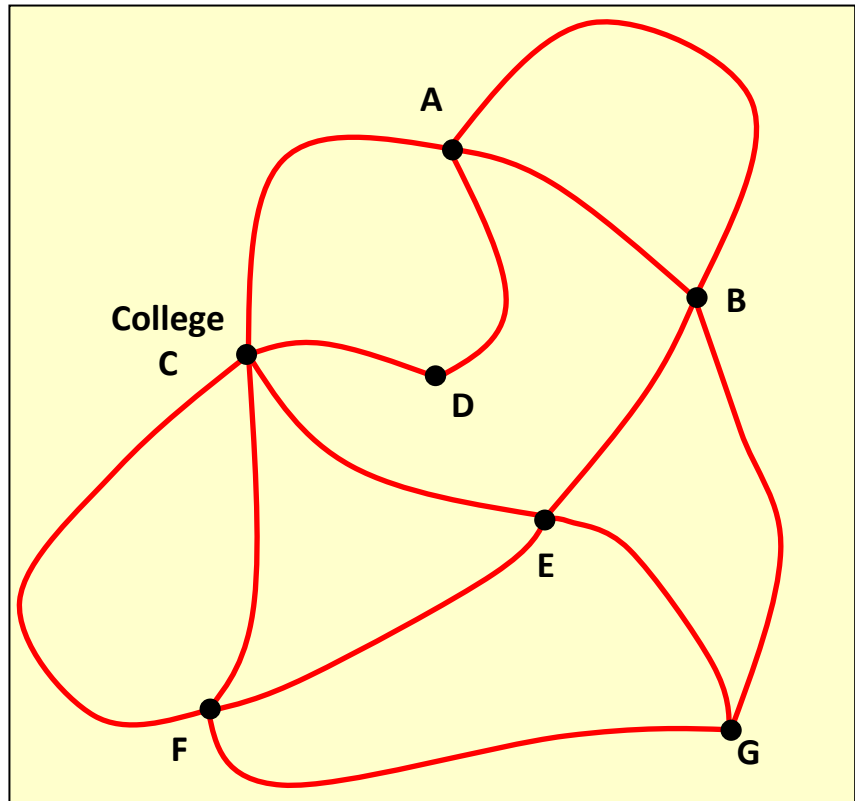
Try out some routes on the map. Start from the college.

The solution of this type of problem depends on whether the vertices in the network are **odd** or **even**. A vertex is odd or even depending on the number of edges that meet there. The number of edges that meet at the vertex is called the **order** or **degree** of the vertex.

To answer

1 The vertices in the network are listed below.
For each vertex, state the order and whether it is even or odd.

- A
- B
- C
- D
- E
- F.....
- G.....



Think about...

A graph is traversable if all its vertices are **even**, or it has just two **odd** vertices.

If a graph has all even vertices, it is possible to traverse the graph starting and finishing at the same vertex.

Can you explain why?

In such a case, the graph is said to have an **Eulerian trail**. An Eulerian trail is a path which starts and ends at the same vertex and includes every edge just once.

If a graph has two odd vertices, it is possible to traverse it by starting at one of the odd vertices and finishing at the other. In this case the graph is said to be **semi-Eulerian**.

The network of streets near the college has two odd vertices, C and G. It is possible to find a **semi-Eulerian trail** that starts at C, passes along each edge just once, and ends at G.

Think about...

Can you find a path that that starts at C, passes along each edge just once, and ends at G?

To deliver the leaflets to all the streets and return to the college, some streets will be travelled along more than once when the 'postman' returns from G to C.

