

## Dynamic modelling 2 - Using spreadsheets

### Introduction

Students use simple models on Excel spreadsheets to look at linear and exponential growth in Activity 2. In Activity 4 they manipulate a simple predator-prey simulation to show the effects of positive and negative feedback. The model gives a very good illustration of feedback, which is one of the most important concepts needed for understanding climate models.

The activities illustrate the advantages and limitations of computer models. Actual climate models are too complex for students to explore but the gradually increasing complexity of the systems studied here illustrate well the potential use of models and their sensitivity to small changes in assumed conditions.

### The activities

Activity 2 only takes a few minutes for each model.

Activities 3 and 4 should be allowed more time, about 15 minutes each, so that students can play with changing the variables in each model and answering the questions. If time is short Activity 4 is probably the more engaging.

Activity 5 consists of short questions testing understanding of the effects of positive and negative feedback. They can be discussed in class or done for homework.

For students who have time and interest Activity 6 contains two extension activities which allow a more detailed understanding of the role of feedback in simple systems. These modelling concepts are related to the complex systems of climate change.

Note: Activity 1 is in Dynamic Modelling 1

#### How Science Works

**Da** A dynamic model is a set of proposed inter-relationships between key variables in a situation. Dynamic models can be constructed and implemented on a computer. They are widely used in science to make predictions and to test explanations in complex situations.

**Db** A dynamic model incorporates hypotheses about the important variables in a situation, and the

way they inter-relate. The outputs from the model will depend on the assumptions built into it and the data used to set the initial conditions. Data derived from a computer model are therefore less trustworthy than data that have been measured directly.

**Dc** A dynamic model shows negative feedback if a change in the input tends to be cancelled out by changes in other variables, so that the output remains relatively constant. A model may behave in this way for small changes in input, but not if the change in inputs is large.

**Dd** A dynamic model shows positive feedback if a small change in the input tends to be magnified by changes in other variables, leading to a large change in the output.

**De** Dynamic models also differ in their sensitivity to small changes in the nature of the inter-relationships processed. So we cannot ever be entirely certain of the initial conditions, predictions from a sensitive model should be treated with more caution than those from a more stable model.

## Answers to questions

### Activity 2

- 1 The gradient of the graph is the rate at which water is added, so the higher this value the steeper the straight line.
- 2 In all cases the gradient of the graph increases sharply as time proceeds. This is because the growth increases as more bacteria are present, positive feedback. Starting with a higher initial number of bacteria gives a steeper graph. A more rapid rate of growth, say 2.2 instead of 2 is initially very similar to growth rate 2. However as time proceeds the graph becomes much steeper.

### Activity 3

- 1 Over almost 1000 years the model shows quite a good fit to the data. Changing the growth rate does not improve the model over the whole period though it gives a closer final value.
- 2 Exponential growth shows a poor fit after the first few years. Reducing the growth rate improves the fit only slightly. These data suggest that negative feedback is involved - the higher the population the slower the growth rate.
- 3 This fits quite well. A growth rate of 2.4 and a limit of 51 improves the fit significantly.
- 4 Increasing the growth rate from 1.5 to 2.0 means that population takes a shorter time to reach the limit.

Raising it further produces an 'overshoot' followed by oscillation – at 2.7 the oscillations die out, but at 3.3 they get bigger in each cycle. Raising the growth rate to 3.6 produces a more complex oscillation, and further increase in the growth rate lead to dramatic and unpredictable changes in the population.

With a growth rate of 4, the model shows chaotic behaviour. A very small change of initial population in from 8.9 to 8.901, results in a very different behaviour and a completely different predicted population size 200 years later.

At growth rate 3.6 the population is projected to swing rapidly between high and low values the greatest change being from 71 and 25 million over one time unit. It is extremely unlikely that this would happen so regularly and dramatically. The model does not include factors which would operate at these high populations and positive and negative feedback are greatly exaggerated.

### Activity 4

- 1 The rabbits increase in population but this allows an increase in the fox population. Positive feedback. As the fox population increases the rabbit population growth slows and the rabbit population declines. Negative feedback. A decline in rabbits leads to a decline in foxes. Positive feedback.

Answers are not given for questions 2 - 4. Students can check their own predictions by using the model.

### Activity 5

- 1 This is positive feedback, a rise in atmospheric carbon dioxide causes a further increase in atmospheric carbon dioxide.
- 2 This is negative feedback. Plants lock up atmospheric carbon dioxide reducing the total.
- 3 This is negative feedback. More vegetation means more atmospheric carbon dioxide locked up, reducing the total.

