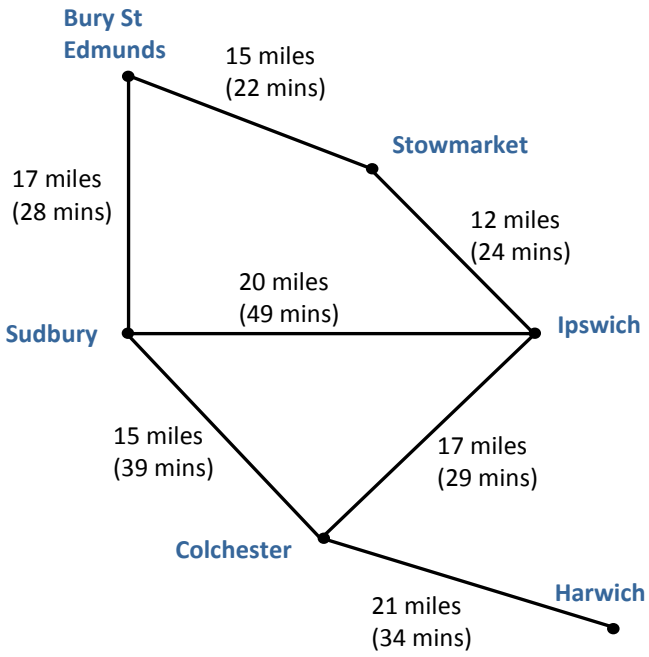
Networks are used to model many real-life situations.

For example, this network is a simple map showing estimated distances and times of travel by bus or car between some English towns.



Information sheet

Network definitions

A **graph** is a diagram consisting of **edges** and **vertices** representing how objects are related to each other.

A **vertex** (or node) is a point where edges meet. The vertex (plural vertices) is **even** or **odd** according to whether an even or odd number of edges meet there.

The **degree** of a vertex is the number of edges that meet at the vertex.

Think about …

Why must the sum of the degrees of the vertices in any graph always be even?

What can you say about the number of odd vertices in a graph?

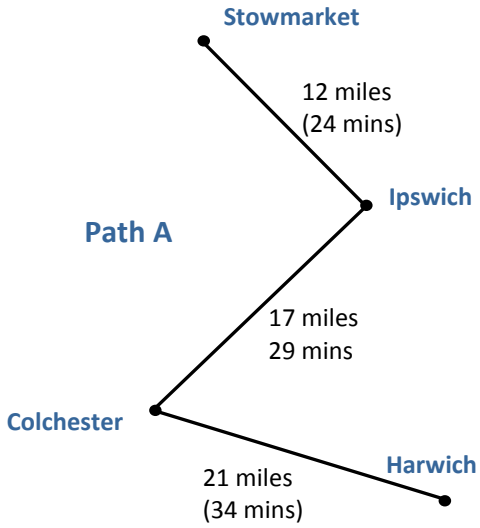
An **edge** is a line joining two vertices. It can be **directed** (one-way) or **undirected** (two-way).

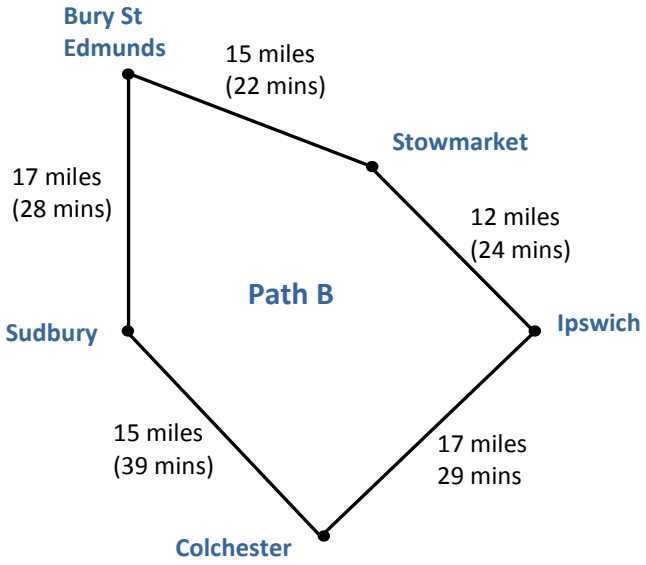
A **weight** can be allocated to an edge. This may represent distance, time or costs.

