Students interpret a speed–time graph for a car during a test run, and fit linear and quadratic models to the graph.

A spreadsheet contains the actual data from a test run; students add their quadratic models to the spreadsheet and to a graph for comparison.

Suitability and Time

Level 3 (Advanced); 1 hour, or longer if used in a more open-ended way

Resources

Information sheet, worksheet, spreadsheet

Equipment

Calculator, spreadsheet facility

Key mathematical language

graph, gradient, maximum, model, estimate, fit, linear, quadratic, acceleration, deceleration

Notes on the activity

The teacher's spreadsheet can be used to introduce the activity and to aid an initial class discussion about the data and graph. The graph is also provided on the students' information sheet.

After reading values form the graph, students are asked to model three sections of the graph using linear functions. If you decide to demonstrate this, it would be better to use the second or third time intervals, rather than the very simple first case.

The teacher's spreadsheet includes the actual data and linear models, with the gear changes represented by constant velocity – you may decide to discuss this with students before they attempt to find quadratic models in the last part of the activity.

In question 5 it is intended that students find their own quadratic model from the graph, by using the coordinates of three points, or by using other features of the graph such as the maximum point. In questions 6 and 7 they can then use the students' spreadsheet to check how well their model fits the graph.

As there is a lot of data on the spreadsheet, students should be warned not to print it all. Graphs of two possible quadratic models are given on the teacher's spreadsheet. These graphs, or some of the students' own graphs, could be used to motivate a discussion on the suitability of the different models and how the models could be refined.

During the activity

Students could work individually or in pairs. It may be necessary to emphasize that questions 1 to 5 on the worksheet should be answered using information from the graph, rather than the data in the spreadsheet. Alternatively, you may decide that some students could use the spreadsheet, and these models could be compared with models produced from the graph.

If graphic calculators are used to produce the models, these could also be compared with those produced by hand and by the spreadsheet.

Points for discussion

Whether the models fit the data well, and over what range of data.

Variation in models.

The answers below are given to 3sf because this is usually what awarding bodies require, but it is worth discussing reasonable levels of precision – in practice, values would be rounded so that the models have no greater degree of precision than the data used to find them

Use of the maximum in fitting a quadratic can be connected with completing the square.

Extensions

How can the models be refined?

Combining several functions might produce a model that fits the data better than a single function.

Is there a trade-off between the simplicity of use of a single function model and how well it fits the data?

Answers

Information from the graph

1a 67.5 m.p.h. **b** 10.7 seconds

2 4.8 – 5.2 seconds, 7.9 – 8.6 seconds, 12.4 – 13.0 seconds (all approx)

Modelling the data with linear functions

In the answers below, the co-ordinates of the end points of each section have been estimated from the graph as (0, 0), (4.6, 35), (5.2, 35), (7.8, 52), (8.6, 52) and (12.2, 65) and used to find the linear functions. Values from the spreadsheet would give slightly different results.

3a $0 \le t \le 4.6$ $v = 7.61t$

b $5.2 \le t \le 7.8$ $v = 6.54t + 1.0$

c $8.6 \le t \le 12.2 \quad v = 3.61t + 20.9$

4a When t = 2.3, v = 17.5 Value from graph is approx 16.5 mph

b When t = 6.5, v = 43.5 Value from graph is approx 44 mph

c When t = 10.4, v = 58.4 Value from graph is approx 59 mph

Values predicted by the linear models are all reasonably near to the values shown on the graph.

Modelling the data with a quadratic function

5 Substituting co-ordinates from three points on the curve into the general quadratic form $v = at^2 + bt + c$ gives three equations from which values for a, b and c can be found.

For example, using points (2.3, 16.5), (6.5, 44) and (10.4, 59) from the graph gives the quadratic function $v = 9.48t - 0.334t^2 - 3.55$, whilst the points (0, 0), (7.5, 50) and (15, 67.5) give the quadratic function $v = 8.83t - 0.289t^2$.

For the majority of times in the interval $0 \le t \le 15$, the function $v = 8.83t - 0.289t^2$ gives higher values for the speed than the actual values, whilst the function $v = 9.48t - 0.334t^2 - 3.55$ is a better model of the actual data up to t = 11, but poor thereafter.

If students use the spreadsheet or graphic calculators to find models, they will get different quadratic functions that are likely to fit the data better than those given above.