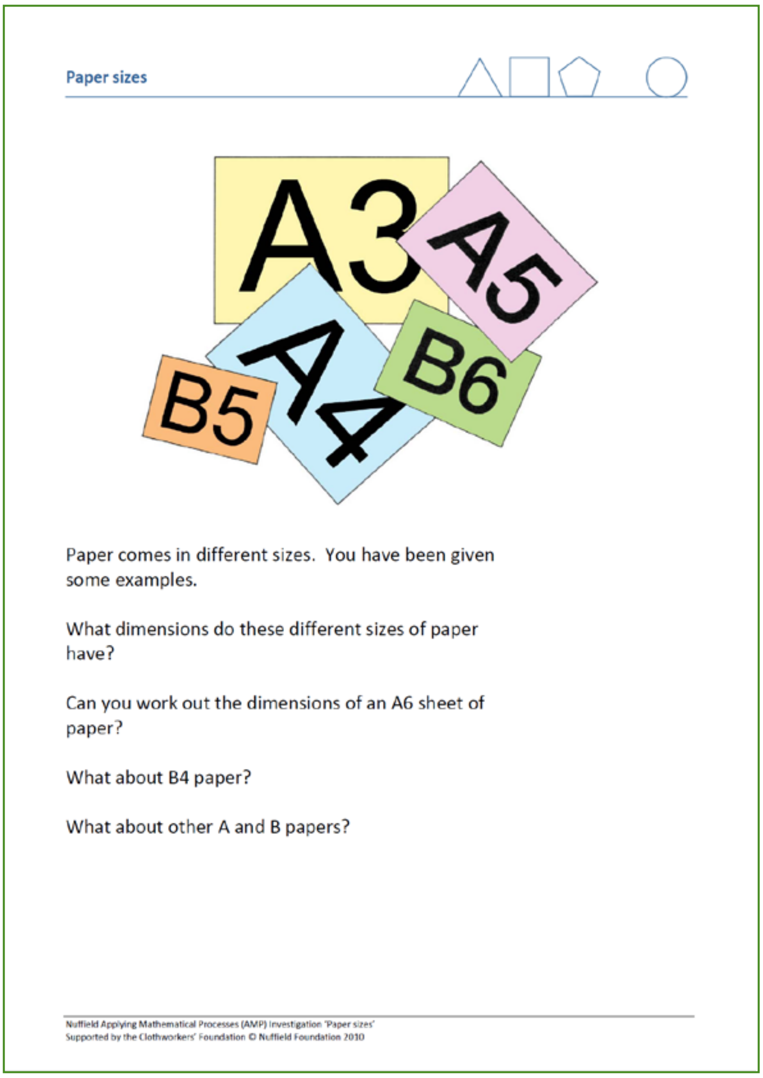
Activity description

Pupils investigate paper sizes in the A and B international series. They can explore the relationships within each series and between the series.

If pupils access information on paper sizes on the web, the focus of their work will need to be on interpreting and explaining their research.


Paper sizes

Paper comes in different sizes. You have been given some examples.

What dimensions do these different sizes of paper have?

Can you work out the dimensions of an A6 sheet of paper?

What about B4 paper?

What about other A and B papers?

Nuffield Applying Mathematical Processes (AMP) Investigation 'Paper sizes'
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Suitability

Pupils working at all levels; individuals or pairs

Time

1 to 2 hours

AMP resources

Pupil stimulus

Equipment

1 sheet of each of A3, A4, A5, B5 and B6 paper, labelled appropriately.
Rulers

Key mathematical language

Dimension, length, width, area, measurement, centimetre, millimetre, double, half, ratio, proportion, scaling, surd, series, similar, congruent, upper bound, lower bound

Key processes

Representing Identifying the mathematics involved in the task and developing appropriate representations.

Analysing Working systematically; identifying patterns; beginning to make generalisations.

Interpreting and evaluating Considering the findings to form convincing arguments.

Communicating and reflecting Explaining the approach taken and the outcomes achieved at each stage of the work.

