Activity description

This level 3 activity shows students how to find the mean values of quantities that vary with time.

This involves finding the area under graphs. Initially this is done by using geometrical formulae for areas.

This is followed by the use of integration in a variety of real life situations.

Suitability

Level 3 (Advanced)

Time

2–3 hours

Resources

Student information sheet and worksheet
Optional: slideshow

Equipment

Graphic calculators and computer spreadsheet access

Key mathematical language

Function, estimate, graph, area, trapezium, mean value, integration

Notes on the activity

The slideshow can be used in conjunction with the information sheet, leaving the students to complete some tasks, which are then checked using the slideshow. This is explained further in the next section.

During the activity

Slides 1, 2 and 3 introduce the activity, using the context of a car travelling along a road in town.

Slide 4 shows how the area under a speed–time graph gives the distance travelled, for a simple case involving constant speed. Pages 1 and 2 of the student sheets also contain this information.

Slide 5 shows how the area under the original speed–time graph can be split into triangles and trapezia.

You could use some of Slide 6 go through part of the working with students before asking them to complete it using page 3 of the student sheets. Use the rest of slide 6 to check their work.

Slide 7 shows how integration can be used to find the area under one section of the graph.

Students can then find the area for all the other sections to check their previous answers. You may decide that students should share this work and discuss the functions they find to model each section.

Finally the mean value of a more complex function is found using integration. Students can then attempt the questions in the 'Try these' section on pages 4 and 5.

Points for discussion

Encourage students try to explain what the graphs and functions indicate about the real life contexts. Discuss what the 'mean value' stands for in each example, and how this relates to the area under the curve.

Encourage students to compare the two methods used to find the area under a straight line graph. They should realise that integration, whilst effective, may not always be the quickest or easiest method.

The reflection questions at the end of the worksheet and slideshow ask students to think through the methods they have used, and consider which they prefer.

You may also wish to ask students other questions about the formulae used. For example, that given in the last question:

$$\text{Mean depth} = \frac{\int_0^{12} \left(5.4 + 0.6x - 0.702x^2 + 0.1x^3 - 0.0038x^4\right) dx}{12}$$

is an integral for $0 \le x \le 12$.

Ask about what difference it would make if the integral had been over the interval $6 \le x \le 15$ and what the divisor would have been. [Would give the mean depth between 9 am and 6pm, with divisor 9]

Extensions

Students could apply this work in other topic areas such as geography and economics if possible. However, they are likely to need help with finding suitable functions to model the situation.

Answers

Page 3

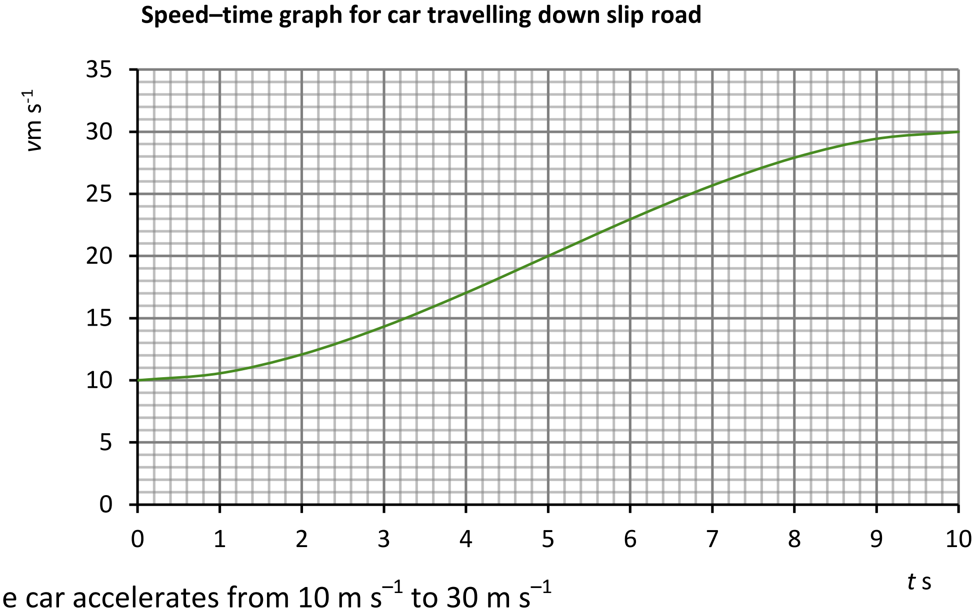
A = 11.2 B = 4.8 C = 29.295 D = 26.8 E = 11.135

Total distance = 83.23 m Average speed = 8.5 m s^{-1} (to 1 dp)

