



Coastal erosion is the process by which cliffs and rocks along the coast are broken up by the action of waves, wind and landslides. The Holderness coastline of Yorkshire suffers from some of the worst coastal erosion in Europe. During the last 2000 years the coastline has retreated by almost 400 metres and over 30 villages between Bridlington and Spurn Head have been lost to the sea.

Geologists study the coast to predict how it will change over time. While they use increasingly sophisticated methods to address a range of questions, we will see how, given a set of measurements along the coastline, one can approximate land area and the amount of area lost over a period of time.

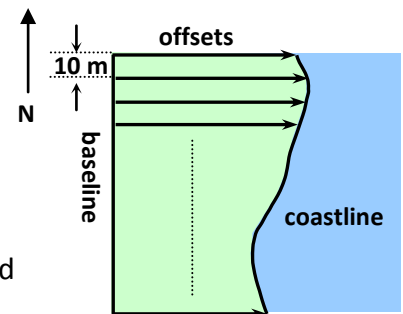


Information sheet

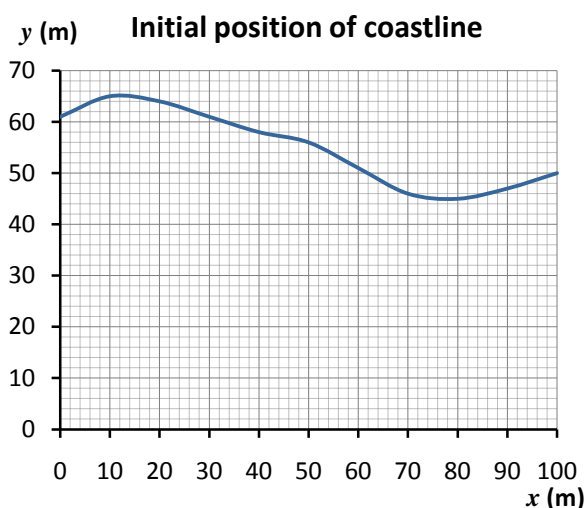
A Estimating land area

The area of a piece of land with an irregular coastline can be estimated using a variety of methods.

Suppose we wish to estimate the area of the piece of the land shown in the sketch. Then we want to estimate how much of this land will be lost by coastal erosion over a 20 year period.



The initial position of the coastline can be defined by measuring a series of perpendicular distances (offsets) from a baseline as shown. In this case the baseline has been taken to be a line drawn from North to South and the measurements have been taken at intervals of 10 metres. The baseline can then be taken as the x axis as shown below. The table gives the offsets as y co-ordinates taken at 10 metre intervals of x .



x (metres)	y (metres)
0	61
10	65
20	64
30	61
40	58
50	56
60	51
70	46
80	45
90	47
100	50

Think about

How could you estimate the area of the coastal land shown above?
 Would it be possible to use integration? What function might give the best fit – quadratic, cubic, sine wave?

B Using Integration

The coastline can be modelled by a cubic function.

A cubic function has been superimposed on the original graph. It was found using a trendline in Excel.

If you have a graphic calculator, use it to find a cubic model. It should be similar to this, but may not be exactly the same.

The area of land between the base line and the coastline is approximately equal to the integral of the cubic function between 0 and 100.

