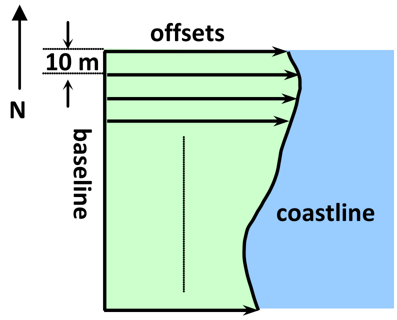
Coastal erosion is the process by which cliffs and rocks along the coast are broken up by the action of waves, wind and landslides.

The Holderness coastline of Yorkshire suffers from some of the worst coastal erosion in Europe. During the last 2000 years the coastline has retreated by almost 400 metres and over 30 villages between Bridlington and Spurn Head have been lost to the sea. Geologists study the coast to predict how it will change over time.

Geologists use increasingly sophisticated methods to address a range of questions. However this activity will show you how, given a set of measurements along the coastline, you can approximate land area and the amount of area lost over a period of time.

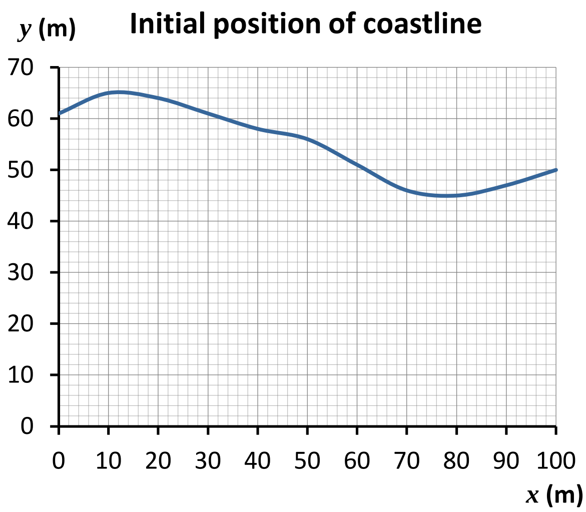
Information sheet A Estimating land area



The area of a piece of land with an irregular coastline can be estimated using a variety of methods.

Suppose we wish to estimate the area of the piece of the land shown in the sketch. Then we want to estimate how much of this land will be lost by coastal erosion over a 20 year period.

The initial position of the coastline can be defined by measuring a series of perpendicular distances (offsets) from a baseline as shown. In this case the baseline has been taken to be a line drawn from North to South and the measurements have been taken at intervals of 10 metres. The baseline can then be taken as the x axis as shown below. The table gives the offsets as y co-ordinates taken at 10 metre intervals of x.



x (metres)	y (metres)
0	61
10	65
20	64
30	61
40	58
50	56
60	51
70	46
80	45
90	47
100	50

Think about

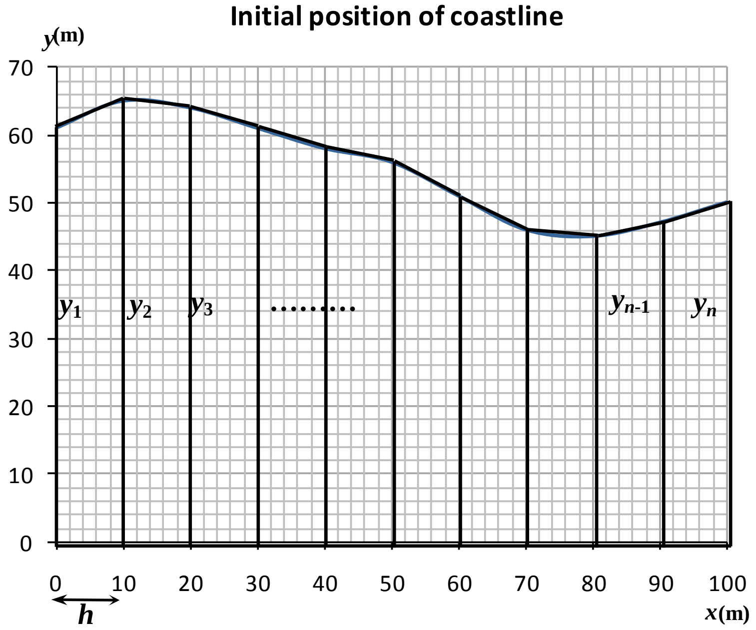
Estimate the area of the coastal land shown above. How could you obtain a better estimate? What can you say about the accuracy of the curve giving the coastline?