The answers can also be found on slide 6 of the slideshow

Pages 5 and 6

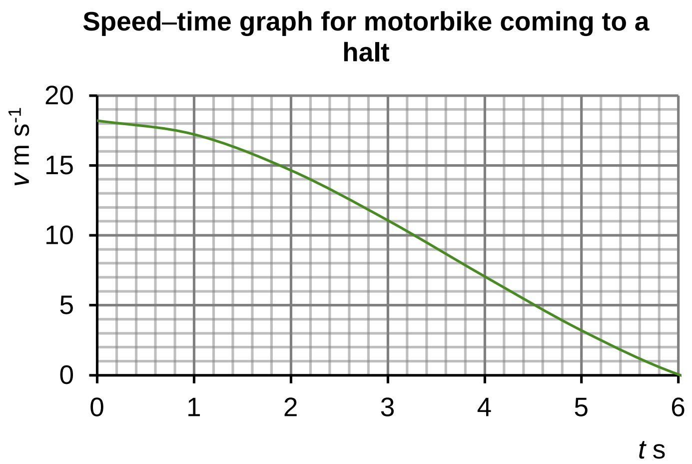
1a



The car accelerates from 10 m s^{-1} to 30 m s^{-1}

Area under curve = 200 Average speed = 20 m s^{-1}

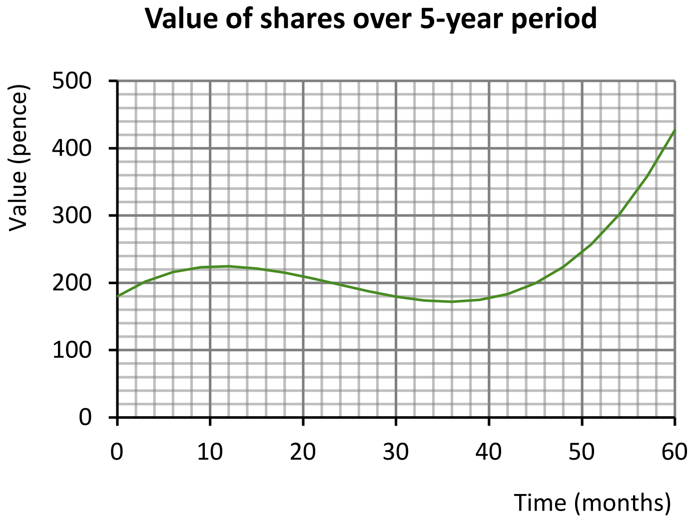
1b



The motorbike decelerates from 18.2 m s^{-1} to 0 m s^{-1}
in approximately 6 seconds.

Area under curve = 62.5 Average speed = 10.4 m s^{-1} (1 dp)

2a i

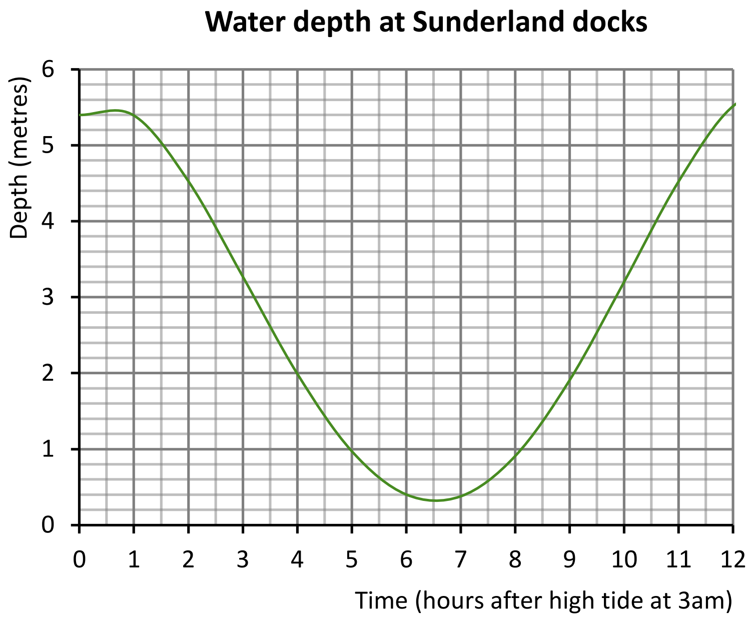


ii The value of the shares increases from 180 p to 225 p after 11 months, then falls to approximately 175 p after 36 months.

The value then rises sharply to 427 p at the end of the 5-year period.

b 221 p (nearest pence)

3a i



ii Approximately 9:30 am with a depth of approximately 0.3 m

b 2.7 metres (to 1dp)