Teacher guidance

Pupils may have relevant prior knowledge, so the activity should start with a class discussion of paper sizes. Which ones are pupils familiar with? What are they used for, and why? If printing templates to create any of the sheets, please take care that the paper sizes print accurately.

Consider the following starter: provide paper sizes A3 to A6 inclusive and ask pupils to write what size they think it is. Choose some of the probing questions to display around the room to prompt discussion. Open the discussion with: 'How can you find the size of A2 paper and A7 paper without physically having the paper in front of you?'

The activity works best if pupils work together, pooling their resources. Consider a mini-plenary during the activity to share results, using this as an opportunity to check accuracy to allow pupils to take the activity further.

During the activity

Encourage pupils to measure as accurately as possible, but discuss with them the idea that measurements are approximate and that there may be small errors in the rulers, the paper and/or their readings. Support pupils in focusing on the relationships between sizes.

Allow sufficient time for pupils to discover these relationships. Most begin by noticing very general features, such as that the letters A and B alternate when the pieces are arranged in order of size. Even when they measure, most pupils are likely to focus on additive relationships such as 'B5 is 4 centimetres longer than A5' and 'the difference goes up by a half each time'.

Use probing questions to encourage them to progress to multiplicative relationships, such as 'the length of A4 is double the width of A5', and to recognising that the various sizes are *similar*.

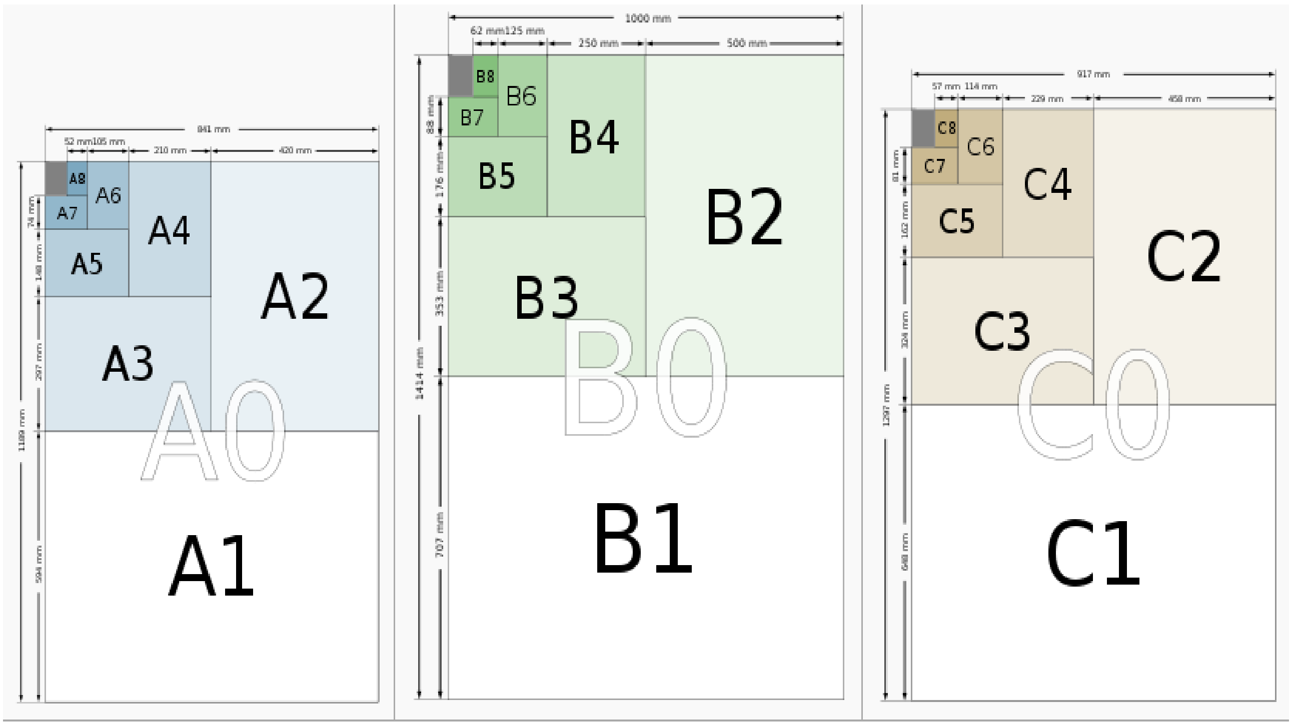
Probing questions and feedback

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- What units are you using to measure the dimensions? Why?
- If you only had sheets of A4 paper, how could you create a sheet of A3 paper? A5 paper? What does that tell you about the relationships between the dimensions of these paper sizes?
- What do you think the dimensions of A7 paper are? Why?

