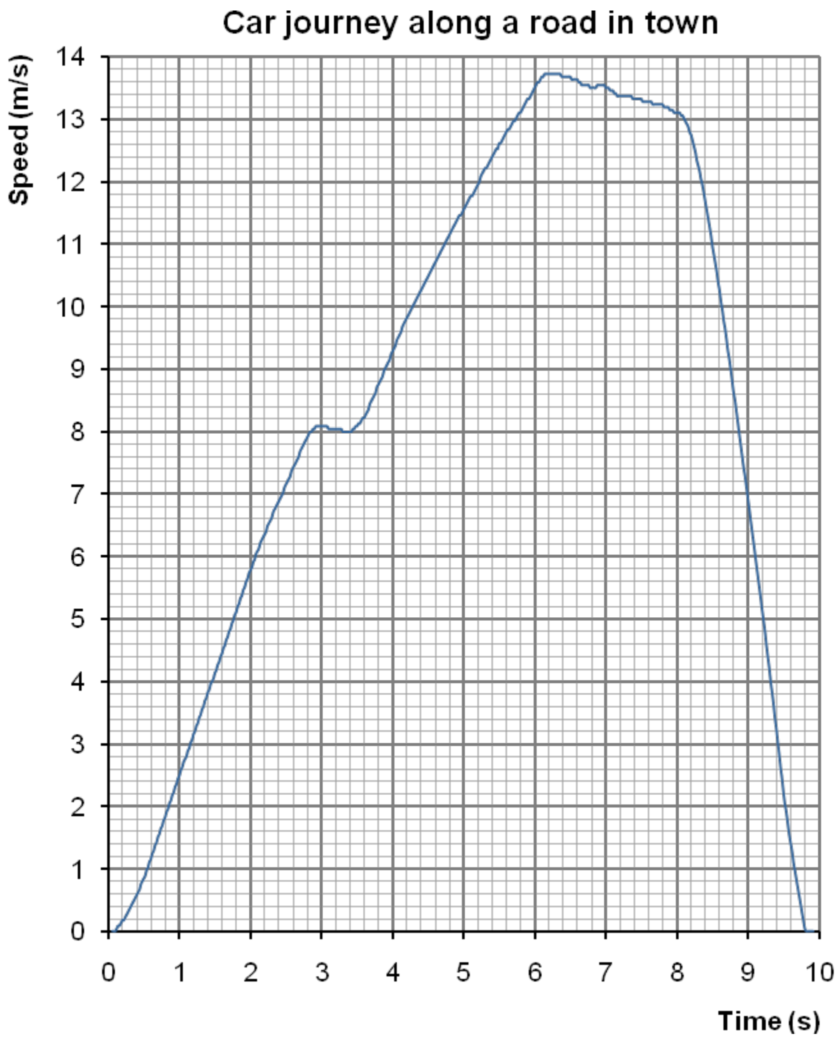
This activity is about finding the mean (average) value of a quantity which varies with time. You will use the area under a graph to find mean values such as the average speed of a car or the average price of a share on the stock market over a period of time.

Information sheet

The graph shows data collected from an accelerometer as a car travels along a road in town.



Think about…

Describe what you think happens during this short journey.

How could you find the car's average (mean) speed?

A Finding the average speed from a graph

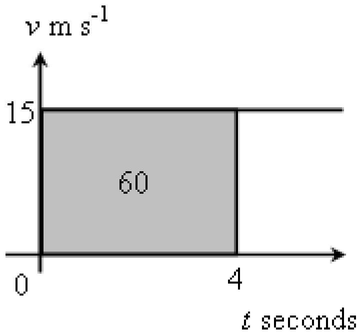
$$\text{Mean speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