- 4 This is positive feedback. Rising temperature causes more of the Sun's energy to be absorbed by the Earth, further warming the Earth.
- 5 This is negative feedback. If many parts of the world are less habitable then the overcrowding will lead to reduction of the population. (There may be terrible unrest and violence as competition for the remaining habitable land increases.)
- 6 The increase in standard of living will increase the population in the short term but this may have negative feedback as family size decreases, stabilising the population.

**Activity 6 - the extension activities**

- 1 Increasing the velocity increases the slope of the line. Increasing the acceleration also increases the slope, but makes the line bend upwards more sharply. You can see these effects more easily by using model 5.2 to compare the two cases. So, a car with a high initial velocity and a low acceleration can be overtaken by a car with a low initial velocity and a high acceleration (i.e. the two lines will cross each other).
- 2 Increasing the distance of the mass from the centre at the start means that the oscillations will be bigger but will still be the same frequency (i.e. the amplitude will increase but the period will stay the same). Increasing the mass means that the frequency will decrease but the size of the oscillations (i.e. the amplitude) will stay the same.

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### Introduction

You may have discussed the outcomes of some models predicting population and climate in Dynamic Modelling–1. This second activity helps you understand the principles used in modelling by working with some simple models. You will start by looking at simple systems, such as a bath filling up with water or predator prey relationships. The same basic kinds of changes that govern these simple systems can be found in more complex physical systems such as weather and climate, biological systems such as changes in human population size, and even social systems such as the financial markets.

### Building computer models

To build a computer model to make predictions about the future of a system, you need to identify:

- the variables of the system – these can be quantities (e.g. population size, carbon dioxide level) or rates (e.g. birth rate)
- the relationships between the variables (e.g. the birth rate increases the population).

Once you have written all the equations into the computer model, you can then run it. For this you would need to know:

- the starting conditions (e.g. the present sea level)

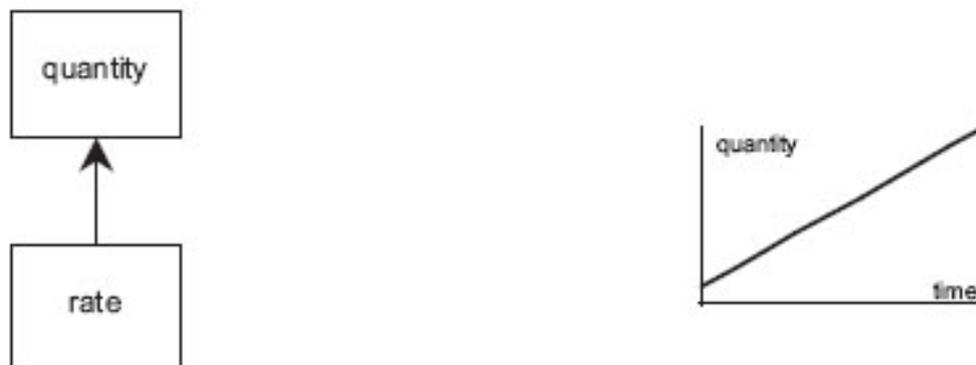
After you have entered these values into the computer model, it then carries out the calculations, and out come the results. In the following sections you will explore these ideas about how a computer model is constructed.

### Activity 2 Linear change and exponential change

- 1 Open the spreadsheet 'models1'. This represents water flowing into a bath, and is a linear change. The starting conditions consist of two values – the initial volume and the flow rate. Look at the line on the graph – what happens to it if you change the initial volume? You can use model 1.2 to compare the effects of two sets of values on the same graph. What happens if you make the flow rate bigger or smaller? You could use this model to predict how long it would take to fill your bath.
- 2 Open the spreadsheet 'models2'. This represents an exponential change. The starting conditions consist of two values – the number of bacteria at the start and the growth during one time unit. Model 2.2 allows you to compare two sets of values. Look at the line on the graph – describe what happens to it if you change the initial number. Then using 1 again as the initial number describe what happens if you make the growth rate bigger or smaller. Start with values between 1.8 and 3.

**Linear change**

Filling a bath up with water is one of the simplest examples of how a system can change over time. We can work out how much water there will be in the bath at any time in the future if we know just two things: how much water we have now and how fast it is filling up. This is represented by figure 5.

**Figure 5**

The diagram shows that a quantity (how much water in the bath) is affected by a rate (how fast the water is flowing from the tap). The graph shows how the volume of the water in the bath changes. It rises steadily, and the graph is a straight line. It is called a linear change. (This type of change is shown in the spreadsheet 'models1' in activity 2).

