Using the *SOROBAN* to develop strategies for mental calculation

The purpose of this research project has been to explore whether learning about and using a *soroban* (Japanese abacus) can enhance students' learning experiences and understanding of number concepts. Particular attention has been paid to students' choice of strategy for mental calculation, to determine whether exposure to the operations of the *soroban* influenced their adopted methods and abilities for addition and subtraction involving up to 3 digits. The motivational aspects arising from this practical approach and interest in cultural issues have also been a feature of the project.

The project proceeded as outlined in the proposal with some minor modifications in light of experiences and feedback from participating staff. The *soroban* activities were restricted to addition and subtraction rather than attempting to pursue all four operations, acknowledging this as a more realistic target within the fairly short period of intervention. The study was extended to three intervention and three control classes since one of the participating staff was teaching two classes of differing abilities in Year 8 – this was felt to be a useful addition to the overall study and did not lead to any resource difficulty. A period of staff absence interrupted the sequence of *Soroban* lessons for one class but every effort was made to recover the situation by devoting additional time later in the study and also by utilising one of the researchers as a 'teacher'. Overall, the *soroban* lessons lasted approximately 12 hours over the 12-week period. The control classes in both schools were studying a block of work on *Fractions* during the period of intervention. They did no work on mental *addition* or *subtraction* of whole numbers, other then through some whole class oral starters that were a feature of delivery in both schools.

Research Questions

In what way does work on the *soroban* enhance student understanding of number concepts, including place value and mental calculation strategies?

Does 'novice' user status on the abacus subsequently enhance the speed and accuracy levels of students work in either 'mental' or 'pencil & paper' calculation exercises?

What motivational effects, with respect to students' engagement with the underlying concepts and attitude towards mathematics, does the initiative provide?

How can teachers best develop number concepts through using the *soroban*?
What support and guidance for teachers is appropriate?

What arrangements are best for learners, in terms of group composition, length and frequency of sessions, support materials etc.?

Findings

Speed and Accuracy

All pupils were tested on speed and accuracy of mental calculation (all four operations) before and after the intervention. The control group performed slightly better than the intervention group in the initial tests. Both the intervention and control groups increased, on average, the number of mental calculations attempted in 6 minutes when tested at the end of the study. While the number of correct answers achieved increased slightly, the rate of accuracy fell.

Intervention group:

Mean number of questions attempted increased by 9.3% from 32.4 to 35.4;

Mean number of questions correct increased by 5.8% from 24.0 to 25.4 (but percentage of answers correct fell from 74.1% to 71.8%)

Control group:

Mean number of questions attempted increased by 12.5% from 34.5 to 38.8;

Mean number of questions correct increased by 6.3% from 26.8 to 28.5 (but percentage of answers correct fell from 77.7% to 73.5%

The tests included six *addition* and six *subtraction* questions at the start of each section. Following completion of the timed element of the test, pupils were asked to jot down some indication of the strategy they had used for each of these twelve items.

<u>Addition Questions</u>

Performance in addition of 2 and 3 digit numbers was generally good. Over 80% of pupils answered at least 5 out of 6 questions correctly. Little change was evident over the period of the intervention. Very little difference between the performance of the groups was evident

Intervention group:

Before intervention:	mean addition score was 5.30
After intervention:	mean addition score fell slightly to 5.23 out of six.

Control group:

Before intervention:	mean addition score was 5.25
After intervention:	mean addition score virtually unchanged at 5.26 out of six.

<u>Subtraction Questions</u>

Many pupils performed poorly in subtraction involving up to 3 digits. Over 50% of pupils answered three or more of the six questions incorrectly. One quarter of all pupils scored 0 or 1 out of 6. It was evident that the pupils struggled with subtraction as soon as any 'exchange'[1] was required.

The difference between the performance of the intervention and control pupils in the subtraction calculations before intervention was noted and taken into account in subsequent analysis.

Intervention group:

Before intervention:	mean subtraction score was 2.89 (s.d.1.80) 31.8% of pupils scored 0 or 1 out of six.
After intervention:	mean subtraction score increased by 9.0% from 2.89 to 3.15 out of six.

