When something is measured, the measurement is subject to uncertainty. This uncertainty is called 'error' even though it does not mean that a mistake has been made. The size of the error depends on the sensitivity of the measuring instrument and how carefully it is used.

Often when measurements are given they are quoted to a particular degree of accuracy. This gives a range of values in which the actual value could lie.

This activity will help you understand how big errors may be, and how errors accumulate when measurements are used in calculations. This is particularly important in scientific contexts.

Information sheet : Section A Errors

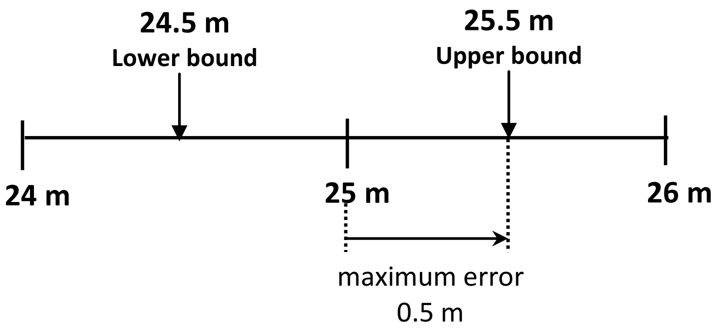
Example: swimming pool

The length of a swimming pool is given as 25 metres to the nearest metre.

Think about…

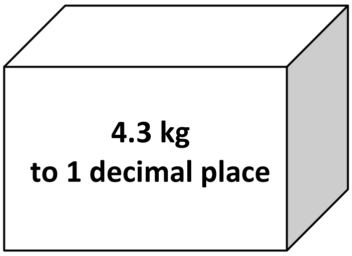
What is the shortest possible length?
What is the longest possible length?

The length is nearer to 25 metres than it is to either 24 metres or 26 metres.
24.5 metres is the shortest length that can be rounded to 25 metres.
This smallest possible value is called the **lower bound** of the measurement.
The largest possible value, in this case 25.5 metres, is called the **upper bound**.



When a value is given to the nearest whole number, the error can be as large as 0.5

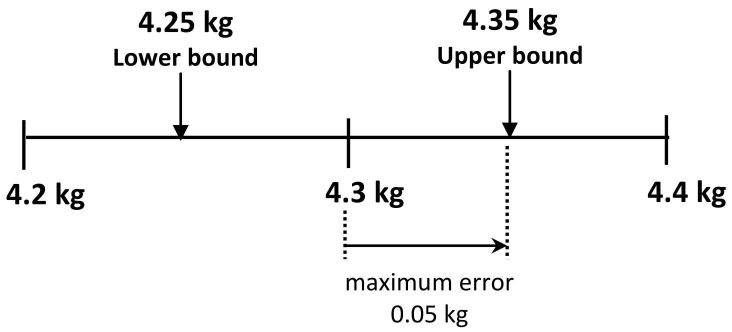
Example: weight of a package



Think about…

What is the smallest possible weight?
What is the largest possible weight?

When the weight of a package is 4.3 kg correct to 1 decimal place, the true weight could be anywhere between 4.25 kg and 4.35 kg.



When a value is given to 1 dp, that is to the nearest 0.1, the error can be as large as 0.05

Think about…

When a measure is expressed to a given unit, the maximum error is half of this unit.

Examples

Accuracy of measure	**Maximum error**
Nearest 100	50
Nearest 10	5
Nearest whole number	0.5
To 1 decimal place (nearest 0.1)	0.05
To 2 decimal places (nearest 0.01)	0.005

Example: journey length

The length of a journey is estimated to be 250 miles to the nearest 10 miles.

Think about…

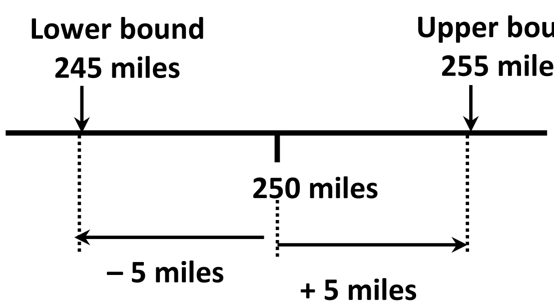
What is the maximum error?

The estimated length is 250 miles, but there could be an error of up to 5 miles.

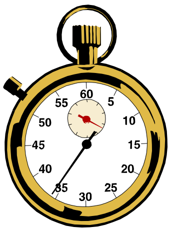
The **upper bound** = 250 + 5 = **255 miles.**

The **lower bound** = 250 – 5 = **245 miles.**

The length of the journey could be written as 250 $\pm$ 5 miles.