A **network** is a graph with weighted arcs. In the graph shown here, the vertices represent towns and the edges represent roads with weights representing the distances (and times) between the towns.

A **path** is a route through the graph which does not visit any vertex more than once, and does not go along any edge more than once.

The graph on the previous page includes lots of paths.
Two examples are given below.





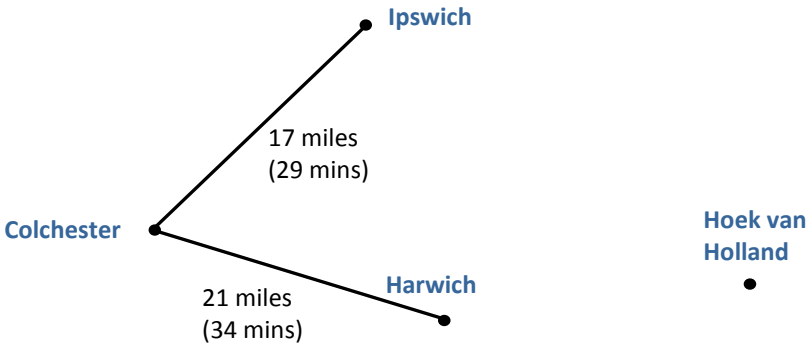
A **cycle** is a path that forms a loop by returning to its starting point.

Path B above is a cycle, but Path A is not a cycle.

A graph is **connected** if there is at least one route between each pair of vertices.

All the graphs above are connected, but the one on the right showing a road network is not.

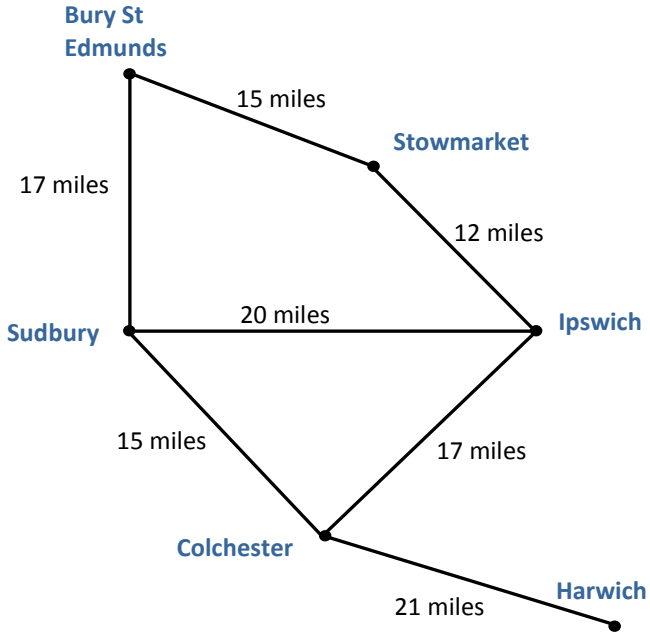
There is no road joining any of the English towns to Hoek van Holland – though you can get there by ferry from Harwich.



A **matrix** is an array of numbers.

In an **adjacency matrix** these numbers represent the number of edges that directly join each pair of vertices. In a **distance matrix** the numbers give the distance between each pair of vertices.

The graph of the road network from page 1, and its adjacency and distance matrices, are given below.