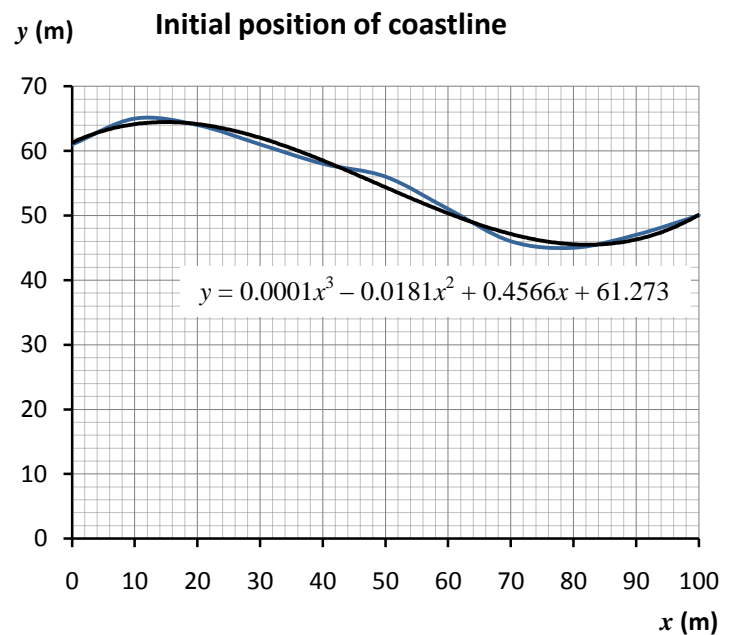
Area

$$\approx \int_0^{100} (0.0001x^3 - 0.0181x^2 + 0.4566x + 61.273) dx$$

$$= [0.000025x^4 - 0.00603x^3 + 0.2283x^2 + 61.273x]_0^{100}$$

$$= 2500 - 6033 + 2283 + 6127 = 4877 \text{ m}^2$$

The initial area of the land is approximately 4900 m².



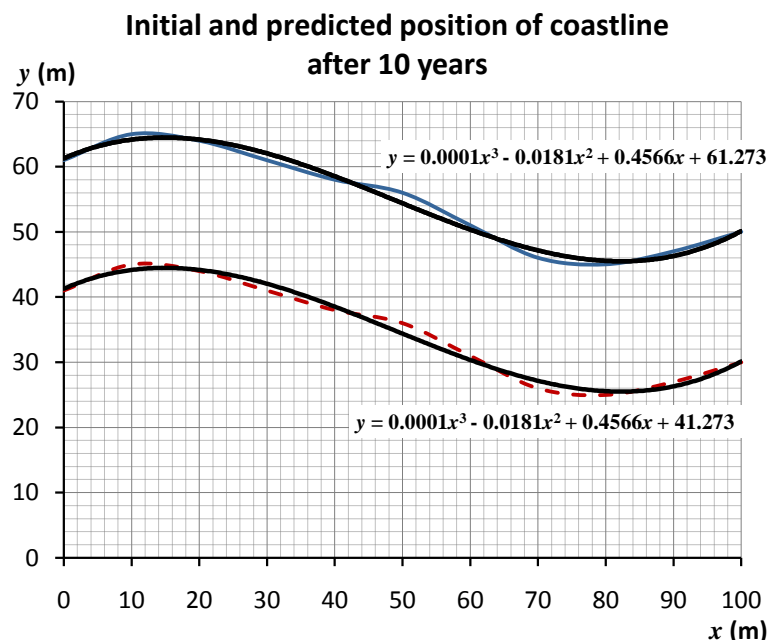
If you have found your own cubic model, use integration to find an estimate of the area. Compare your result with the estimate given here.

C Prediction of land loss

Assume that the coastline erodes at a rate of 1 metre per year – in 20 years it will recede by 20 metres. The graph shows both the initial position of the coastline and its predicted position after 20 years. The cubic function that models the new position of the coastline is almost identical to that which models its initial position.

Think about

What is the only difference between the two functions? Why is this?



Remaining area $\approx \int_0^{100} (0.0001x^3 - 0.0181x^2 + 0.4566x + 41.273) dx$

$$= [0.000025x^4 - 0.00603x^3 + 0.2283x^2 + 41.273x]_0^{100}$$

$$= 2500 - 6033 + 2283 + 4127 = 2877 \text{ m}^2, \text{ approximately } 2900 \text{ m}^2$$