Example: winning time



The winning time in a race is 36.32 seconds to the nearest 0.01 seconds.

Think about…

What is the maximum error?

Maximum error = 0.005

Upper bound = 36.32 + 0.005 = 36.325 seconds.

Lower bound = 36.32 – 0.005 = 36.315 seconds.

The winning time could be given as 36.32 $\pm$ 0.005 seconds.

Example: furnace temperature

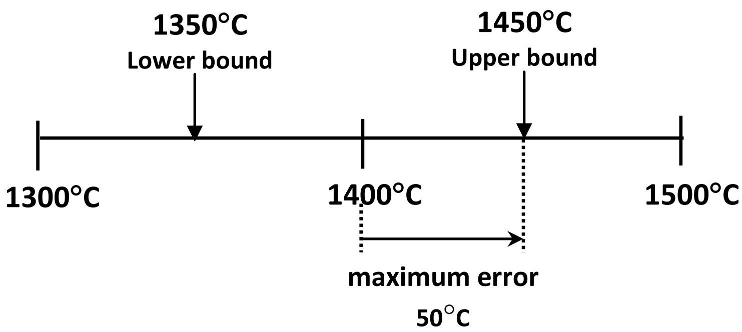
If the temperature of a furnace is 1400 °C correct to 2 significant figures,
this means the temperature is nearer to 1400 °C than to 1300 °C or 1500 °C

Think about…

What is the highest possible temperature?
What is the lowest possible temperature?

Maximum error = 50 °C.
Upper bound = 1450 °C.
Lower bound = 1350 °C.

The temperature could be given as 1400 $\pm$ 50 °C



Think about…

What would be the highest and lowest possible temperatures if the temperature was given as 1400 °C correct to **3 significant figures?**

This would mean the temperature is nearer to 1400 °C than to 1390 °C or 1410 °C.

In this case the maximum error is 5°C and the temperature can be given as 1400$\pm$ 5 °C.

The **upper bound** = **1405 °C**. The **lower bound** = **1395 °C**.

Try these *Complete the following table:*

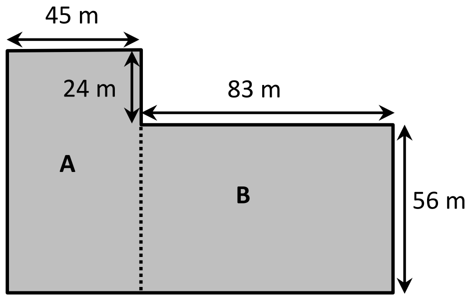
	Measurement	Largest possible error	Upper bound	Lower bound
Height of a tree	50 m to the nearest m			
Mid-day temperature	28°C to the nearest degree			
Weight of a letter	32 g to the nearest g			
Time to complete task	40 minutes to the nearest minute			
Length of caterpillar	3.4 cm to 1 decimal place			
Patient's temperature	38.6°C to 1 decimal place			
Weight of parcel	2.9 kg to 1 decimal place			
Time to reach 60 mph	6.2 seconds to 1 decimal place			
Length of shelf	2.75 m to 2 decimal places			
Weight of fish	1.64 kg to 2 decimal places			
Sprint time	10.27 seconds to 2 decimal places			
Height of a hill	480 m to the nearest 10 m			
Width of drive	560 cm to the nearest 10 cm			
Weight of cake	1200 g to the nearest 10 g			
Weight of cake	1200 g to the nearest 100 g			
Length of a runway	1900 m to 2 significant figures			
Length of a runway	1900 m to 3 significant figures			
Weight of an aircraft	170 000 kg to 2 significant figures			
Weight of an aircraft	170 000 kg to 3 significant figures			

Information sheet Section B Combining errors

When measurements are used in calculations, the errors accumulate so that the end result may be less accurate than you might expect. This is illustrated by the following examples.

Example

The diagram shows a car park.
The lengths given on the diagram are each correct to the nearest metre.



Think about…

How do you find the perimeter and area of the car park?

Using the dimensions on the diagram:
the length of the left hand side of the car park is 24 m + 56 m = 80 m
and the length of the bottom of the car park is 45 m + 83 m = 128 m
So the perimeter = 45 + 24 + 83 + 56 + 128 + 80 = 416 m

The area of section A = 80 × 45 = 3600 m^2
and the area of section B = 83 × 56 = 4648 m^2
So the total area = 3600 + 4648 = 8248 m^2

Think about…

How accurate are these values?