Control group:

Before intervention: mean subtraction score was 3.49 (s.d.1.69)
20.8% of pupils scoring 0 or 1 out of six.

After intervention: mean subtraction score fell by 17.2% from 3.49 to 2.89 out of six.

In order to explore the data further, a between-subjects analysis of variance was conducted to investigate this observed difference, taking account of variable performances across the groups in the initial tests. Although the interaction between success in the initial subtraction test (test 2[2]) and the group (Intervention or Control) is not statistically significant (p=0.08) in the linear model explored, there are confounding issues that should be considered. The discrete nature of the data, reporting the number of successes out of a possible six, imposes limitations when it comes to any model being developed. The context suggests that any model to be used needs to take into account the initial score from Test 2. This is because the potential change in any individual's score will be dependent on the initial score e.g. an initial score of 0,1 or 2 offers greater scope for change than an initial score of say 6, which could only stay the same or fall. With that in mind, the model that takes account of the covariate Test 2 has been used to give the following predictions that are graphically illustrated in Figure 1.

Control status: $Test4 = 2.022 + 0.730 * Test2 - 1.683$

Intervention status: $Test4 = 2.022 + 0.390 * Test2$

It is certainly not a black and white situation but there is evidence to suggest the members of the Intervention group that started from a weak position are able to improve more noticeably than their counterparts in the Control group. It is worth noting that apart from the first test item the subtractions all involved a potential 'exchange', so any success beyond a score of one, indicates an ability to cope with a non-trivial mental subtraction. The model suggests that the biggest differences between the two groups appear in the lower abilities, where initial level of success was 3 or fewer.

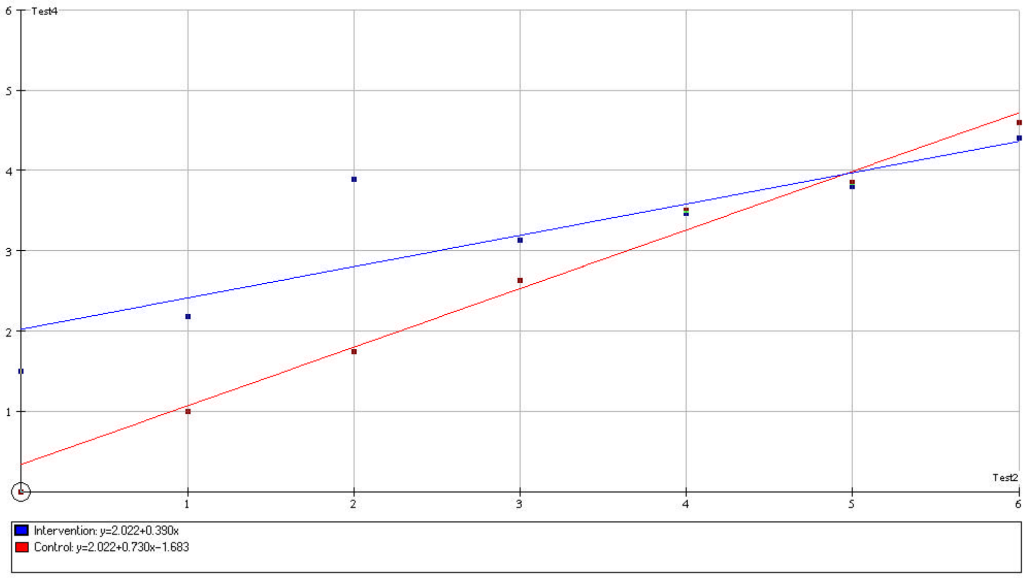


Figure 1 Control group and Intervention group

These results now need to be interpreted in the knowledge of the intervention activities in an attempt to explain why there is such a difference.