The loss of land is approximately 4877 – 2877 i.e **2000 m²**

Think about

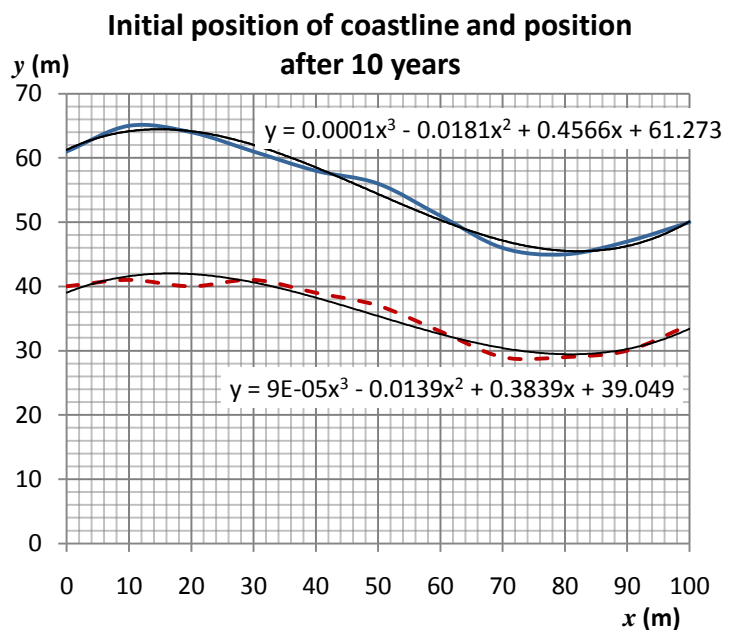
The loss of land is equal to the length of the baseline (100 metres) multiplied by the reduction in the lengths of the offsets (20 metres).

Can you explain this using the graph?

Can you show that this is true by combining the two tasks into a single integration?

In practice the coastline will recede more at some points than others. Suppose the actual offsets from the baseline after 20 years are as shown in the table below. The initial position of the coastline and its actual position after 20 years are shown on the graph, together with the cubic models given by Excel.

x (m)	y (m) initial	y_a (m) after 20 yrs
0	61	40
10	65	41
20	64	40
30	61	41
40	58	39
50	56	37
60	51	33
70	46	29
80	45	29
90	47	30
100	50	34



Using the cubic model given for the new position of the coastline:

$$\begin{aligned}
 \text{Remaining area} &\approx \int_0^{100} (0.00009x^3 - 0.0139x^2 + 0.3839x + 39.049) dx \\
 &= \left[0.0000225x^4 - 0.00463x^3 + 0.19195x^2 + 39.049x \right]_0^{100} \\
 &= 2250 - 4633 + 1920 + 3905 = 3442 \text{ m}^2, \text{ approximately } 3400 \text{ m}^2
 \end{aligned}$$

The loss of land is approximately $4877 - 3442 = 1435$ i.e **1400 m²** (to 2 sf)

If you have a graphic calculator, use it to find a cubic model for the position of the coastline after 20 years.

Integrate the function to estimate the remaining area and the loss of land. Compare your results with the estimates given above.

Think about

Simplify the integration of the two functions by combining them into a single function, containing only 4 terms.

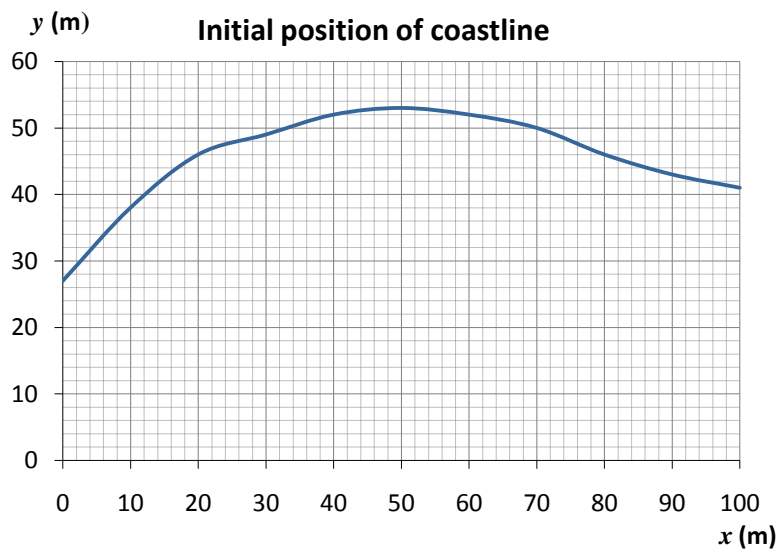
Integrate this function and evaluate it over the interval 0 to 100.