Information sheet B Using the Trapezium Rule

We can approximate the area shown below by a series of trapezia.

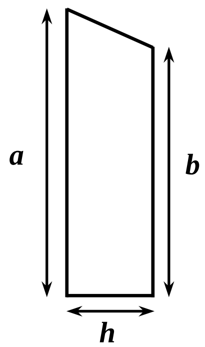
Think about

If there are n values, y_1, y_2, ... y_n, how many trapezia are there?



To think about

What does the formula $A = \frac{h(a+b)}{2}$ give for the first two trapezia on the graph?



In general terms, the total area of such a series of constant-width trapezia is given by:

$$
\begin{aligned}
A &= \frac{h(y_1 + y_2)}{2} + \frac{h(y_2 + y_3)}{2} + \frac{h(y_3 + y_4)}{2} + \ldots.. + \frac{h(y_{n-1} + y_n)}{2} \\
&= h\left(\frac{y_1}{2} + \frac{y_2}{2} + \frac{y_2}{2} + \frac{y_3}{2} + \frac{y_3}{2} + \frac{y_4}{2} + \ldots.. + \frac{y_{n-1}}{2} + \frac{y_{n-1}}{2} + \frac{y_n}{2}\right) \\
&= h\left(\frac{y_1}{2} + y_2 + y_3 + \ldots.. + y_{n-1} + \frac{y_n}{2}\right)
\end{aligned}
$$

This gives the Trapezium Rule. It can be written in words as shown on the next page.

The Trapezium Rule

Area ≈

interval width × (half first y value + half last y value + remaining y values)

Substituting the y-values from the table into the trapezium rule gives an estimate for the area of the given region:

Area (in m^2) ≈ 10 × (30.5 + 25 + 65 + 64 + 61 + 58 + 56 + 51 + 46 + 45 + 47)

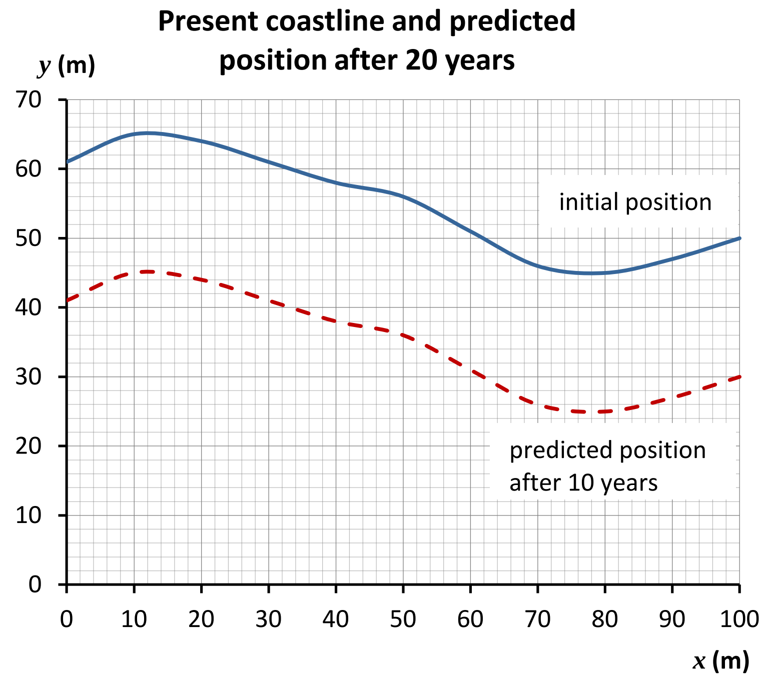
= 5485

The area of the given region is approximately 5500 m^2 (to 2 sf)

x (metres)	y (metres)
0	61
10	65
20	64
30	61
40	58
50	56
60	51
70	46
80	45
90	47
100	50

