Activity description

Pupils investigate what effect the route taken through a series of mazes has on the number of gold coins that can be collected.

Suitability

Pupils working at all levels; individuals or pairs

Time

1 hour upwards

AMP resources

Pupil stimulus, Flash interactive

Equipment

Squared or dotty paper
Coins / counters / multi-link cubes
Calculator

Key mathematical language

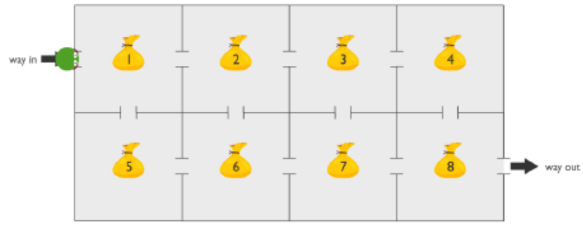
Odd/even, parity, path, sum, maximum, conjecture, proof

Golden Mazes

Here is a maze of rooms.

Each room contains a bag of gold coins.

The numbers tell you how many gold coins there are in each bag.



You have to find your way through the maze, collecting bags of gold as you go.

You have to try to get as many coins as you can, but you are only allowed to go into each room once.

How many coins can you get?

Investigate for different-sized mazes.

Nuffield Applying Mathematical Processes (AMP) Investigation 'Golden mazes'
Supported by the Clothworkers' Foundation © Nuffield Foundation 2010

Key processes

Representing Identifying the mathematics involved in the task and deciding on an appropriate form of representation.

Analysing Working systematically, identifying patterns and relationships – leading to the making of conjectures which can be tested.

Interpreting and evaluating Considering their observations, and forming, testing and justifying generalisations.

Communicating and reflecting Explaining their methodology, and effectively communicating their findings.

Teacher guidance

Using a prepared slide with the pupil stimulus, or the Flash interactive, the teacher or a pupil 'describes' a route through the maze. Each pupil keeps a running total of the number of coins collected as the route is developed. The class then discusses:

- whether the route fits the constraints as stated in the task sheet (for instance that a room can only be entered once)
- the accuracy of the pupils' calculations in terms of the number of coins collected
- options that may increase the total.

Pupils can use the Flash interactive, coins, discs or multi-link cubes to investigate the problem.

During the activity

Ensure that the pupils appreciate there are a variety of routes that may be taken.

Support the pupils in developing their recording of results.

Prompt or ask pupils questions that encourage them to express their findings as a generalisation, using precise language and illustrative examples and, where appropriate, using algebra.

Encourage the pupils to look at alternative designs of maze beyond the original.

Probing questions and feedback

AMP activities are well suited to formative assessment, enabling pupils to discuss their understanding and decide how to move forward. See www.nuffieldfoundation.org/whyAMP for related reading.

- How do you know for certain whether you have collected as many coins as possible?
- Show whether / how the pattern you have found works in other situations.
- Is there a general rule that describes your findings?
- How does the design of the maze influence your findings?
- Are there configurations that make it impossible to visit every room? Why?

Extensions

- Do the investigation for different designs of maze (such as triangular, hexagonal, or three-dimensional blocks).
- Consider different positions of the ways in and out of the maze.

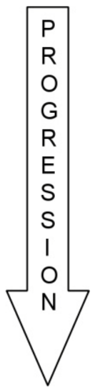
Progression table

The table below can be used for:

- sharing with pupils the aims of their work
- self- and peer-assessment
- helping pupils review their work and improve on it.