- In addition to two A5 sheets joining lengthwise to give an A4 sheet, can you find further mathematical relations between them?
- How do you know that the A series of papers are all similar? Are the B series of papers similar to the A series?
- Why can't the A series of paper, that is A3, A4, A5, etc. continue forever?
- What is the area of an A0 piece of paper? Why do you think that size was chosen? What about B0?
- Suppose you have two pieces of paper, An and An+1 (where n is an integer). If you write the ratio of the sides of An as 1 : *y*, what are the side lengths of An+1 in terms of 1 and *y*? Can you then work out the value of *y*?

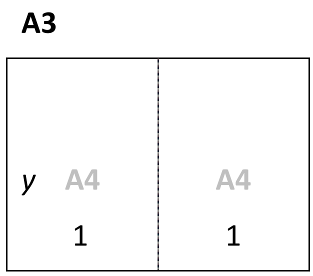
Additional information: International Standard (ISO) paper sizes



A,B,C series images reproduced from the Wikipedia page http://en.wikipedia.org/wiki/A4_paper under the Creative Commons (CC) Attribution-Share Alike license.

Successive paper sizes in the series A0, A1, A2, A3, … are defined by requiring that they be *similar*, that a given size is obtained by halving the preceding paper size along the larger dimension, and that A0 has area 1m^2.

The first two conditions result in the following diagram:



Similarity implies *y* : 1 is the same as 2 : *y* and therefore it follows that $y^2 = 2$.

Thus the common aspect ratio for the A series is 1: √2.

The advantage of this system is its scaling – the compatibility of the sizes with doubling.

A4 (approx. 210 mm x 297 mm) is the most commonly used size.

The area of the less common B series sheets is the geometric mean (GM) of successive (equivalently numbered and preceding) A series sheets. Thus, the area of B1 is the GM of the areas of A1 and A0. This implies that area B1 = area A1 x√2, area B0 = area A0x√2, and so on. Consequently the shorter lengths of the B series sheets are 1m for B0, 0.5m for B1, and so on.

The area of C series sheets is the GM of the areas of the A and B series sheets of the same number. This means that C4 is slightly larger than A4, and B4 slightly larger than C4.

Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

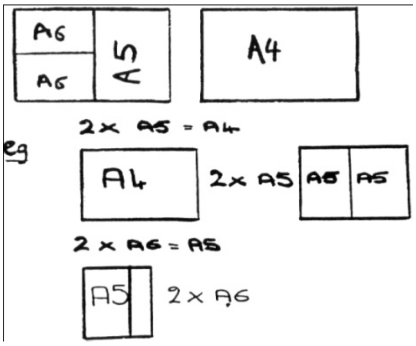
The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

Representing *Selecting a mathematical approach and identifying what mathematical knowledge to use*	**Analysing** *Calculating accurately and working systematically towards a solution*	**Interpreting and evaluating** *Interpreting the results of calculations and graphs in developing the final solution*	**Communicating and reflecting** *Explaining the approach taken and the outcomes achieved at each stage*
Shows minimal understanding of the given problem, e.g. makes a visual comparison with given paper Pupil A	Recognises the systematic numbering and order of paper	Makes a simple observation Pupil A	Sufficient information for someone else to understand their comparisons of different paper sizes Pupil A
Shows fuller understanding of the problem, e.g. chooses to measure dimensions of given paper Pupil B, D	Uses appropriate units of measurement consistently and systematically Pupils B, D	Identifies simple relationship(s) Pupils B, C, D	Presents a simple solution, e.g. dimensions tabulated or clearly labelled diagrams Pupil B
Chooses to research and / or find other paper sizes from those provided	Performs relevant mathematical calculations to an appropriate degree of accuracy Pupil E	Uses the relationship between sizes to make a general statement Pupil E	Mathematically justifies relationship(s) found Pupil D
Identifies other relevant mathematical aspects to explore Pupil F	Calculates and analyses more than one attribute of the paper sizes	Finds more complex relationships between dimensions of the paper sizes Pupil F	Expresses clearly a justification for a complex relationship Pupil E
Identifies and makes connections between several different factors	Organises the activity to explore in depth several different factors	Explains more complex relationships mathematically	Effectively and efficiently explains and mathematically justifies complex relationships between paper sizes