**Exponential change**

If you look at the way that indicators of global development have changed over the last few hundred years, you will not see this kind of change. The size of populations, energy usage, pollution – even the numbers of scientific journals and papers – have grown at an increasingly rapid rate. Again we will look at a simple system to bring out the principle behind this kind of change. Suppose we have a single bacterium. Bacteria reproduce by dividing:

$A \rightarrow 2A$

These can then divide again

$2A \rightarrow 4A$

giving four bacteria. These in turn would lead to eight and then to sixteen bacteria.

The rate of change does not stay the same; it gets faster and faster because it is affected by the number of bacteria present at any time.

**Figure 6**

Figure 6 shows that a quantity (number of bacteria present) is affected by a rate (the rate of division), but also that the rate is itself affected by the quantity. This is a feedback loop. Because making more bacteria increases the rate at which further bacteria are produced this is called positive feedback.

The graph shows how the quantity rises slowly at first, but then rapidly increases. It is called an exponential change. (This type of change is shown in the spreadsheet 'models2' in activity 2).

*A note about the spreadsheets: The spreadsheet workbooks are 'protected' so you can only enter values in the correct cells. These cells are shaded in blue. The first cell of each spreadsheet has a 'comments' box, which you can see by pointing the cursor to the cell. Most of the workbooks consist of several sheets, which you can access by clicking on the tabs at the bottom of the screen.*

### Limitations of models

All models are simplifications, and therefore have their limitations. They may work well within certain constraints, but may break down if they are used beyond what they were intended to do. A simple illustration is provided by the model of a bath filling up. There is nothing in this model that tells us that there is a limit to how much water a bath can contain before it overflows!

The growth of bacteria shows an exponential rise as a colony begins to develop (typically, the number of bacteria doubles every half-hour), but eventually slows as they become overcrowded. Yeast cells in a sugar solution multiply rapidly at first, but cannot survive in too high a concentration of alcohol; it is this that limits the strength of wine or beer.

### Activity 3 Population of England and Wales

In this activity you will look at some data about the population changes in England and Wales, and the way in which models can be varied to fit the data that describe their behaviour.

You will meet the concept of negative feedback in this activity. If the population grows too high a range of factors tends to decrease the rate of population growth. The effect of a high population is to slow the growth rate, this is an example of negative feedback.

- 1 Open 'models3'. This shows the population of England and Wales from 1086. It also shows an exponential model line on the same graph. Can you make a better fit by changing the growth rate? How well do you think that exponential growth describes the way that the population has changed?
- 2 Turn to the next sheet of the same workbook (model 3.2). This focuses on the population change from 1801. Does the exponential model line fit the data well? Can you make it fit any better by changing the growth rate? Describe the changes you tried. Do you think this shows negative feedback?
- 3 Turn to the next sheet (model 3.3). This also looks at the population change from 1801, but with a modification to the exponential model which places an upper limit on the population size. Does this model describe the data better?
- 4 Turn to the next sheet (model 3.4). This shows the same model as on the previous sheet (model 3.3), but the data has been omitted. The purpose of this is to illustrate how the model used can lead to chaotic behaviour with certain starting conditions.

The initial values are those which give a good fit to the actual data. Increase the growth, trying the following values in turn, and observe the shape of the graph: 1.5, 2.0, 2.5, 2.7, 3.0, 3.3, 3.6, 3.8, 4.0.

The graph now shows very unpredictable or chaotic behaviour. With higher growth rate it is now very sensitive to even small changes in the starting conditions. For example, notice the effect of changing the initial population from 8.9 to 8.901. Describe the behaviour of the model with a growth rate of 3.6. Does this suggest any limitations in the model?

### Chaos

The last model you looked at in activity 3 illustrates a very important point about complexity. We can have complexity in the structure of a model in which there are a large number of variables and relationships. But more fundamentally, we can have complexity in the behaviour of even very simple models.

This understanding of the way that systems can behave is relatively recent, and has led to the opening up of a whole new branch of research on chaos theory. This studies systems in which the behaviour is very sensitive to the starting conditions. The weather is an example of such a system – it can make it very difficult to produce accurate weather forecasts because of this sensitivity. It is sometimes called the 'butterfly effect'; to say that a butterfly flapping its wings on one side of the world could lead to a storm on the other is a striking way of making the point about the sensitivity of chaotic systems.

In an ecosystem, for example, there are many different species, each with their own feeding relationships to other species; the sizes of the populations of these are affected by many positive and negative feedback loops.

### Activity 4 Predator-prey populations

In this activity, you will look at a highly simplified model of part of an ecosystem in which there are just two species – a predator (fox) and prey (rabbit) – and to try to interpret the behaviour of this system in terms of its positive and negative feedback loops.