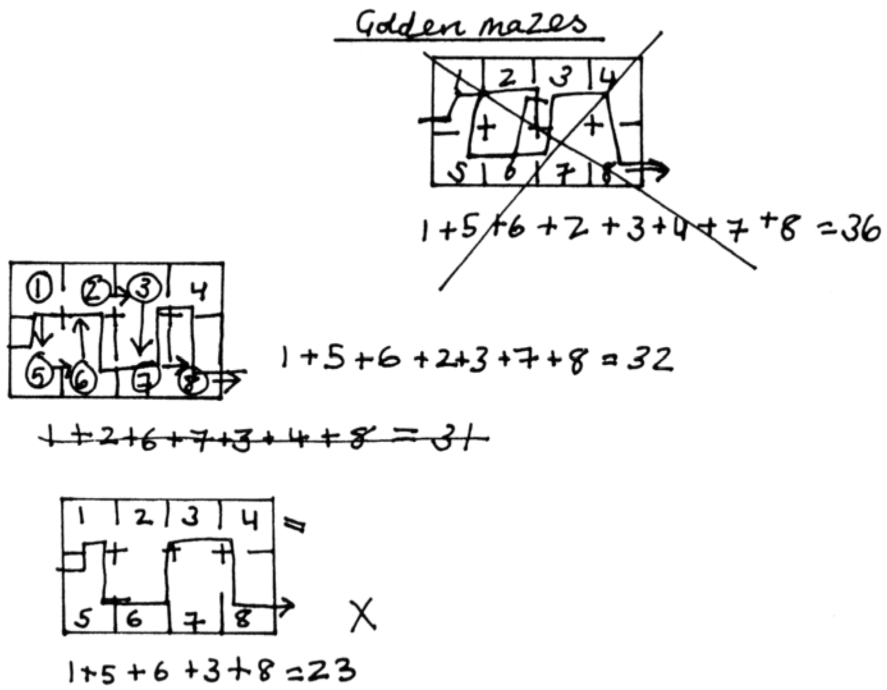
The table supports formative assessment but does not provide a procedure for summative assessment. It also does not address the rich overlap between the processes, nor the interplay of processes and activity-specific content. Please edit the table as necessary.

Representing *Clear choice of appropriate forms of representation and use of them to develop ideas and solutions*	**Analysing** *Systematic approaches allowing the development of generalisations and reasoning*	**Interpreting and evaluating** *Relating findings to the original task and justifying the approaches taken*	**Communicating and reflecting** *Effectively reviewing, refining and communicating findings and approaches*
Shows understanding of the situation by producing a representation of at least one route through the given maze, and calculates the total number of coins Pupils A,C	Presents one or more solutions to find the most coins collected for the given maze Pupil A	Makes a simple observation based on initial findings	Communicates how answer was obtained Pupil B
	Tests different cases albeit randomly, or recognises a strategy for optimising the total Pupil B	Recognises there are different routes for the same maze Presents a solution for the most coins collected with some justification Pupil B	Explains why number of coins is the maximum Pupil C
Chooses to present findings in a more sophisticated representation Pupil D	Chooses to vary the size of the maze systematically, identifying key features of the mazes that determine the solutions Pupil C	Makes sense of findings and starts to justify them Pupils C, D	Uses a variety of forms to communicate effectively Pupil D
Chooses to present generalisations in math-ematically sophisticated form, e.g. uses algebraic expressions for maximum coins in mazes with odd or even numbers of columns	Examines the results obtained, refining arguments and making generalisations Pupil D	Justifies generalisations and solutions, e.g. uses their own findings to provide reasons, or through testing of generalisations	Reviews and refines their approaches; communicates how they handled variation in routes and sizes