PROGRESSION

Sample responses

Pupil A

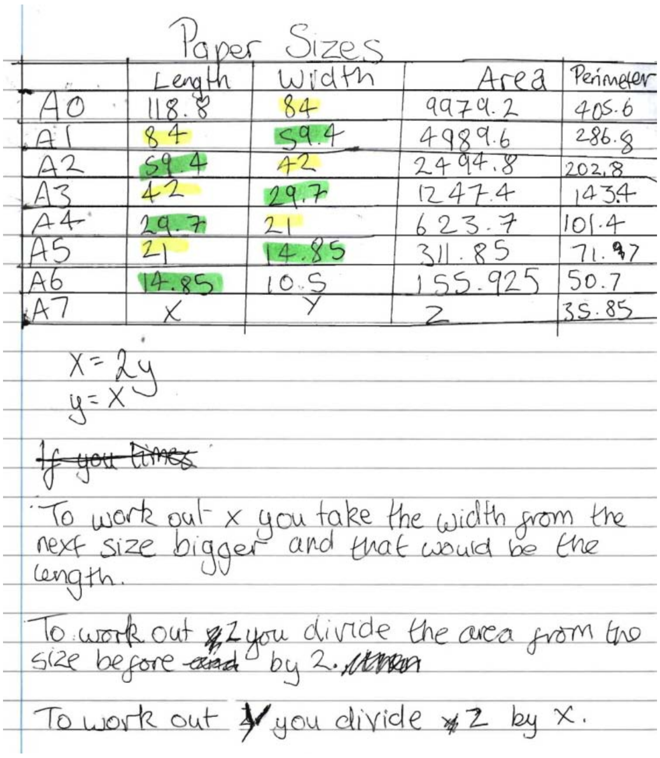


Pupil A uses diagrams effectively to show simple relationships between adjacent paper sizes.

Probing question

- How can you use the relationship you have found to predict the dimensions of an A3 piece of paper?

Pupil B



Paper Sizes

	Length	Width	Area	Perimeter
A0	118.8	84	9979.2	405.6
A1	84	59.4	4989.6	286.8
A2	59.4	42	2494.8	202.8
A3	42	29.7	1247.4	143.4
A4	29.7	21	623.7	101.4
A5	21	14.85	311.85	71.97
A6	14.85	10.5	155.925	50.7
A7	x	y	z	35.85

x = 2y
y = x

~~If you times~~

To work out x you take the width from the next size bigger and that would be the length.

To work out z you divide the area from the size before ~~and~~ by 2.

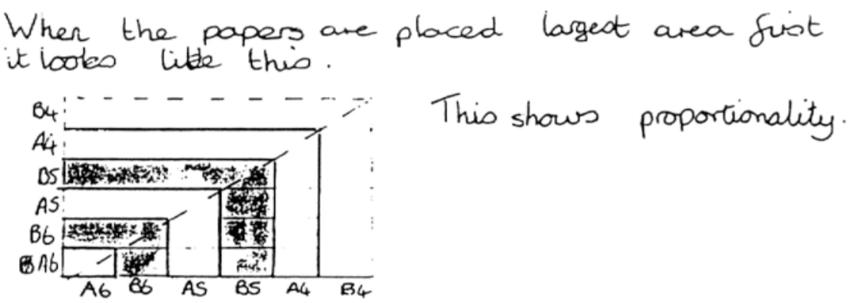
To work out y you divide z by x.