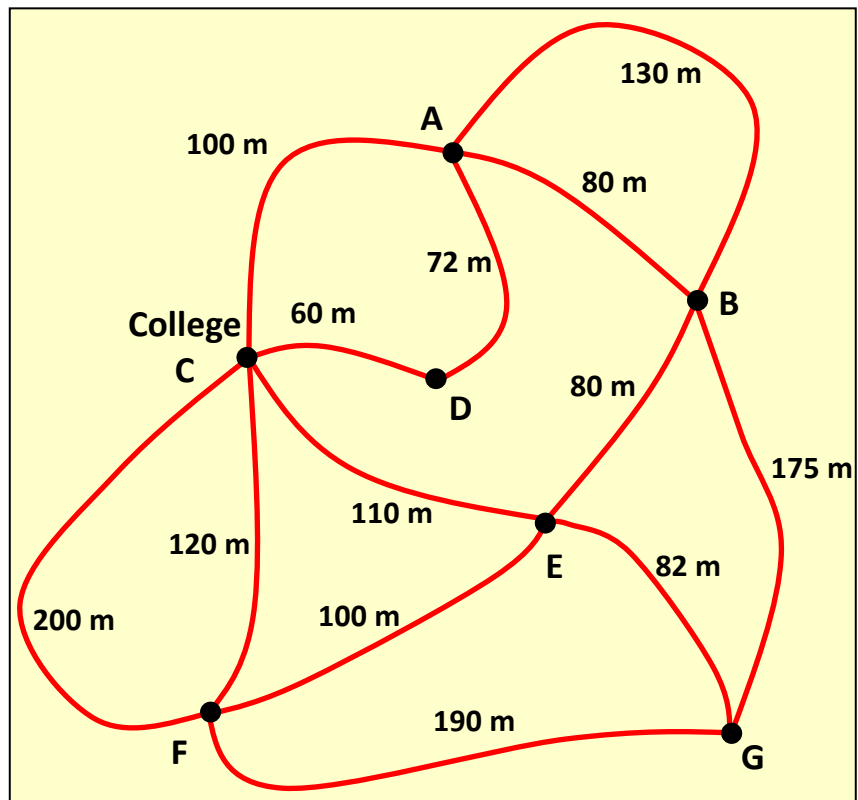
The most efficient route depends on the lengths of the streets.

Suppose the lengths of the streets are as given on this map (in metres).

To answer

2 Which is the shortest route from G back to C?

3 What is the shortest possible distance the 'postman' needs to travel to deliver the leaflets?



Solving the Chinese postman problem

Here is the general method for solving the Chinese postman problem.

- a** Identify all the odd vertices in the network.
- b** List all the possible ways to pair the odd vertices.
- c** For each pair of odd vertices, find the edges with the minimum weight that connect the vertices.
- d** Find the pairings for which the total weight is as small as possible.
- e** Add these edges onto the original network. You will now have a network with only even vertices.
- f** Add up the weights of all the edges in the original network and add to this the total weight of the edges you have added. This is the minimum weight of an Eulerian trail.
- g** Find a route for the new network – there are usually many possibilities.

Try this ... Easter Parade

An Easter Parade is to travel along each street in the network shown below.

The estimated time it will take to travel along each street is shown on the network.

1 For each vertex in the network, state the order and whether it is even or odd.

A.....

B.....

C.....

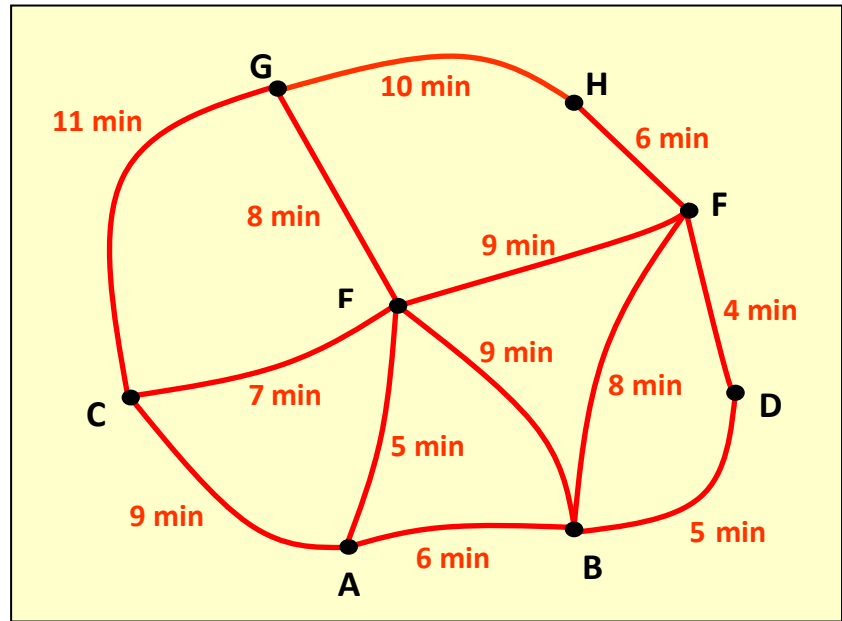
D.....

E.....

F.....

G.....

H.....



2 List the vertices that are odd

3 Pair the odd vertices to give the smallest total time

Add in these edges again on the diagram.

4 Write down a route for the Easter Parade that starts and ends at A and goes along each edge once.

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5 Calculate an estimate for the total time this will take.

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Think about...

What else might you consider in the real situation?

Reflect on your work

An Eulerian trail is a path which starts and ends at the same vertex and includes every edge just once.

Euler discovered that such a trail will exist if and only if **all vertices are even**. Can you explain why?

In any network, the sum of the orders of the vertices is even. Can you explain why?

In any network, the number of odd vertices must be even. Can you explain why?