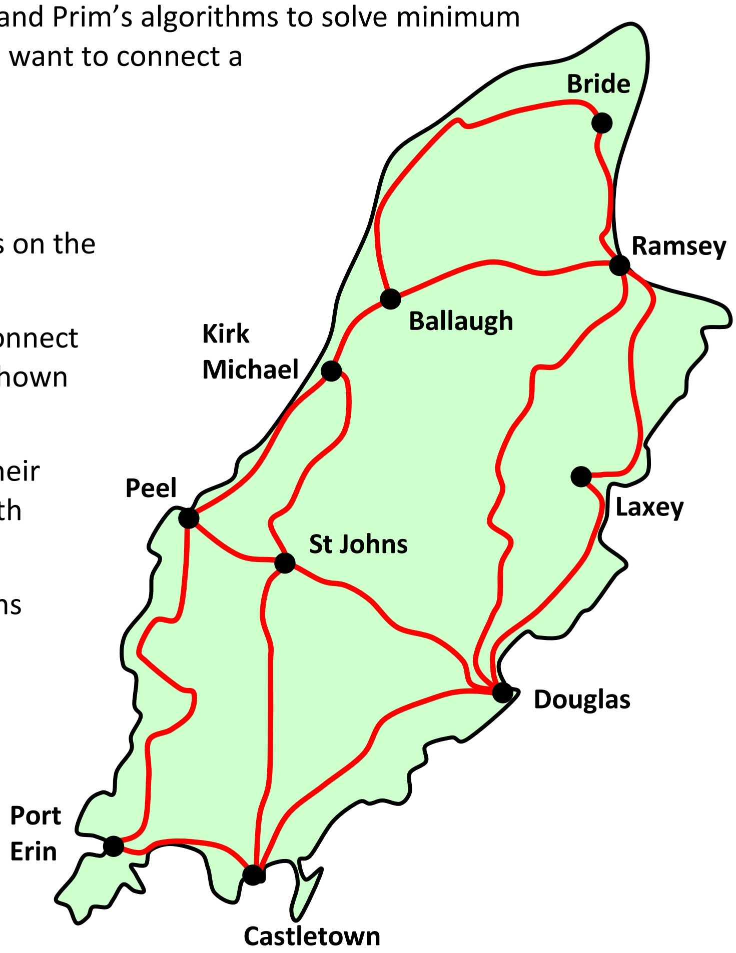
This activity shows you how to use Kruskal's and Prim's algorithms to solve minimum connector problems. This happens when you want to connect a network and minimise the cost.



Information sheet

The map shows some of the towns and roads on the Isle of Man.

A cable TV company wants to lay cables to connect the towns, laying the cable along the roads shown on the map.

They want to connect all of these towns to their cable network using the **minimum** total length of cable.

The length of the roads joining adjacent towns is given in miles in the chart below.

Note that a dash in the table (–) means there is no direct route between the towns.

Ballaugh									
9.3	**Bride**								
–	–	**Castletown**							
–	–	10.2	**Douglas**						
2.8	–	–	–	**Kirk Michael**					
–	–	–	7.7	–	**Laxey**				
–	–	–	–	6.8	–	**Peel**			
–	–	4.7	–	–	–	13.9	**Port Erin**		
6.5	4.6	–	15.4	–	9.3	–	–	**Ramsey**	
–	–	9.3	8.2	7.4	–	2.7	–	–	**St. Johns**

To solve the problem you need to find a **spanning tree** of **minimum** length.

A spanning tree is a tree that connects all the vertices together.

A **minimum spanning tree** is a spanning tree of minimum length

Think about…

What is the connection between the number of towns and the number of edges in the minimum spanning tree?

The type of problem involved in this activity is often called a **minimum connector problem**. **Kruskal's algorithm** is one method of solving such problems.

Kruskal's Algorithm

This is for finding a minimum connector (that is minimum spanning tree).

Step 1 List the edges in order of increasing weights.

Step 2 Start with the edge with the smallest weight.

Step 3 From the remaining edges, choose the one with the smallest weight which does not form a cycle. (If there are 2 shortest edges, choose either.)