A closer analysis of the subgroups of pupils who answered subtraction questions incorrectly at the start of the study was carried out. For example, in Figure 2, of the 16 pupils in the control group who were initially wrong on subtraction ‘64-25’ (subtraction item 2.3 before), only 18.8% went on to give the correct answer to the corresponding item ‘74-35’ (subtraction item 2.3 after) post intervention. This group can be compared with the 28 pupils in the intervention group who initially failed to answer ‘64-25’ correctly. 42.9% of those pupils correctly answered ‘74-35’ after taking part in *soroban* lessons.

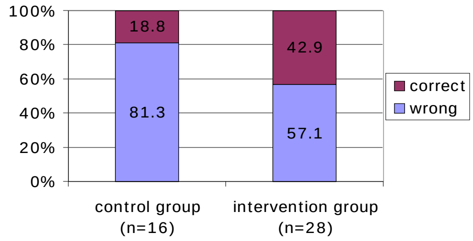


Figure 2 Performance in subtraction item 2.3(after) by those who answered subtraction item 2.3 (before) incorrectly

Corresponding results for the other subtraction questions are given in Table 1 which indicates that the *soroban* experience was of benefit to those who failed to answer subtraction questions 2.1 to 2.5 correctly in the initial test.

Table 1 Analysis of subtraction performance after intervention, for pupils who gave <u>wrong</u> answer initially.

I =Intervention group **C**= Control group

	2.1		2.2		2.3		2.4		2.5		2.6	
	I	C	I	C	I	C	I	C	I	C	I	C
wrong	3 19%	4 50%	26 59%	21 88%	16 57%	13 81%	22 58%	15 62%	23 68%	18 78%	36 80%	28 74%
correct	13 **81%**	4 **50%**	18 **41%**	3 **12%**	12 **43%**	3 **19%**	16 **42%**	9 **38%**	11 **32%**	5 **22%**	9 **20%**	10 **26%**

There were other pupils who answered subtraction questions correctly at the start of the project went on to answer incorrectly at the end. Details are given in Table 2.

Table 2 Analysis of subtraction performance after intervention, for pupils who gave <u>correct</u> answer initially

I =Intervention group **C**= Control group

	2.1		2.2		2.3		2.4		2.5		2.6	
	I	C	I	C	I	C	I	C	I	C	I	C
correct	45 90%	35 78%	15 68%	20 69%	30 79%	23 62%	16 57%	15 52%	19 59%	16 53%	9 45%	10 67%
wrong	5 **10%**	10 **22%**	7 **32%**	9 **31%**	8 **21%**	14 **38%**	12 **43%**	14 **48%**	13 **41%**	14 **47%**	11 **55%**	5 **33%**

Those taking part in the *soroban* lessons had a better rate of continuing success in subtraction in four of the six types of question assessed. The fact that a significant proportion of pupils who had correctly carried out a mental subtraction in the initial assessment could not do the same thing three months later is striking.

The data presented above highlights the fact that many pupils struggle with subtraction, particularly when an exchange is involved. The results also indicate a difference in achievement across the two groups, with those pupils following the *soroban* activities demonstrating a greater measure of success on mental subtraction of 2-digit numbers; at least their performance was not as poor as that of the control group. Some of this difference may well be attributable to the *soroban* activities.

Strategies for Mental Calculation

A classification of strategies used by pupils was developed, based on the work of Thomson & Smith (1999) and Klein & Beishuizen's (1998). A category of 'enhanced flexibility' in number work was introduced to capture sophisticated approaches including compensation strategies. For this study it was felt that a classification as represented in Table 3 could be used to distinguish between pupils' approaches to the addition and subtraction tasks.

Table 3 Classification of strategies

Strategy	Description	Example: 37+64
Counting on	Counting on (Ones and Tens)	
Digit manipulation	Manipulating digits (including standard algorithmic approach)	7+4=11, 3+6 +1=10 =101
Splitting	10 10 Mainly Left to Right computation but can be Right to Left if explicit reference to values is evident	30+60=90, 7+4=11 90+11=101
Mixed	Mixed method involving 10 10 and then sequential adding rather than a combination of the Ones being added to the sub-total	30+60=90, 90+7=97 97+4=101
Sequencing	N10 Sequencing with a strict use of Tens and Ones or multiples – nothing fancy!	37+60=97, 97+4=101
Flexible	Flexible thinking being demonstrated through: inventive use of number enhanced number sense compensation method Basically anything more sophisticated than other categories, indicating a full appreciation of the process and use of number	See Figure 3 below