Pupil B finds and writes down the relationship between sizes to continue to find the dimensions within the A series of paper.

Probing questions

- See if you can find another way to find y, the width of the paper.
- You've written down the perimeter of A7; how did you work that out?
- What is the connection between the areas of different-sized items in a paper size series?

Pupil C



Pupil C combines the A-sized and B-sized paper. Whilst the diagram has not been drawn to scale, the comment is valid.

Probing question

- Suppose you drew this information on a graph.
  How could the equation of the diagonal line help you to find the relationship between the length and width of different sizes?

Pupil D

Pupil D tabulates length and width using measures consistently.

The prediction is actually an observation with no indication of checking.

However a reason is given for the observation made from graphing the results.

Probing question

- Can you find the gradient of the line on your graph? What would that tell you about the relationship between length and width of A-sized paper?

Paper Investigation

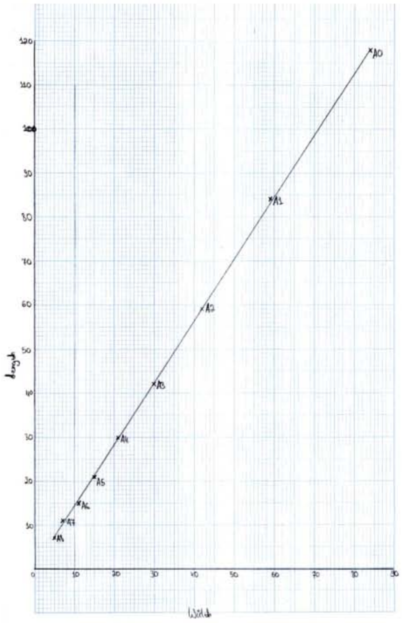
Size of Paper	Measurements	
	Length	Width
A0	118.4 cm	84 cm
A1	84 cm	59.2 cm
A2	59.2 cm	42 cm
A3	42	29.6 cm
A4	29.6 cm	21 cm
A5	21 cm	14.8 cm
A6	14.8 cm	10.5 cm
A7	10.5 cm	7.4 cm
A8	7.4 cm	5.25 cm

Predictions: the length of a size will be the same as the width of the one before.

* the width of the size of paper is half the length of the size of paper above.

① We measured the length and width of the papers that we had.
② We put the results into a table.
③ We found the patterns between the lengths and the widths. (see predictions)

The Graph shows that the smaller the number after the A, the bigger the paper. There is a pattern between sizes because the line is straight.



Pupil E

Paper Investigation

Name	Sketch + sizes (cm)
A4	29.8cm, 21cm
A5	14.8, 21
A3,	42.1, 29.8

We have also drawn a graph.

We measured all of the papers we had!

We predict A6 will be 14.8cm by 10.5, basically half of A5 on one side.

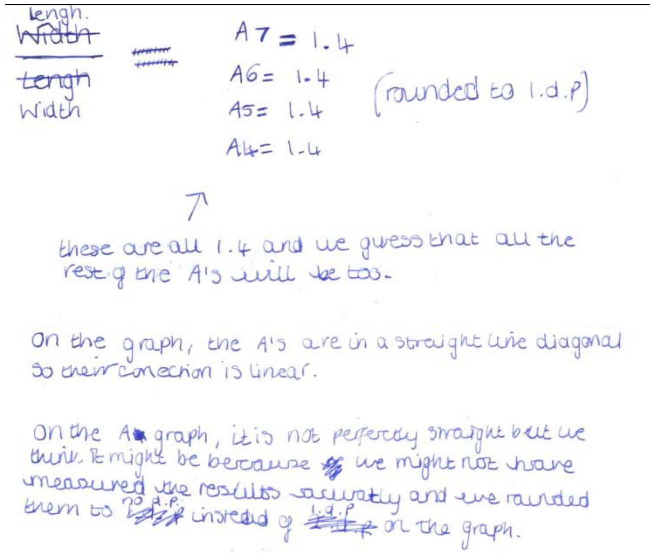
We predict A7 will be 10.5cm by 7.4cm.

We predict A2 will be 42.1cm by 59.6cm.