Sample responses

Pupil A

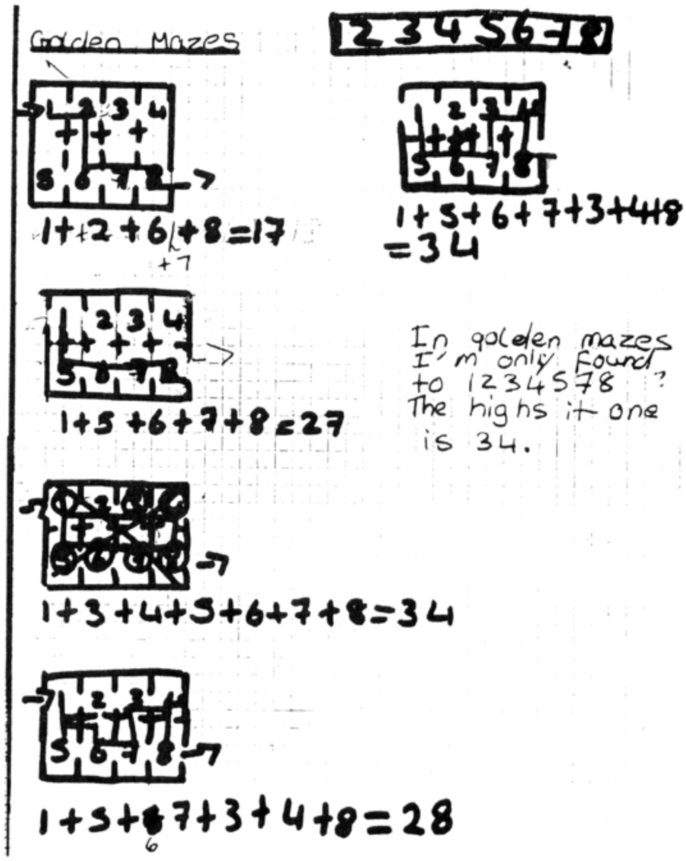


Pupil A has some understanding of the task, and has attempted to calculate the number of coins collected, choosing to represent the work diagrammatically with lines and arrows indicating route.

Probing questions

- How could you show that you are only visiting each room once?
- How does your route affect the total number of coins you collect?
- How can you guarantee that your chosen route gives the maximum number of coins?

Pupil B



Pupil B demonstrates different routes through the maze and correctly concludes that 34 is the maximum. This was supported by a verbal justification ‘I think 34 is the highest because I’ve used the big numbers’. The verbal justification is necessary evidence.

Probing questions and feedback

- It would be good to have a discussion with the pupil as to how ‘I’ve used the big numbers’ can be translated into a precise strategy, accounting for further choices that may need to be made, and possible generalisations to other mazes.