Note

In the matrices below:

B represents Bury St Edmunds
Su represents Sudbury
C represents Colchester
H represents Harwich
I represents Ipswich
St represents Stowmarket

Adjacency matrix

	B	Su	C	H	I	St
B	0	1	0	0	0	1
Su	1	0	1	0	1	0
C	0	1	0	1	1	0
H	0	0	1	0	0	0
I	0	1	1	0	0	1
St	1	0	0	0	1	0

Distance matrix

	B	Su	C	H	I	St
B	0	17	0	0	0	15
Su	17	0	15	0	20	0
C	0	15	0	21	17	0
H	0	0	21	0	0	0
I	0	20	17	0	0	12
St	15	0	0	0	12	0

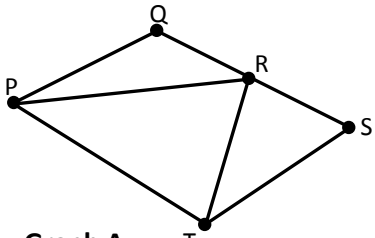
Think about…

What do you notice about the pattern in the adjacency and distance matrices?

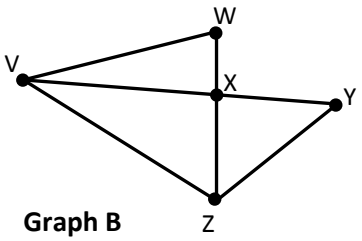
Try this

1 Copy and complete the table for the graphs shown below it.

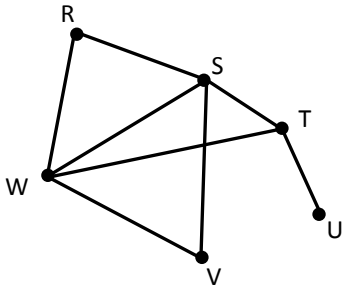
Graph	Number of edges	Even nodes	Odd nodes
A			
B			
C			



Graph A



Graph B



Graph C (The intersection of SV and WT is not a node)

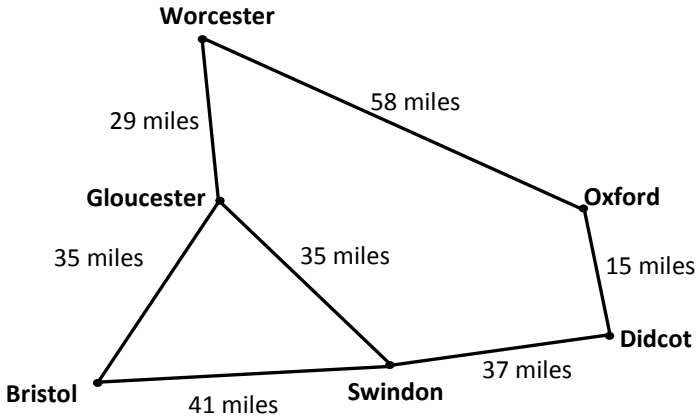
2 The graph shows the rail network connecting six places in England.

a Draw an adjacency matrix.

b Draw a distance matrix.

c Sketch two paths through the graph.

d List nodes that give a cycle in the graph.



3 This distance matrix gives the mileages between Manchester (M), Leeds (L), Sheffield (S), Doncaster (D) and Kingston-upon-Hull (K).

a Draw the corresponding graph of the network.

b Show the distances given in the matrix on your graph.

	M	L	S	D	K
M	0	40	38	0	0
L	40	0	33	29	55
S	38	33	0	18	0
D	0	29	18	0	47
K	0	55	0	47	0

4a Draw a sketch of the road network joining towns in your local area. Maps and distances (and estimated times) can be found on the internet.

b Draw the corresponding adjacency and distance matrices.

Reflect on your work

- Explain what is meant by the following terms: network, graph, edge, vertex, node, degree, directed, undirected, weighted, path, cycle, connected, adjacency matrix, distance matrix.
- If a road is one way, then when planning a driving route it may be possible to get from A to B, but not from B to A. What difference would this make to the adjacency matrix?