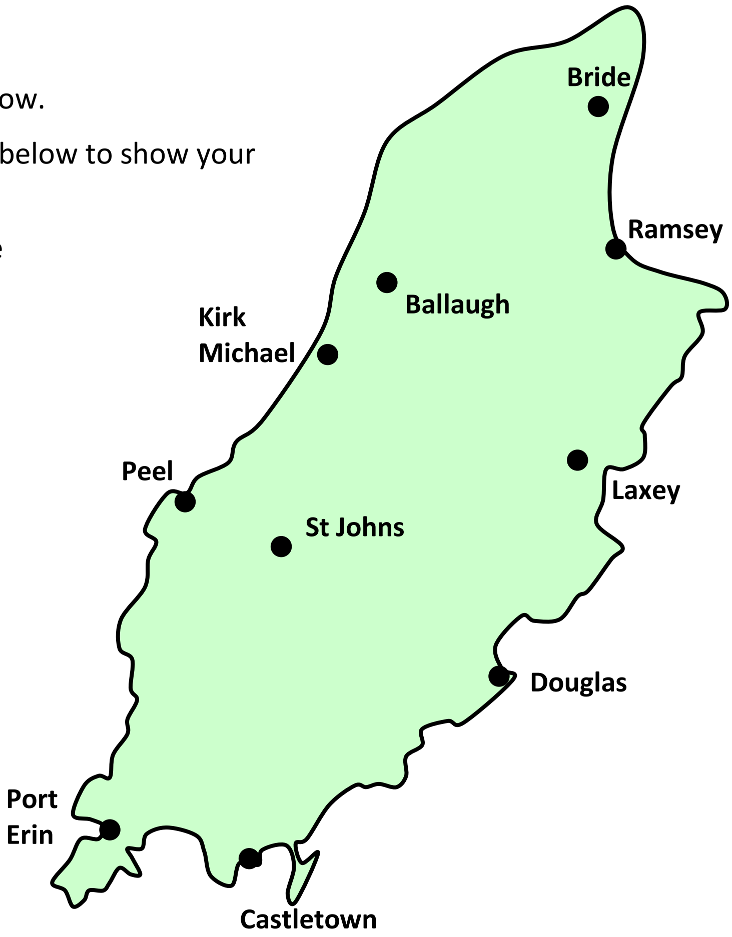
Step 4 Repeat Step 3 until all the vertices are connected.

Try this: Cable TV problem

Carry this out for the Cable TV problem.
First list the edges in order of length below.

Then follow steps 2 to 4, using the map below to show your solution.

Write down the order you introduce the edges into the minimum spanning tree.



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Prim's Algorithm

This gives an alternative method for solving minimum connector problems.

Prim's Algorithm for finding a minimum connector (i.e. minimum spanning tree):

Step 1 Starting from any vertex, join it to the nearest adjacent vertex.

Step 2 Join the next nearest vertex to those already included, provided that this does not form a cycle. (If there are 2 nearest vertices, choose either.)

Step 3 Repeat Step 2 until all the vertices are included.

Try this … using the adjacency matrix (table) below.

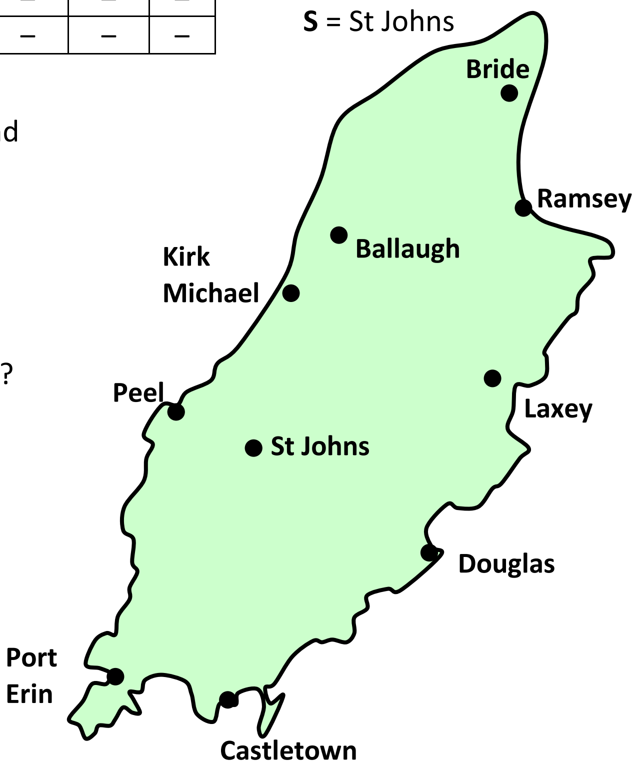
First choose a starting vertex, say B_1, and label it as 1 above the table.
Delete row B_1 and look for the smallest entry in column B_1. This is the 2.8 in row K.

This means B_1K is in the solution. Label K as 2 (in table) and draw B_1K (on map).

Now delete row K and look for the smallest entry in column B_1 or K.
This is 6.5 in column B_1 and row R. Label R as 3 and draw B_1R on the map.
Continue in this way until all the vertices are connected.

	B_1	**B_2**	**C**	**D**	**K**	**L**	**P_1**	**P_2**	**R**	**S**
B_1	–	9.3	–	–	2.8	–	–	–	6.5	–
B_2	9.3	–	–	–	–	–	–	–	4.6	–
C	–	–	–	10.2	–	–	–	4.7	–	9.3
D	–	–	10.2	–	–	7.7	–	–	15.4	8.2
K	2.8	–	–	–	–	–	6.8	–	–	7.4
L	–	–	–	7.7	–	–	–	–	9.3	–
P_1	–	–	–	–	6.8	–	–	13.9	–	2.7
P_2	–	–	4.7	–	–	–	13.9	–	–	–
R	6.5	4.6	–	15.4	–	9.3	–	–	–	–
S	–	–	9.3	8.2	7.4	–	2.7	–	–	–

B_1 = Ballaugh
B_2 = Bride
C = Castletown
D = Douglas
K = Kirk Michael
L = Laxey
P_1 = Peel
P_2 = Port Erin
R = Ramsey
S = St Johns



You should find the solution is identical to that found using Kruskal's algorithm.