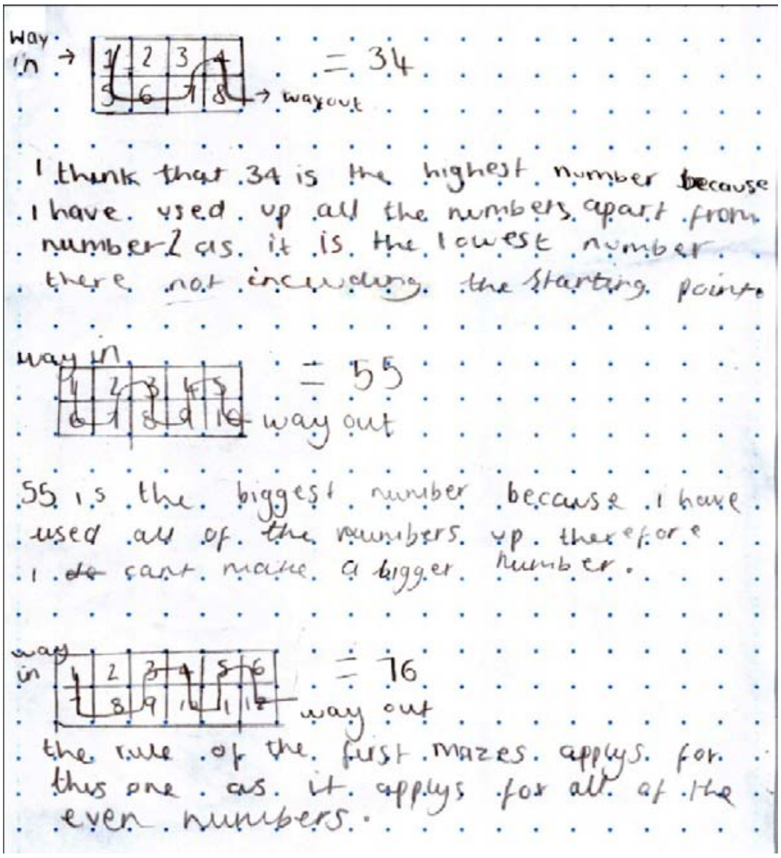
Pupil C

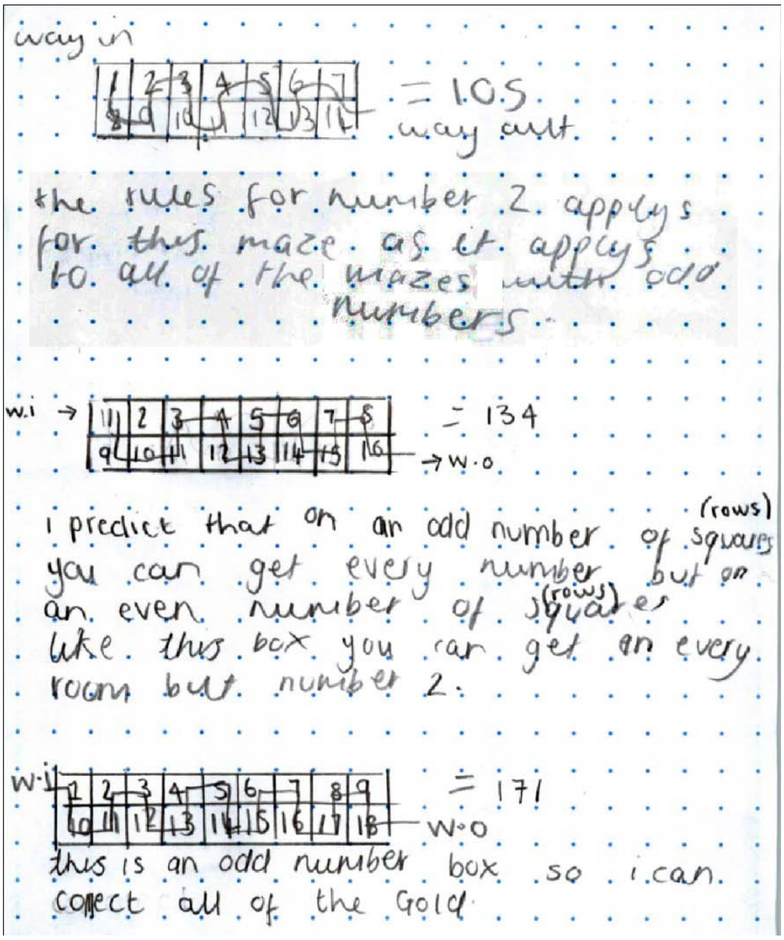
Pupil C has analysed the arrangement of the maze and explained how they know that as many coins as possible have been collected.

The numbers of columns in the maze has been increased systematically and the significance of the odd and even number of columns has been correctly identified, although in the written explanation there is some confusion between number of squares, rows and columns.

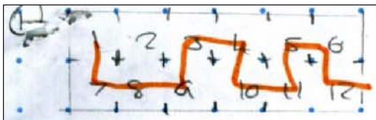
Probing questions

- What happens if you vary the number of rows?
- See if you can work out how many coins you could collect for a maze of any size.





Pupil D



Routes

① 1+7+8+9+3+4+10+11+5+12= 70

If the across dimension is odd then you can go into every room and get the highest number, so if its even you go into every room except the lowest possible one (2) to get the highest number.

e.g. 10 across

Add every number together from 1–20 except 2 to get your highest number.

e.g. 12 across

Add every number together and you'll get the highest number.

Rule =. if the numbers odd, leave out 2, if its even add everyone together = highest number.