We predict A1 will be 59.6cm by 84.2cm

We predict A0 will be 84.2 by 119.2 cm.

We got these by doubling and halving. If you start with A4, A3 will be double A4 on one side, A2 would be double A3 on one side e.t.c. A5 would be half A4 on one side, and A6 would be half A5 on one side e.t.c.



length / width

A7 = 1.4
A6 = 1.4
A5 = 1.4
A4 = 1.4

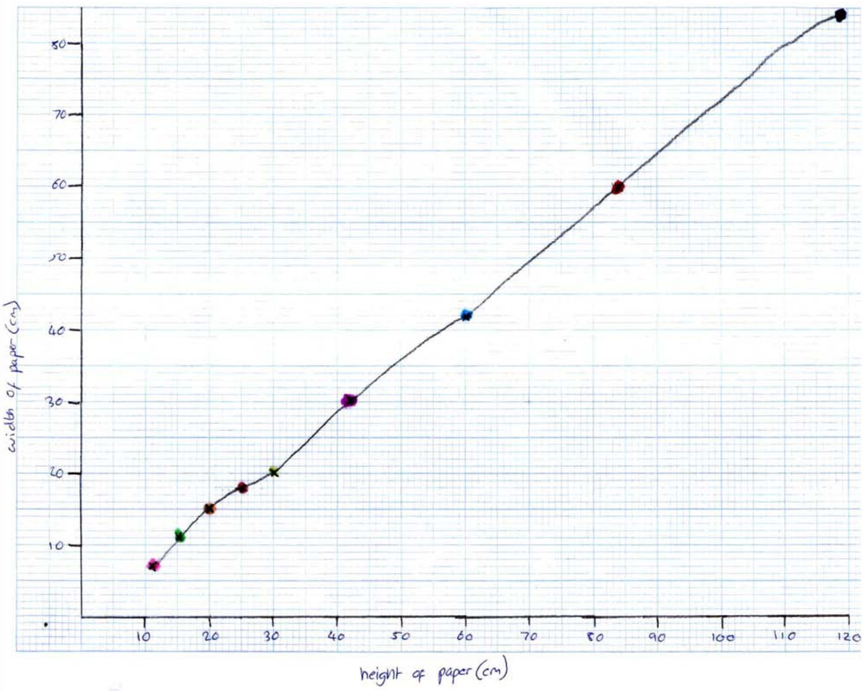
(rounded to 1.d.p)

these are all 1.4 and we guess that all the rest of the A's will be too.

On the graph, the A's are in a straight line diagonal so their connection is linear.

On the A graph, it is not perfectly straight but we think it might be because we might not have measured the results accurately and we rounded them to 2 d.p. instead of 1 d.p on the graph.