C Prediction of land loss

If we assume that the coastline erodes at a rate of 1 metre per year, then in 20 years it will recede by 20 metres. The graph shows both the initial position of the coastline and its predicted position after 20 years. The table gives the present and predicted values of the offsets from the baseline.



x (m)	y (m) initial	y_p (m) predicted
0	61	41
10	65	45
20	64	44
30	61	41
40	58	38
50	56	36
60	51	31
70	46	26
80	45	25
90	47	27
100	50	30

Using the Trapezium Rule gives an estimate of the area of the land remaining after 20 years:

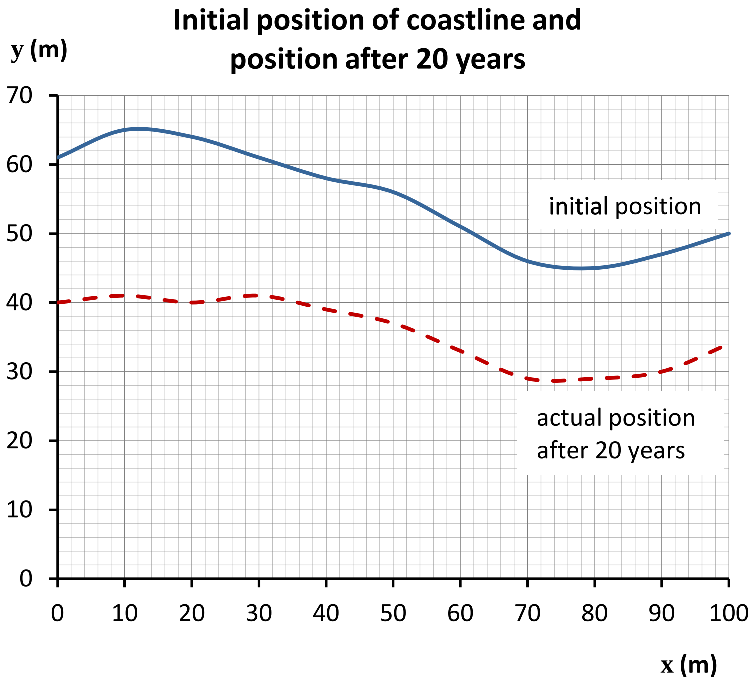
Predicted area (in m^2)
≈ 10 × (20.5 + 15 + 45 + 44 + 41 + 38 + 36 + 31 + 26 + 25 + 27) = 3485

The loss of land is approximately (5485 – 3485) m^2 = **2000 m^2**

Think about

Note that this value is equal to the length of the baseline (100 metres) multiplied by the reduction in the lengths of the offsets (20 metres).
Can you explain this using the graph?

In practice the coastline will recede more at some points than others. Suppose that more accurate offsets from the baseline after 20 years are as shown in the graph and table below.



x (m)	y (m) initial	y_a (m) after 20 yrs
0	61	40
10	65	41
20	64	40
30	61	41
40	58	39
50	56	37
60	51	33
70	46	29
80	45	29
90	47	30
100	50	34

New area after 20 years
$\approx 10 \times (20 + 17 + 41 + 40 + 41 + 39 + 37 + 33 + 29 + 29 + 30) = 3560$

