There may be times when you need extra money for unexpected or unplanned expenses. It is always best to plan ahead and save for such eventualities, but sometimes that may not be possible. Sometimes you may have no alternative to taking out a loan to cover these expenses.

There are many ways of borrowing and paying back a sum of money. There may be administration fees, regular instalments, insurance payments, or other fees which a borrower is required to pay.

The Annual Percentage Rate (APR) is a way of comparing the costs of different schemes. It is, by law, given in all advertisements for borrowing money.

This activity shows how APR is calculated in the simplest cases. These are where a sum of money is borrowed at a particular time and paid back, with interest, in a single payment at a later date.

Information sheet A Loan paid back after a whole number of years

APR formula for a single repayment

You may have met the idea of the present and future value of a sum of money before.

The future value after n years is given by

$FV = PV\,(1 + r)^n$

where PV is the present value and r is the annual interest rate (written as a decimal).

This formula can be rearranged to give:

$$PV = \frac{FV}{(1+r)^n}$$

Replacing PV by the size of a loan, C, FV by the amount repaid, A, and r by the annual rate, i, gives the following formula.

$$C = \frac{\boldsymbol{A}}{(\boldsymbol{1}+\boldsymbol{i})^n}$$

This gives the APR, i, as a decimal when a loan of £C is paid back by a single repayment £A after n years.

To find the APR, i, you need to rearrange this formula as shown in the following example.

Example

A loan of £2000 is repaid by a payment of £2500 made 3 years later

$$2000 = \frac{2500}{(1+i)^3} \qquad \Rightarrow \qquad (1+i)^3 = \frac{2500}{2000}$$

$$\Rightarrow \qquad (1+i)^3 = 1.25 \qquad \Rightarrow \qquad 1+i = \sqrt[3]{1.25} = 1.25^{\frac{1}{3}}$$

$$\Rightarrow \qquad 1+i = 1.07721... \qquad \Rightarrow \qquad i = 0.07721...$$

The APR is 7.7% (to 1 dp).

Think about…

What does $1.25^{\frac{1}{3}}$ mean? Can you find it on your calculator?

Why does a value of 0.07721 for i suggest that the APR is 7.7% (to 1 dp)?

Try these A

Find the APR for each of the following loans:

a £1000 repaid by single repayment of £1200 made 2 years later.

b £4000 repaid by single repayment of £6000 made 4 years later.

c £5000 repaid by single repayment of £7200 made 5 years later.

d £12 000 repaid by single repayment of £15 000 made 1½ years later.

e £6400 repaid by single repayment of £8000 made 2½ years later.

f £25 000 repaid by single repayment of £26 000 made 6 months later.

Think about…

The example says if $(1 + i)^3 = 1.25$ then $1+i = \sqrt[3]{1.25} = 1.25^{\frac{1}{3}}$

What key sequence do you use on your calculator if the number of years is 4 or 1½ instead of 3?

Information sheet B Loan paid back after other times

The arithmetic is more complex when the loan period is not an exact number of years.

Example

A borrower is lent £8750 on 10 December 2011 and agrees to repay this loan by a single repayment of £10 000 on 13 May 2013.

The time period from 10 December 2011 to 13 May 2013 is:

1 year and 154 days (22 weeks) = 1 + 154 ÷ 365 = 1.4219178… years.

When the numbers are difficult, use the calculator's memory or work to 6 decimal places. At the end of the calculation, round the APR to 1 decimal place.

$$8750 = \frac{10\,000}{(1+i)^{1.421918}} \quad \Rightarrow \quad (1+i)^{1.421918} = \frac{10\,000}{8750}$$

$$\Rightarrow \quad (1+i)^{1.421918} = 1.142\,857 \quad \Rightarrow \quad 1+i = 1.142\,857^{\frac{1}{1.421918}}$$

$$\Rightarrow \quad 1+i = 1.098\,460 \quad \Rightarrow \quad i = 0.098\,460$$

The APR is 9.8% (to 1 dp).

Think about…

Which key sequence do you use on your calculator to work this out?

Try these B

Find the APR for each of the loans described in the table below:

	Amount borrowed	Date of Loan	Amount repaid	Date of repayment
a	£1000	3 December 2011	£1150	17 December 2012
b	£3500	22 December 2011	£4200	26 January 2013
c	£5600	11 January 2012	£6400	11 March 2013
d	£7500	3 March 2012	£10 000	26 June 2013
e	£20 000	1 April 2012	£24 800	2 August 2014
f	£25 000	16 April 2012	£32 500	30 November 2014

Think about…

Which years are leap years?
Does this matter if you are working to 1 decimal place?

Reflect on your work

Show that the formula $C = \dfrac{A}{(1+i)^n}$ can be rearranged to give $i = \left(\dfrac{A}{C}\right)^{\frac{1}{n}} - 1$

Use this formula to check your answers to 'Try these A'.