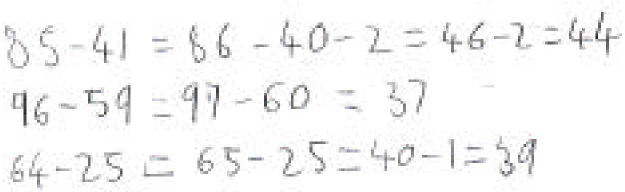


Figure 3 Pupils work illustrating variety of flexible strategies

Differences in the distributions of strategies used by the two groups were evident at the start of the project.

For addition, while ‘splitting’ strategies dominated in both groups, the control group included many instances of a 'flexible' strategy being used and few pupils resorted to 'digit manipulation'. After the period of the intervention, the control group made less use of ‘splitting’ in addition calculations, and there were more examples of the use of 'flexible' strategies.

In contrast many pupils in the intervention group initially used 'digit manipulation' and instances of any 'flexible' strategy were few. After the *soroban* lessons, the intervention group increased their use of the ‘splitting’ strategy and there was less ‘digit manipulation’.

Strategies used for subtraction also varied between the two groups. The control group used a mixture of strategies. No consistent pattern of change in subtraction strategies over all six questions was evident after the period of the intervention.

‘Digit manipulation’ and ‘splitting’ were the dominant strategies in the intervention group across all six types of subtraction at the beginning of the project. After the *soroban* lessons there was a move away from ‘digit manipulation’ towards ‘splitting’ strategies for all six types of subtraction. This suggests that the *soroban* lessons may have improved pupils’ ability to work with the holistic value of numbers, entering the values from 'left to right' rather then considering the 'units, tens and hundreds' from 'right to left'. Such a change is likely to be beneficial in terms of promoting number sense. A small increase in use of ‘flexible’ strategies was also found for almost all questions.

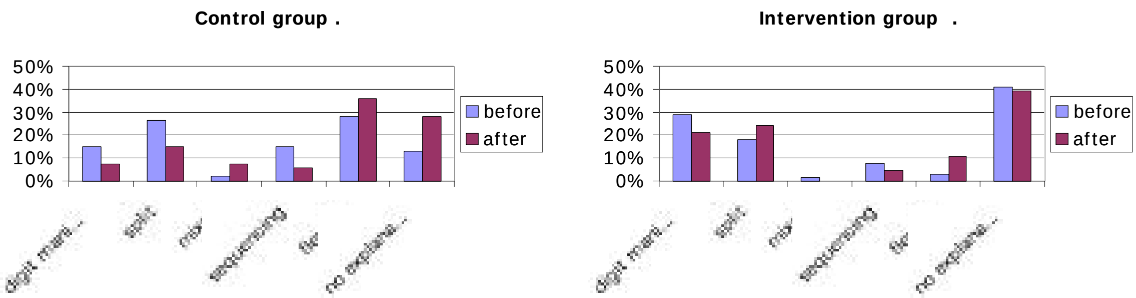


Figure 4 Strategies used for subtraction item 2

Many pupils gave poor or no explanations of their mental calculations. The control group were initially much better than the intervention group at explaining their strategies but at the end of the project the groups were broadly comparable in providing adequate explanations.

A connection between the selected *strategy* and the *success rate* for mental calculation was evident. This depended very much on the nature of the question posed. For a question such as 85-41, 'splitting' seems an appropriate strategy and was rewarded by a considerable measure of success.

In contrast, if a splitting strategy is used to calculate a subtraction that requires exchange success is less likely as illustrated in Table 4.