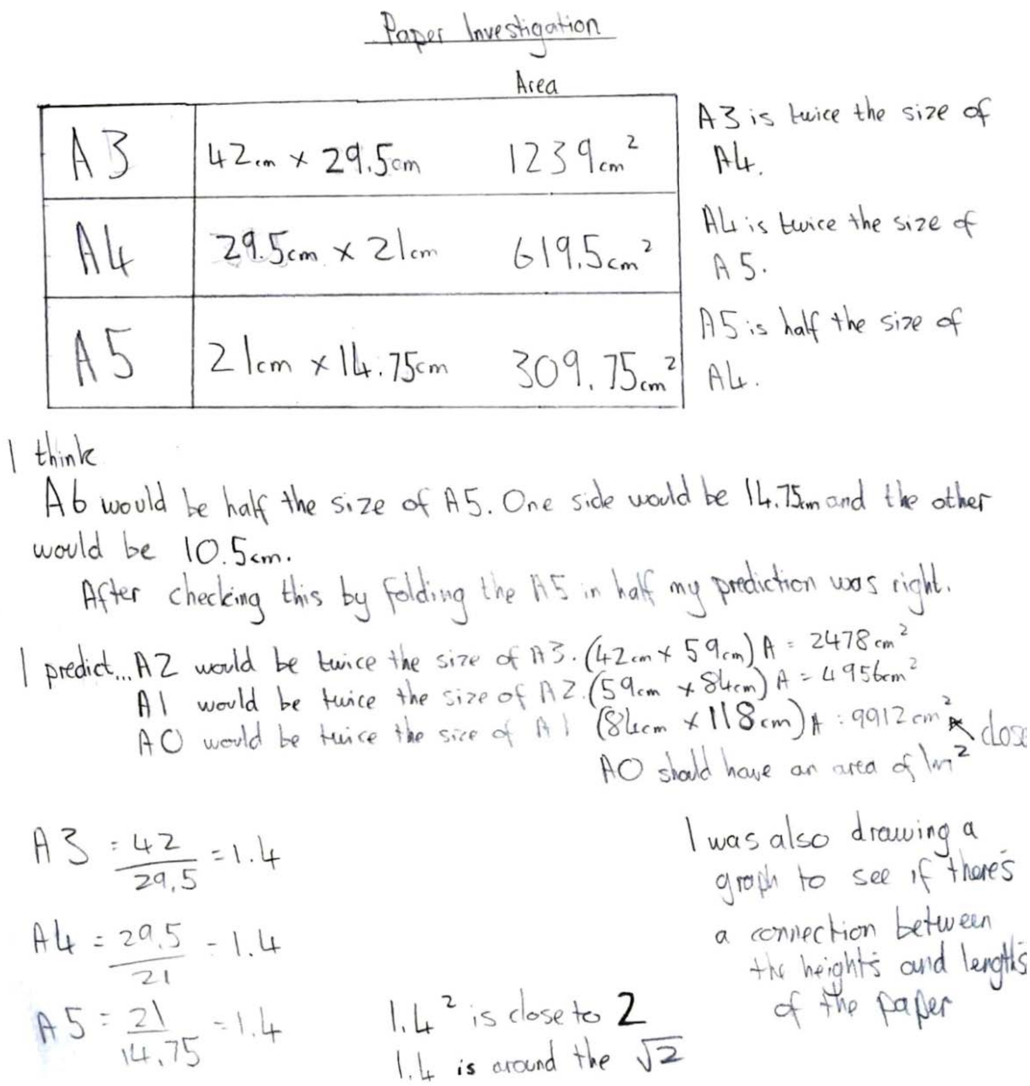
Pupil E establishes the common ratio of length to width for A-size paper. Results are graphed with a valid observation, but no connection is made between the ratio and the graph.



Probing questions

- How could you check that the A series of paper sizes all have the same length/width ratio?
- See if you can find the gradient of your line.
  How is this connected to the length/width ratio you have already found?
  Can you explain why?
- Use the ‘doubling and halving’ relationship to find a more accurate ratio than 1.4.

Pupil F



Paper Investigation

		Area
A3	42cm × 29.5cm	1239cm²
A4	29.5cm × 21cm	619.5cm²
A5	21cm × 14.75cm	309.75cm²

A3 is twice the size of A4.
A4 is twice the size of A5.
A5 is half the size of A4.

I think
A6 would be half the size of A5. One side would be 14.75cm and the other would be 10.5cm.
After checking this by folding the A5 in half my prediction was right.

I predict... A2 would be twice the size of A3. (42cm × 59cm) A = 2478cm²
A1 would be twice the size of A2. (59cm × 84cm) A = 4956cm²
A0 would be twice the size of A1 (84cm × 118cm) A = 9912cm² ← close
A0 should have an area of 1m²

A3 = 42/29.5 = 1.4
A4 = 29.5/21 = 1.4
A5 = 21/14.75 = 1.4

1.4² is close to 2
1.4 is around the √2

I was also drawing a graph to see if there's a connection between the heights and lengths of the paper

Pupil F finds the common ratio and makes the link to √2, but has not made the connection between this value and the lengths of the sides.

Probing questions

- You say that your area for A0 is close to 1m^2. Why do you think this may not be accurate?
- Explain how '1.4^2 is close to 2' may link with A3 being twice the size of A4.
- You found the fraction length/width for A3, A4 and A5. How would your graph have helped you to explore this connection? What would be the connection between this and your statement '1.4 is around the √2'?