In this activity you will practise the technique of completing the square, and consider how the graph of a quadratic function is related to the completed square form.

Information sheet

Completing the square means writing a quadratic in the form of a squared bracket and adding a constant if necessary.

For example, consider $x^2 + 6x + 7$.

Start by noting that

$(x + 3)^2 = (x + 3)(x + 3) = x^2 + 6x + 9$.

This is 2 more than our expression, so $x^2 + 6x + 7 = (x + 3)^2 – 2$

Minimum or maximum

One application of completing the square is finding the maximum or minimum value of the function, and when it occurs.

From above $x^2 + 6x + 7 = (x + 3)^2 – 2$

As $(x + 3)^2 \geq 0$, $(x + 3)^2 – 2 \geq –2$, so the minimum value of $x^2 + 6x + 7$ is $–2$

This occurs when $(x + 3)^2 = 0$, that is when $x = –3$.

Think about

How do you know whether the function has a minimum or a maximum?

Graphs of quadratic functions

Think about

What shape is the graph of $y = x^2 + 6x + 7$?

Can completing the square help you to sketch the graph of a quadratic function?

Example

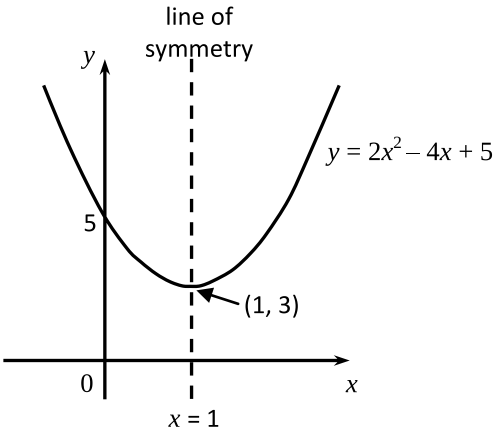
$2x^2 - 4x + 5 \;= 2[x^2 - 2x] + 5$

$= 2[(x-1)^2 - 1] + 5$ because $(x-1)^2 = (x-1)(x-1) = x^2 - 2x + 1$

$= 2(x-1)^2 + 3$

Therefore the graph of $y = 2x^2 - 4x + 5$ has a minimum point at $(1, 3)$.

Putting $x = 0$ gives the value of the y intercept to be 5.



Example

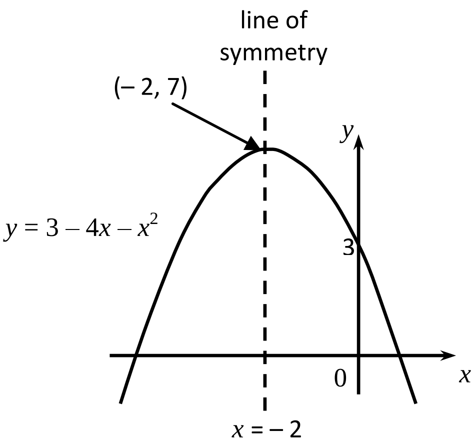
$3 - 4x - x^2 \;= 3 - [x^2 + 4x]$

$= 3 - [(x+2)^2 - 4]$ because $(x+2)^2 = (x+2)(x+2) = x^2 + 4x + 4$

$= 7 - (x+2)^2$

Therefore the graph of $y = 3 - 4x - x^2$ has a maximum point at $(-2, 7)$.

Putting $x = 0$ gives the value of the y intercept to be 3.



Reflect on your work

How does completing the square help you to sketch the graph of the function?

Can you use completing the square to tell you whether the quadratic function has any real roots?