83-27	Intervention group			Control group		
	wrong	correct	total	wrong	correct	missing
Digit manipulation	10	6	16	2	5	7
Split	**8**	**2**	**10**	**10**	**4**	**14**
Mixed	0	1	1	1	0	1
Sequencing	2	3	5	2	6	8
Flexible	0	1	1	2	5	7
No explanation	18	15	33	7	9	16
Totals	38	28	66	24	29	53

Table 4 Success with strategies chosen to calculate ‘83-27’ before intervention

It is noted that the intervention group used the splitting strategy more successfully than the control group after using the *soroban*, as illustrated in

Table 5

62-38	Intervention group			Control group		
	wrong	correct	total	wrong	correct	missing
Digit manipulation	8	6	14	2	1	3
Split	**6**	**5**	**11**	**9**	**2**	**11**
Mixed	0	0	0	2	2	4
Sequencing	0	4	4	1	0	1
Flexible	1	3	4	4	10	14
No explanation	19	14	33	11	9	20
Totals	34	32	66	29	24	53

Table 5 Success with strategies chosen to calculate ‘62-35’ after intervention

It is clear that pupils who make use of ‘sequencing’ or ‘flexible’ strategies are more successful in subtraction of this type.

<u>Motivational Effects</u>

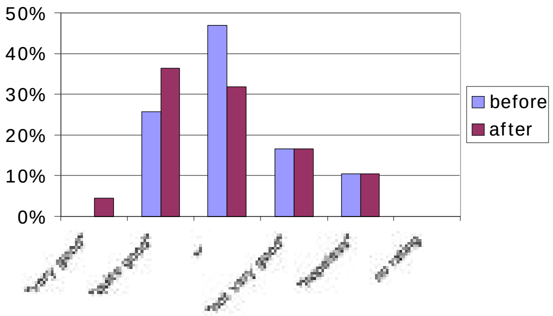
Both groups of pupils were asked ‘How much do you like mental calculation?’ at the start and end of the study. Replies at the end of the programme were less positive than those received at the outset.

The percentage of the intervention group putting forward a negative view rose from 30% to 38%, with an increase of 7 pupils in the number choosing ‘I hate it’ (23%)

The percentage of pupils in the control group expressing negative attitudes to mental calculations was initially larger and rose from 40% to 51%, though the numbers expressing the most extreme view were smaller than in the intervention group (17%).

Pupils were also asked to rate themselves on mental calculation.

Figure 5 Mental calculation rating – intervention group



Many members of the intervention group were clearly more confident in their own abilities after the *soroban* lessons. The percentage of those rating themselves as 'quite good' or 'very good' at mental calculation rose from 26% to 41%. (The proportion rating themselves as 'not very good' or 'hopeless' remained unchanged.)

There was little change in the responses given by the control group (apart from a small move from 'not very good' to 'hopeless'). Ratings were broadly comparable to those of the intervention group at the start of the study.

Pupils who took part in the *soroban* lessons were also asked to respond to statements relating to their experiences. The results are summarised in the table below. Original responses from a five-point likert scale have been conflated to the following categories:

	Strongly agree / Agree	Don't know	Disagree/ Strongly disagree
'I prefer lessons without the *soroban*'	38%	30%	32%
'I can picture the *soroban* beads to work out calculations in my head'	28%	20%	52%
'Using a *soroban* made me think about numbers in a different way'	41%	24%	35%
'A *soroban* is difficult to operate'	29%	13%	58%
'I might do some mental calculations differently now that I have learned to use a *soroban*'	32%	35%	32%
'All pupils would benefit from learning to use a *soroban*'	30%	41%	30%
'I can do calculations in my head faster now because of the *soroban* lessons'	21%	31%	48%
'I enjoyed learning about Japanese use of number'	60%	13%	27%

Table 6 Responses from pupils' questionnaire

Interviews with staff and pupils provided an opportunity to explore some of the respondents' thinking in more detail. A common response in support of the *soroban* activities related to the cultural dimension and relationship to Japanese lifestyle and use of number. The pupils were very enthusiastic and quick to recall some of the basic 'background' information that was shared with them - including the Japanese numbers, use of the *soroban* in everyday situations and some of the basic Japanese words associated with learning environments:

> "It was really good at the start, learning about the Japanese numbers and what it meant in Japanese"