Think about…

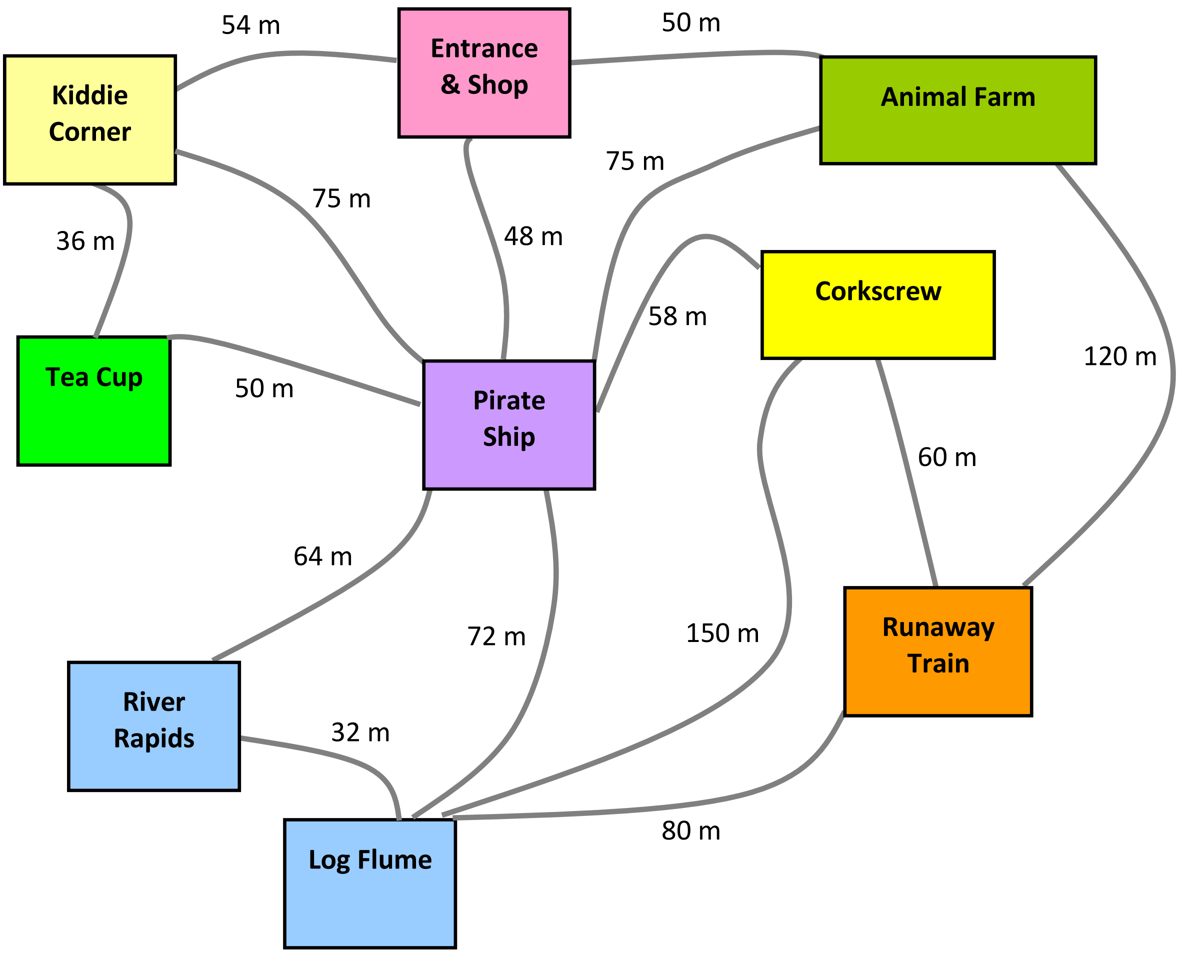
Can you explain how the matrix method works?

Which do you prefer – Kruskal's or Prim's algorithm? Why?

What else would need to be considered in the real situation?

Try this: Theme Park Problem

The plan below shows the lengths of paths linking parts of a theme park.



The manager of the theme park wants to widen and re-surface some of these paths to provide better access to the rides for wheelchairs. His aim is to provide better paths to connect all parts of the theme park, but using the minimum total length possible.

Use Kruskal's and Prim's algorithms to find the minimum spanning tree for this network.

Write a paragraph explaining the advantages and disadvantages that your solution may have in the real situation.

Reflect on your work

How many edges will there be in the minimum spanning tree of a network with n vertices?

What are the main advantages and disadvantages of Kruskal's and Prim's algorithms?

What other sorts of things might need to be considered in a real situation?