where

Distance travelled = Area under the speed–time graph

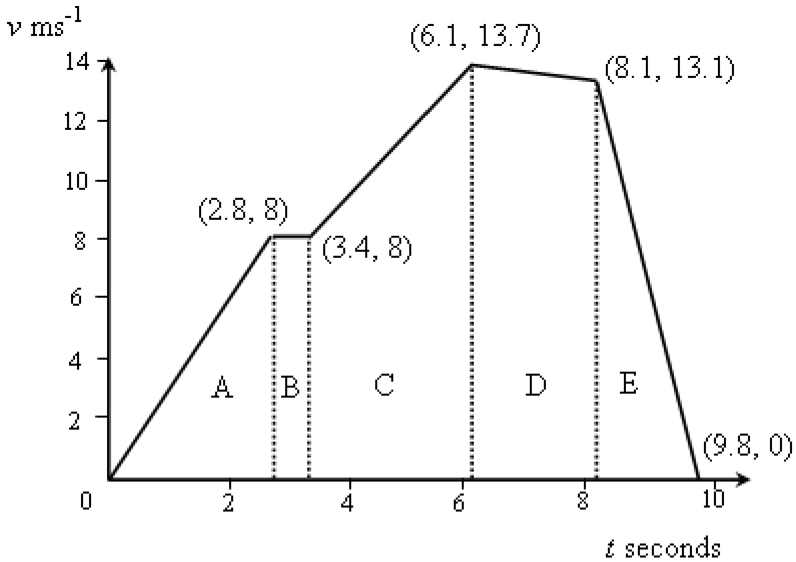
You can see this from the graph that shows a steady speed of 15 m s^{-1}.

The distance travelled in 4 seconds is $4 \times 15 = 60$ m. This is equal to the area of the rectangle.



For the car travelling along the road in town:

- the graph can be modelled by a series of straight lines
- the area under the graph can be estimated using triangles, rectangles and trapezia:

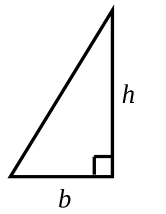


Think about…

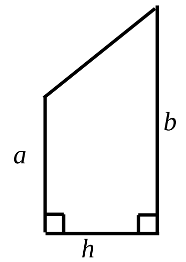
The area under the graph is split into triangles and trapezia.
What are the formulae for the area of these shapes?

B Using geometry to find the area

Use the formulae to complete the following:



Area of a triangle = $\dfrac{b \times h}{2}$



Area of a trapezium = $\dfrac{(a+b) \times h}{2}$

Area of A = ………………………………………………

Area of B = ………………………………………………

Area of C = ………………………………………………

Area of D = ………………………………………………

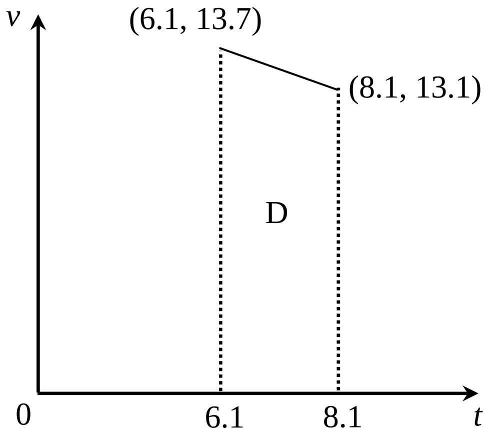
Area of E = ………………………………………………

Total area = Total distance travelled = ……………………………

Average speed = ………………………………………..

C Using Integration to find the area

You can estimate the area under a graph by integrating functions that model the graph.



Example

For section D of the graph on page 2:

The line joining (6.1, 13.7) and (8.1, 13.1)

has gradient: $m = \dfrac{13.1 - 13.7}{8.1 - 6.1} = \dfrac{-0.6}{2} = -0.3$

Substituting (6.1, 13.7) in $y = mx + c$ gives:

$$13.7 = -0.3 \times 6.1 + c$$

and so $c = 13.7 + 0.3 \times 6.1 = 15.53$

The equation of the line is $y = 15.53 - 0.3x$ that is, $v = 15.53 - 0.3t$

Using integration:

Area of D = $\int_{6.1}^{8.1} (15.53 - 0.3t)\,dt = \left[15.53t - \dfrac{0.3t^2}{2}\right]_{6.1}^{8.1} = \left[15.53t - 0.15t^2\right]_{6.1}^{8.1}$

$= [15.53 \times 8.1 - 0.15 \times 8.1^2] - [15.53 \times 6.1 - 0.15 \times 6.1^2]$

$= 115.9515 - 89.1515 = 26.8$

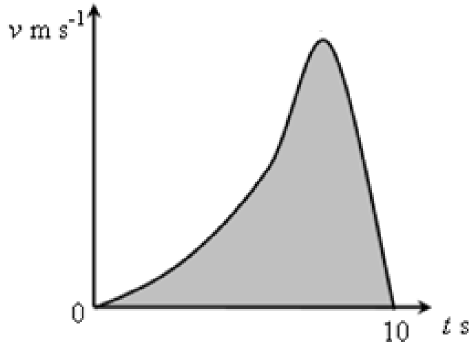
Think about…

Use the area of D found above to find the mean value of the speed in this section. Does the result agree with what you would expect by looking at the graph?

Use integration to find the areas of other sections shown in the graph on page 2.

Compare the answers with the values you found earlier.

Which method do you prefer, and why?



Example
Using integration to find the area under a curve

The speed of a cyclist along a road can be modelled by the function $v = 0.1t^3 - 0.01t^4$

This is shown on the graph.

Think about…

Describe what happens.

To find the cyclist's mean speed, first find the distance travelled:

Area under the graph $= \int_0^{10} \left(0.1t^3 - 0.01t^4\right) dt = \left[\frac{0.1t^4}{4} - \frac{0.01t^5}{5}\right]_0^{10}$

$= \left[0.025t^4 - 0.002t^5\right]_0^{10}$

$= 250 - 200 = 50$

Distance travelled $= 50$ metres

Average speed $= \frac{50}{10} = 5$ m s^{-1}

Try these …

1 For each part:

- use a graphic calculator or spreadsheet to draw a speed–time graph
- describe what happens during the time interval
- use integration to find the area under the curve
- estimate the average speed.

a The speed of a car, v m s^{-1}, as it travels down the slip road onto a motorway can be modelled by $v = 10 + 0.6t^2 - 0.04t^3$ for $0 \le t \le 10$ where t is the time in seconds.

b The speed of a motorbike , v m s^{-1}, as it comes to a halt at a T-junction can be modelled by $v = 0.096t^3 - 1.08t^2 + 18.2$ where t is the time in seconds.

The mean values of other variables can also be found using integration.
Here are two to try.

2 The value of shares in a company over a five-year period is modelled by

$$y = 0.0072x^3 - 0.51x^2 + 8.8x + 180$$

where y is the value in pence and x is the time in months.

a i Use a graphic calculator or spreadsheet to draw the graph of this function for $0 \le x \le 60$.

ii Describe what happened to the value of the shares over this five-year period.

b The mean value of the shares over this period is given by

$$\text{Mean value} = \frac{\int_0^{60} \left(0.0072x^3 - 0.51x^2 + 8.8x + 180\right)dx}{60}$$

Find this mean value, giving your answer to the nearest pence.

3 The depth of water at Sunderland docks over a 12-hour period can be modelled by

$$y = 5.4 + 0.6x - 0.702x^2 + 0.1x^3 - 0.0038x^4$$

where y is the depth in metres and x is the time in hours after 3 am.

a i Use a graphic calculator or spreadsheet to draw the graph of this function for $0 \le x \le 12$.

ii According to your graph, when was low tide, and what was the depth of the water at this time?

b Find the mean depth of water during this period using

$$\text{Mean depth} = \frac{\int_0^{12} \left(5.4 + 0.6x - 0.702x^2 + 0.1x^3 - 0.0038x^4\right)dx}{12}$$

Reflect on your work

Summarise the method for finding the mean value of a quantity from its graph.

In cases where the graph consists of straight lines, you can use either geometrical formulae to find the area or integration.

Which of these methods do you prefer, and why?