Activity description

Students approximate the area of a piece of land with an irregular coastline, and the area lost to coastal erosion over time.

The activity serves as an introduction to estimating the area under a curve using the trapezium rule. Significant material is available online on coastal erosion to run more open-ended contextual variants touching on other aspects of modelling and calculus.

Suitability and time

Advanced (Level 3); 1-3 hours

Resources and equipment

Student information sheet and worksheet
Optional: Slideshow
Optional: internet access

Key mathematical language

Estimating, area, area under a curve, modelling, trapezium rule, integration, offset, rate, coordinates

Notes on the activity

This activity uses the context of coastal erosion to introduce the trapezium rule for estimating the area under a curve. The activity can be presented using the slideshow and related material on the internet.

You could choose to run a more open-ended version where students develop their own approximation methods for the area under a curve, and discuss other rules such as the Mid-ordinate Rule and Simpson's Rule and their relative strengths and weaknesses.

www.shodor.org/interactivate/activities/Integrate/ has an excellent interactive that shows how a variety of integration methods, including the trapezium rule, approximate the integral and how accuracy increases with a greater number of intervals.

Coastal erosion B: Integrating area has the same example and exercises, but with functions modelling the coastline so that the areas can be determined using integration. There is also a natural extension that relates to introducing and using rates of change (land loss due to coastal erosion) and these are discussed further in the extensions.

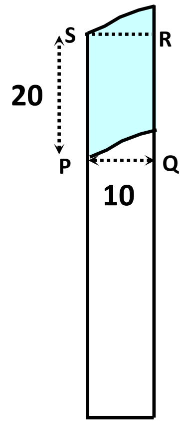
During the activity

Students should recognise that they do not have the data for the full coastline – their approximation should focus on the data points provided. Students should understand the role of the various factors in the trapezium rule formula, why the endpoint y-values are scaled by 0.5, and why the 'interior' y-values are scaled by 1.

Points for discussion

Under-estimation of the area under a curve by a trapezium when it is convex (⌒shaped) and over-estimation when it is concave (◡shaped)

Why the predicted loss of land is equal to the length of the baseline multiplied by the expected reduction in the length of the offsets (see example on page 3 and notes below). This can be extended into a single application of the trapezium rule in such problems rather than carrying out two applications and a subtraction.



This sketch shows the part of the graph between the first two offsets when a consistent loss of 20 m is assumed. The shaded section is the predicted loss of land. It can be seen that this is equal in area to the rectangle PQRS i.e. $10 \times 20 = 200$ m^2

Extending this idea to other sections of the graph gives the result.

Extensions

Coastal erosion B: Integrating areas uses the same example and exercise, but the answers are evaluated using approximating functions and integration rather than using the trapezium rule. Each activity can be used by itself, or the two activities can be combined allowing comparisons to be made between the two methods. If you wish to use the two activities together, you will also need to provide each student with pages 2 and 3 from *Coastal erosion: Integrating area*.

The Coastal Explorer tool www.coastalexplorer.eastriding.gov.uk provided by the East Riding local authority allows students to see various positions of the coastline over time, draw baselines and offsets, and also provides estimates of rate of land loss along a fixed set of points. Students could use these to draw initial and predicted coastlines and then approximate the area lost to coastal erosion.

The British Geological Society www.bgs.ac.uk/ has a wealth of information on coastal erosion, including description of the use of modern laser-based technology (Terrestrial LiDAR Survey Techniques) that enables measurement of not just planar area but the calculation of volume and the recession rates of cliff faces.

A number of related links and further contextual information on the erosion of the Holderness Coast is available via the University of Hull www.hull.ac.uk/erosion. Teachers could also find examples for other coastal areas via the www.bgs.ac.uk/ website.

Answers

1a 4630 = 4600 m^2 (to 2 sf); **b** 1800 m^2

2 11 740 – 7450 = 4290 = 4300 m^2 (to 2 sf)

3 16 300 – 8 300 = 8000 m^2