Squares.	Best routes.	Total
8	~~1+5+6+7+3+4 3+8~~ ~~1+5+6+3+7+8+4~~	

Column Row

4×2 = 8 squares the Sum is = add 1 to 10 −2 = 34

5×2 = 10 squares the sum is = add 1 to 10 55

6×2 = 12 squares the sum is = add 1 to 12−2 = 70

7×2 = 14 squares the sum is = add 1 to 14 = 105

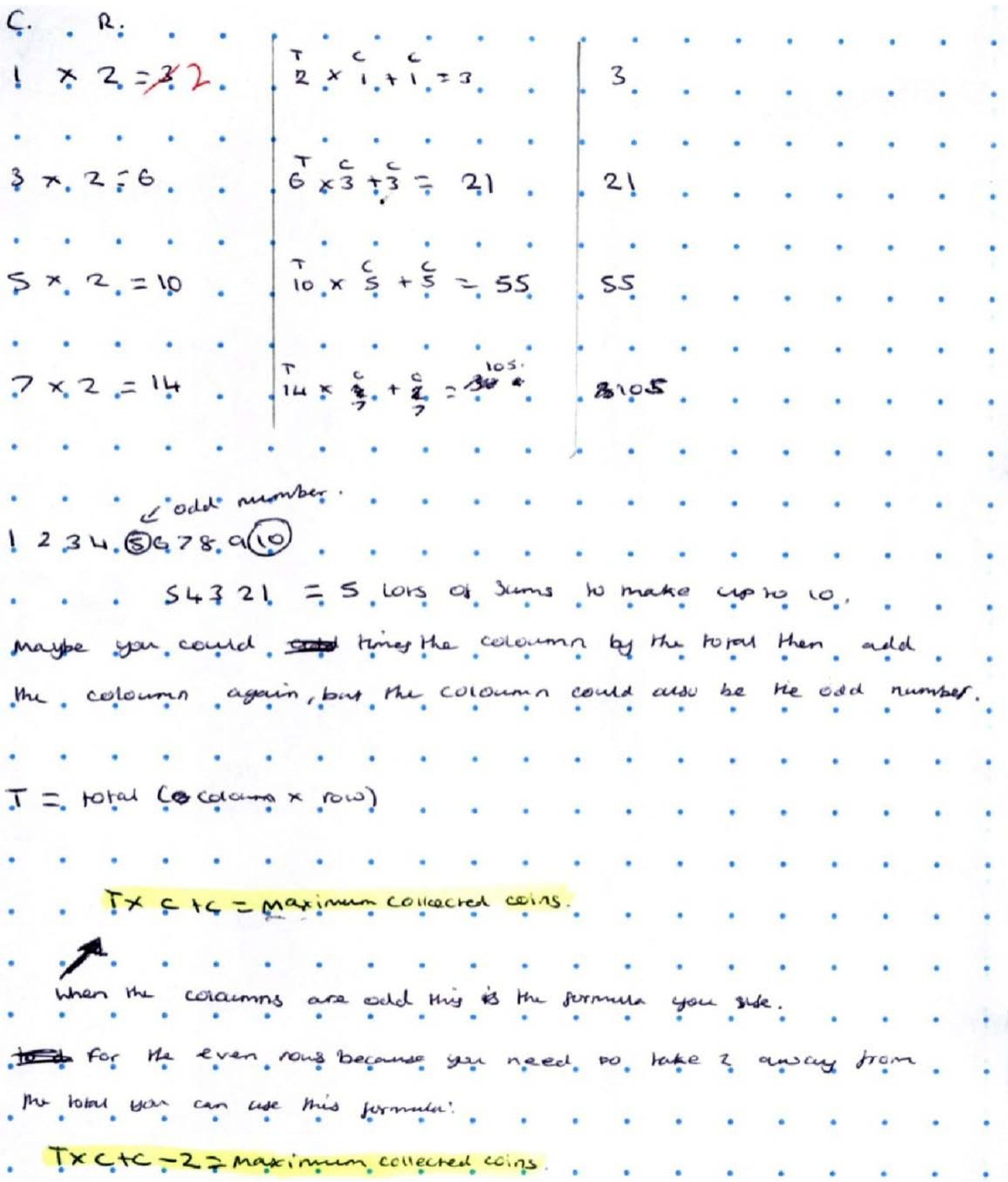
8×2 = 16 squares the Sum is = add 1 to 16 − 2 = 134

9×2 = 18 squares the Sum is = add 1 to 18 = 171

10×2 = 20 squares the sum is = add 1 to 20 − 2. = 208

Pupil D continues on the next page

Pupil D *continued*



C.	R.			
1 × 2 = ~~3~~ 2	T 2 × C 1 + C 1 = 3	3
3 × 2 = 6	T 6 × C 3 + C 3 = 21	21
5 × 2 = 10	T 10 × C 5 + C 5 = 55	55
7 × 2 = 14	T 14 × C 7 + C 7 = ~~30~~ 105	~~8~~105

odd number.
1 2 3 4 5 6 7 8 9 10
5 4 3 2 1 = 5 lots of sums to make up to 10.
Maybe you could ~~add~~ times the coloumn by the total then add the coloumn again, but the coloumn could also be the odd number.

T = total (column × row)

Tx c + c = maximum collected coins.

When the columns are odd this is the formula you use.

~~to~~ For the even rows because you need to take 2 away from the total you can use this formula:

Tx c + c − 2 = maximum collected coins.

Pupil D has considered routes through mazes of various sizes and produced a rule to determine the number of coins collected, based on whether the number of columns is odd or even. The rule is expressed in words, and then the pupil investigates how it can be expressed algebraically. This investigation is based on spotting patterns in numerical results and expressing these algebraically, but the algebra is not interpreted in terms of the original problem.

Probing questions

- Why does your formula work?
- Would moving the exit affect your results?
- How could you extend the investigation?