Do you get the same result as that above?

Try these

In each case, use either Excel or a graphic calculator to find a quadratic or cubic function that best fits the data.

1 The coordinates give the position of a coastline, with offsets from the x axis, at intervals of 10 metres.



x (m)	y (m)
0	27
10	38
20	46
30	49
40	52
50	53
60	52
70	50
80	46
90	43
100	41

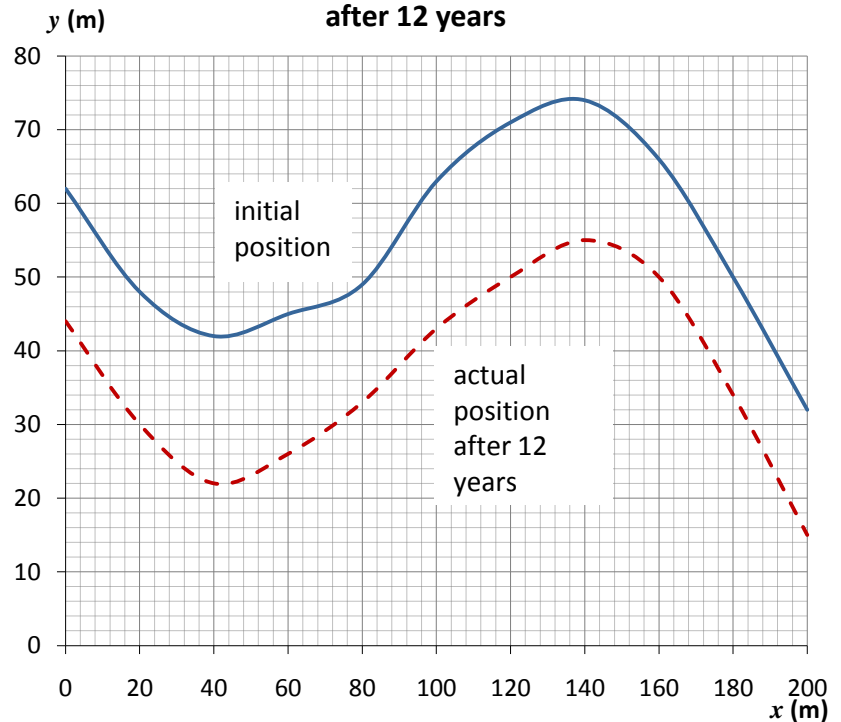
a Find the area of land represented by the area between the curve and the x axis.

b If the coastline recedes at an average rate of 1.8 metres per year, estimate the area of the land lost to the sea in a period of 10 years.

2 The table and graph give the initial position of a coastline and its position 12 years later.

x (m)	y (m) initial	y_a (m) after 12 yrs
0	62	44
20	48	30
40	42	22
60	45	26
80	49	33
100	63	43
120	71	50
140	74	55
160	66	50
180	50	34
200	32	15

Positions of coastline initially and after 12 years



Estimate the area of land lost.

3 The table below gives the lengths of offsets from a baseline to a coastline taken at 50 metre intervals:

Distance along baseline (m)	0	50	100	150	200	250	300	350
Initial offset (m)	76	62	54	46	40	36	34	32
Offset after 20 years (m)	39	33	28	24	20	18	16	15

a By considering a graph of these data points (drawn by hand or on a graphic calculator or spreadsheet) decide which type of function is likely to be the best model.

b Estimate the area of land that has been eroded in the 20 year interval.

Reflect on your work

What features of the coastline indicate whether a quadratic or a cubic function is likely to give the better fit?

What methods could you use to find models?

How would you decide which was the best model?

Describe how integration can be used to estimate land loss due to coastal erosion.