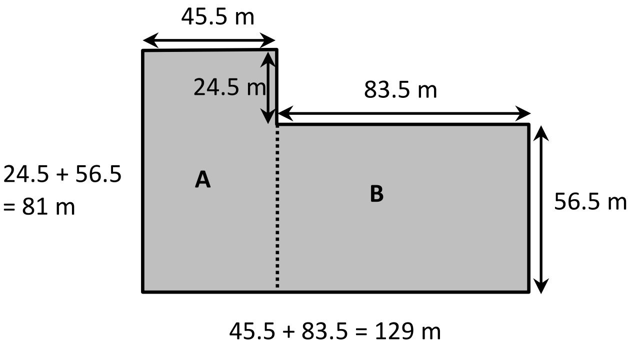
To answer this question, find upper and lower bounds for the perimeter and area of the car park. First note that the maximum error for each side is 0.5 m.

Upper bounds

The second diagram shows the upper bound for each side of the car park.
The upper bound for the perimeter
= 45.5 + 24.5 + 83.5 + 56.5 + 129 + 81
= **420 m**

The upper bound for area A
= 81 × 45.5 = 3685.5 m^2

The upper bound for area B
= 83.5 × 56.5 = 4717.75 m^2



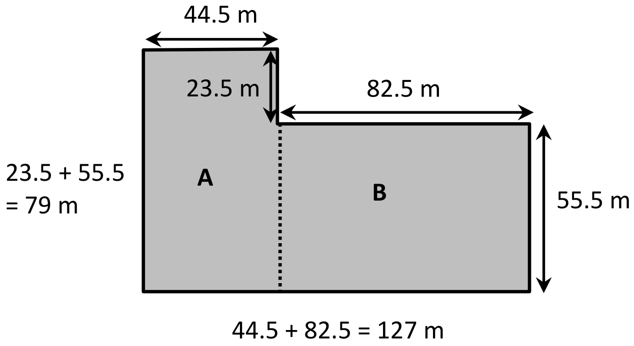
The upper bound for the area of the car park = 3685.5 + 4717.75 = **8403.25 m^2**

Lower bounds

This diagram shows the lower bound for each side of the car park.

The lower bound for the perimeter
= 44.5 + 23.5 + 82.5 + 55.5 + 127 + 79
= **412 m**

The lower bound for area A
= 79 × 44.5 = 3515.5 m^2
The lower bound for area B
= 82.5 × 55.5 = 4578.75 m^2



The lower bound for the area of the car park = 3515.5 + 4578.75 = **8094.25 m^2**

So **the perimeter could take any value from 412 m to 420 m**
and **the area could be anything from 8094 m^2 to 8403 m^2** (to the nearest m^2).

Think about…

What final answers should be given for the perimeter and area of the car park?

Usually answers are given to the same number of significant figures as the least accurate measurement used in the calculation. The dimensions of the car park are all correct to 2 significant figures, so using 2 significant figures:

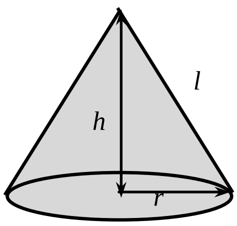
Perimeter = 420 m (to 2 sf) **Area = 8200 m^2 (to 2 sf)**

Think about…

Look at the possible values again. Are the final answers accurate to the number of significant figures that are given?

Example

The volume and surface area of a cone are given by the formulae:



$$V = \frac{1}{3}\pi r^2 h \qquad \text{and} \qquad S = \pi\, r\,(r + l)$$

where r is the radius, h the height and l the slant height.

Suppose a cone has radius 3.5 cm and height 5.2 cm, both correct to 1 decimal place.

Volume

The best estimate of $V = \frac{1}{3}\pi \times 3.5^2 \times 5.2 = 66.7$ cm^3

Upper bound of $V = \frac{1}{3}\pi \times 3.55^2 \times 5.25 = 69.3$ cm^3

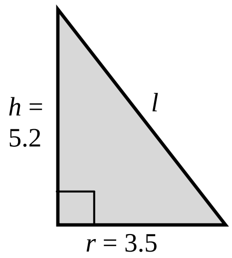
Lower bound of $V = \frac{1}{3}\pi \times 3.45^2 \times 5.15 = 64.2$ cm^3

The actual volume could lie anywhere from 64.2 cm^3 to 69.3 cm^3.
The cone's measurements were correct to 2 significant figures.
The best estimate of the volume of the cone = 67 cm^3 (to 2 sf)

This has been rounded to 1 decimal place, but note how the upper and lower bounds show that this answer may not be very accurate.