The loss of land is approximately $(5485 - 3560)\ \text{m}^2 = 1925\ \text{m}^2$

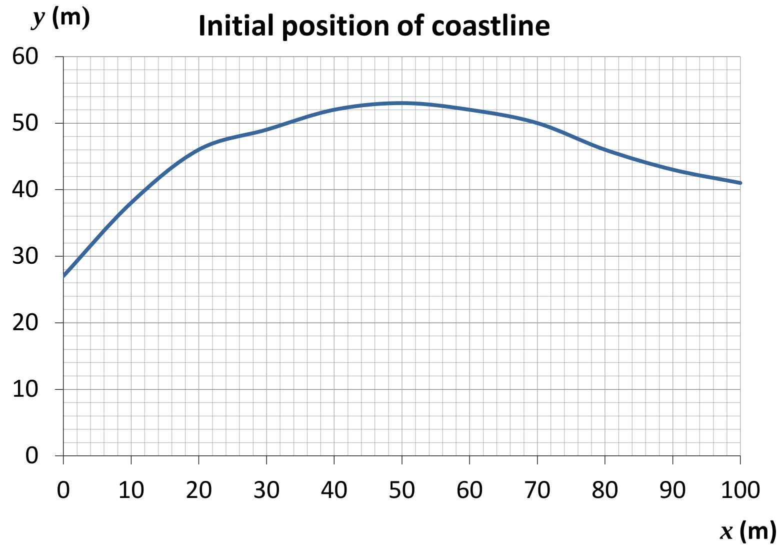
$= 1900\ \text{m}^2$ (to 2 sf)

Think about

If we were only interested in the lost area, how could we simplify the application of the Trapezium Rule?

Try these

1 The coordinates give the position of a coastline, with offsets from the x-axis, at intervals of 10 metres.

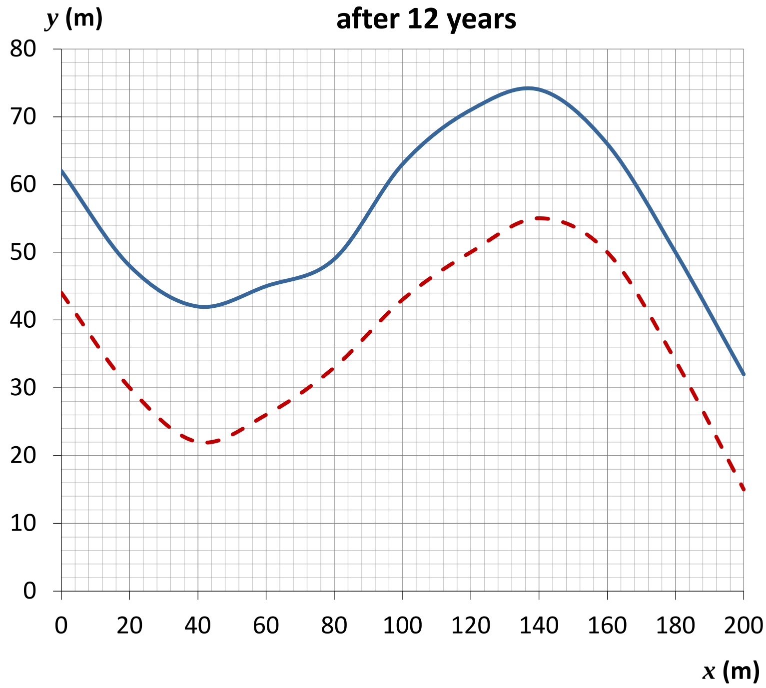


x (m)	y (m)
0	27
10	38
20	46
30	49
40	52
50	53
60	52
70	50
80	46
90	43
100	41

a Find the area of land represented by the area between the curve and the x axis.

b If the coastline recedes at an average rate of 1.8 metres per year, estimate the area of the land lost to the sea in a period of 10 years.

2 The table and graph give the initial position of a coastline and its position 12 years later.



x (m)	y (m) initial	y_a(m) after 12 yrs
0	62	44
20	48	30
40	42	22
60	45	26
80	49	33
100	63	43
120	71	50
140	74	55
160	66	50
180	50	34
200	32	15

Estimate the area of land lost.

3 The table below gives the lengths of offsets from a baseline to a coastline taken at 50 metre intervals:

Distance along baseline (m)	0	50	100	150	200	250	300	350
Initial offset (m)	76	62	54	46	40	36	34	32
Offset after 20 years (m)	39	33	28	24	20	18	16	15

Estimate the area of land that has been eroded in the 20 year interval.

Reflect on your work

What is the trapezium rule for finding the area between a graph and the x axis?

Explain the role of the various factors in this formula.

What would you need to do to have more accurate estimates?

Describe how you can use the trapezium rule to estimate land loss due to coastal erosion.