## Activity description

This activity shows students how to use differentiation to find stationary points on the curves of polynomial functions. It includes the use of $\frac{d^2y}{dx^2}$ to determine the nature of the stationary point. Students will learn how to use this and other information to sketch the curves, then use graphic calculators to check their answers.

### Suitability

Level 3 (Advanced)

### Time

1–2 hours

### Resources

Student information sheet, slideshow *(optional)*

### Equipment

Graphic calculators

### Key mathematical language

Function, gradient, maximum, minimum, point of inflection, differentiate.

## Notes on the activity

Students need to be able to:

- differentiate polynomials
- solve linear and quadratic equations
- sketch curves on a graphic calculator.

## During the activity

The slideshow includes the same examples as the student sheets, and can be used to introduce this topic.

## Points for discussion

Discuss with students the way that the gradient changes from positive through zero to negative at a maximum point.

This means that $\frac{dy}{dx}$ is decreasing in value and so $\frac{d^2y}{dx^2}$ is negative.

From similar reasoning $\frac{d^2y}{dx^2}$ must be positive at a minimum point.

Students should know that substituting $x$ = 0 into the function gives the point where the curve crosses the $y$ axis.

Check that students know how to solve the equations that follow from equating $y$ to zero to find the points at which the curves cross the $x$ axis.

Ask students to think about what the function does as $x \to \pm\infty$.

Knowing whether the function's value tends to $\pm\infty$ (or zero) is another useful aid to sketching the curve.

## Extensions

More able students could investigate the number of stationary points in polynomials of various orders. They may discover that the number of stationary points is usually one less than the order of the polynomial, but that this is not always the case.

## Answers

**1a** Minimum at (2, – 4), meets axes at (0, 0), (4, 0)

**b** Minimum at (3, – 4), meets axes at (0, 5), (1, 0), (5, 0)

**c** Minimum at (– 1, – 9), meets axes at (0, – 8), (– 4, 0), (2, 0)

**d** Maximum at (0, 16), meets axes at (0, 16), (– 4, 0), (4, 0)

**e** Maximum at (3, 9), meets axes at (0, 0), (6, 0)

**f** Maximum at (– 0.25, 1.125), meets axes at (0, 1), (– 1, 0), (0.5, 0)

**g** Minimum (2, – 4), maximum at (0, 0), meets axes at (0, 0), (3, 0)

**h** Maximum at (0, 16), meets axes at (0, 16), (– 2, 0), (2, 0)

**i** Minimum (1, – 2), maximum at (– 1, 2), meets axes at (0, 0), ($-\sqrt{3}$, 0), ($\sqrt{3}$, 0)

**j** Point of inflexion at (0, 1), meets axes at (– 1, 0), (0, 1)

**2a** Minimum at (1, 1), maximum at (– 3, 33), meets axis at (0, 6)

**b** Minimum at (2, – 16), maximum at (– 1, 11), meets axis at (0, 4)

**c** Minimum at (1, – 7), maximum at (– 1, – 3), meets axis at (0, – 5)

**d** Minimum at (– 2, – 76), maximum at (2.5, 106.25), meets axis at (0, 0)

**e** Maximum at (0, 3), minima at (– 1, 2) and (1, 2), meets axis at (0, 3)

**f** Maximum at (1, 6), meets axis at (0, 3)