Surface area

In order to find S we need first to find l



Using Pythagoras $l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 5.2^2}$ = 6.2682 cm

Best estimate of S = $\pi \times 3.5 \times (3.5 + 6.2682) = 107.41$ cm^2

Note it is best not to round values until the end of a calculation.
Here values are given to 5 sf, but in the actual calculation all the figures were stored in the calculator's memories.

Upper bound of l = $\sqrt{3.55^2 + 5.25^2}$ = 6.3376 cm

Upper bound of S = $\pi \times 3.55 \times (3.55 + 6.3376) = 110.27$ cm^2

Lower bound of l = $\sqrt{3.45^2 + 5.15^2}$ = 6.1988 cm

Lower bound of S = $\pi \times 3.45 \times (3.45 + 6.1988) = 104.58$ cm^2

The actual surface area could lie anywhere between 104.58 cm^2 and 110.27 cm^2

The best estimate of the surface area of the cone = 110 cm^2 (to 2sf)

Think about…

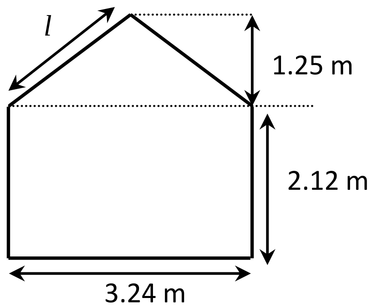
Compare the final answers with the range of possible values. Are the final answers accurate to the number of significant figures that are given?

Try these.....

1 A rectangular field has sides of length 72 metres and 58 metres, each correct to the nearest metre.
a i Find the best estimate of the perimeter of the field.
ii Calculate the upper and lower bounds of the perimeter.
b i Find the best estimate of the area of the field.
ii Calculate the upper and lower bounds of the area.

A circular pond has a radius of 2.75 metres correct to 2 decimal places (that is to the nearest centimetre). Find upper and lower bounds for the circumference and area of the pond. Give your answers to 2 decimal places.

3 The sketch shows the end of a conservatory.



Each length is correct to 2 decimal places.

Give answers to the following questions to 2 decimal places.

a i Find the best estimate of the total area.

ii Calculate the upper and lower bounds of the total area.

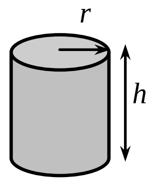
b i Calculate the best estimate of the length of the sloping edge of the roof marked l on the diagram.

ii Calculate upper and lower bounds for this length.

4 The **diameter** of a hemispherical bowl is measured to be 70 mm to the nearest millimetre.
a Use the formula $V = \frac{2}{3}\pi r^3$ (where r is the radius) to find the best estimate of its volume. Give your answer to the nearest mm^3.

b Find **(i)** the maximum possible volume **(ii)** the minimum possible volume.

5 The volume and total surface area of a cylindrical water tank are given by the formulae:

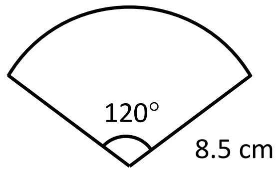


$V = \pi r^2 h$ and $S = 2\pi r(r + h)$

where r is the radius and h is the height.
One tank has radius 1.25 m and height 2.75 m, each correct to 3 significant figures.
Give answers to the following questions to 3 significant figures.

a i Calculate the best estimate of the volume of the tank.
ii Find the upper and lower bounds for the volume of the tank.
b i Calculate the best estimate for the total surface area of the tank.
ii Find the upper and lower bounds for the total surface area of the tank.

6 A metal plate is shaped as a sector of a circle with radius 8.5 cm (to 2 significant figures) and angle 120° (nearest degree).

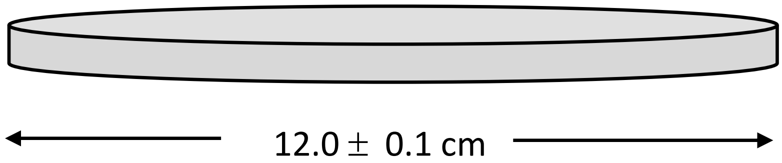


Give answers to the following questions to 3 significant figures.

a Find maximum and minimum possible values for the area of the plate.

b Find maximum and minimum possible values for its perimeter.

At the end of the activity



- What is the maximum value for the diameter of this CD?
- What is the minimum value for the diameter?
- What are the maximum and minimum values for the radius?
- Write the radius in the form $a \pm b$
- Work out a best estimate for the area of the top of the CD.
  How accurately do you think you should give the answer?
- Work out the upper and lower bounds for the area.
  Was the answer you gave reasonable?
- In general, what accuracy should you give in answers to calculations involving measurements?