> "There was this man … it was in the train station, the lady came in to buy her ticket and they had to work it all out and it took him about 4 seconds just to work it out and the change and add it all up"

> "I learned that they call, like their teachers 'Sensi' in the classes"

This enthusiasm rolled over into the operational aspects as well, with the pupils being reasonably well motivated and generally enjoying the *soroban* lessons. It was not all straightforward however with difficulties being experienced when it came to dealing with exchanges, particularly within the subtraction operations:

> "The bits where you've to add on 8 and you've not got 8, that's difficult. That is difficult"

> "64 add 2… 5, but there's nothing else to do. This is a hard one, I cannae dae it"

> "When there's not enough beads to take away you have to work out another way of doing it"

This highlights the misconception that using an abacus is a trivial, low-level activity when in fact it makes greater demands of the operator in terms of number sense and appreciation of operations on number:

> "It's not like a calculator, nothing like a calculator. You have to use your brain. I don't know; you just sort of have to think in a different way from the calculator. You've got to, with a calculator, you just punch in the numbers; with a *soroban* you've got to think where the numbers is and that. It's harder."

The majority of respondents (58%) did not feel the *soroban* was difficult to operate.

The questionnaire responses (Table 6) indicate that 28% of pupils agreed they could use the beads to work out calculations in their head. It is likely that the 52% who disagreed with this statement were referring to the *calculation* element. In individual and focus group interviews, the majority of pupils claimed to be able to visualise the *soroban* and to have alternative representations for number in terms of beads on the *soroban*.

> "You'd pinch the 7, move up with your thumb the 3 and pinch with the 8"

> "What are you picturing in your head?
> - The *soroban* with the numbers and you just picture. But I'm not very good at taking them away, just adding"

Going further than this, to calculate an addition or subtraction, was on the whole beyond their comprehension after such a short period of practice:

When asked about the effects of *soroban* experience on subsequent speed of mental calculation some respondents claimed that they were thinking more about the operations involved at the expense of completing more of the test items:

> "Just after we done the *soroban* it was clear in your head and you were still thinking about the way the beads were moving and it was taking you ages to work it out"

Conclusions

The research project has been generally successful. While the lack of any dramatic improvement in pupils' mental calculation skills was disappointing, it is clear that learning to use a *soroban* has benefits in terms of improving and broadening attitudes and can help some learners to visualise number and to subtract with increased success. The experience has also improved many pupils' ability to work with the holistic value of numbers, dealing with values from 'left to right' rather than from 'right to left' (starting with the units) and therefore promoting number sense.

The study will inform decisions about the timing, structure and value of future *soroban* lessons. The research has also raised awareness of the range of mental calculation strategies necessary for improved teaching and learning of mental skills.

A number of potential strands have emerged from the study that are worthy of further exploration, namely:

- the benefits of alternative classroom arrangements and timing of *soroban* activities, structured in the light of comments from participants (teachers and pupils). e.g. a concentrated period of study followed by frequent short sessions; use of *soroban* lessons as enrichment activities for gifted and talented pupils

- making sense of students' mental calculation strategies following an intensive programme of 'numeracy' lessons. e.g. are strategies that have been explicitly taught used as intended?

- comparative study of mental calculation strategies with Japanese students, including *soroban* 'experts'

<u>References</u>

Klein & Beishuizen, 1998, The Empty Number Line in Dutch Second Grades: Realistic versus Gradual program Design, *Journal for Research in Mathematics Education*, Vol. 29.4, 443-464

Thomson, I. & Smith, F., 1999, *Mental Calculation Strategies for the Addition and Subtraction of 2-digit Numbers* (Report for the Nuffield Foundation), Newcastle upon Tyne, Department of Education, University of Newcastle upon Tyne.

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[1] Exchange may be carried out when calculating 83-27. Since it is impossible to subtract 7 from 3, one of the tens in 83 can be exchanged for ten units (83=70+13). This gives (70+13) minus (20+7) leaving (50+6) or 56.

[2] Test 2 refers to score on six subtraction items before the intervention; Test 4 refers to score on six corresponding subtraction items after the intervention