- 1 Open 'models4'. This is a simple model of a predator-prey system. In this model, rabbits breed and die, and so too do foxes. However the populations of the rabbits and foxes are coupled to each other, since the rate at which rabbits die is related to the number of foxes (they eat them) and the rate at which foxes breed is related to the number of rabbits (they need food to produce offspring.) Look at the shape of the lines on the graph carefully. Can you explain what is happening? Try using the ideas of positive and negative feedback.
- 2 What do you think would happen if you started with double the rabbit population? Make a prediction, again considering positive and negative feedback, and then test it on the model.
- 3 What do you think would happen if you doubled the birth rate of foxes? Make a prediction and then test it on the model.
- 4 What do you think would happen if you doubled the birth rate of rabbits? Make a prediction and then test it on the model.

It is not easy to predict how a system like this will behave. For example, one might imagine that increasing the death rate for rabbits would lead to smaller populations of rabbits. In fact, the peaks of the rabbit population are much higher. The higher death rate for rabbits leads to faster reductions in the population, leading to a crash in the fox population, which allows the rabbit population to grow bigger. Depending on the values used, a system like this might simply oscillate, but it may show chaotic behaviour. This is for a system with just two species; for an ecosystem with thousands of interacting species it will be very difficult to make predictions.

## Activity 5 Questions on interpreting feedback in complex systems

Models of global population and climate changes contain many interacting variables, with positive and negative feedback loops. In these questions you will consider some of these interactions. Think about the following effects and explain them in terms of positive or negative feedback.

- 1 As carbon dioxide levels increase the global temperature will rise. As the oceans become warmer they will be able to hold less dissolved carbon dioxide.
- 2 Increasing levels of carbon dioxide may encourage greater growth of vegetation through photosynthesis.
- 3 As carbon dioxide levels increase the global temperature will rise. This will increase evaporation and lead to more rainfall in some parts of the world. An increase in rainfall may encourage more vegetation to grow.
- 4 An increase in global temperature will mean that sea ice will tend to melt. Less radiation from the sun will be reflected by the white ice and more will be absorbed by the earth's surface.
- 5 An increasing world population will tend to increase energy usage and carbon dioxide levels. The climate change caused by this may make many parts of the world less habitable.
- 6 An increase in the standard of living may result in a lowering of the death rate in the population. This in turn may result in people tending to have smaller families.

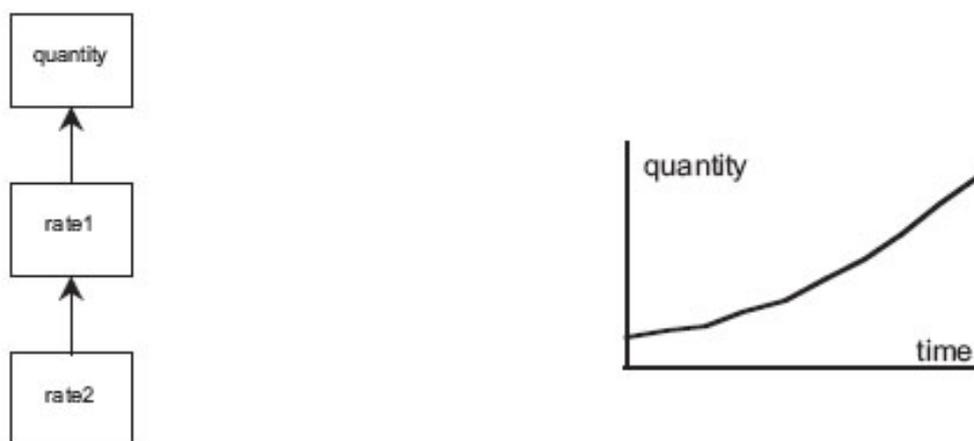
## Extension activities

### Rates that affect rates, negative feedback and oscillation

On the news, you can often find examples of people (particularly politicians) talking about the way something is changing in a way that can be misunderstood. For example, 'the rate of increase in crime is falling' does not mean that crime is falling, though it might sound that way to a casual listener – it just means that it is not increasing as fast as it was. So as well as thinking about rates of change of quantities, we need to think about the rates of change of the rates themselves!

Dropping a ball on the floor seems a simple kind of change. Predicting how far it travels is a little bit more difficult than predicting how a bath fills up with water. This is because the rate at which it falls (its velocity) does not stay the same. It starts slowly and then speeds up – in other words it accelerates. Its acceleration is the rate at which the velocity changes. The change is represented by figure 7.

Figure 7



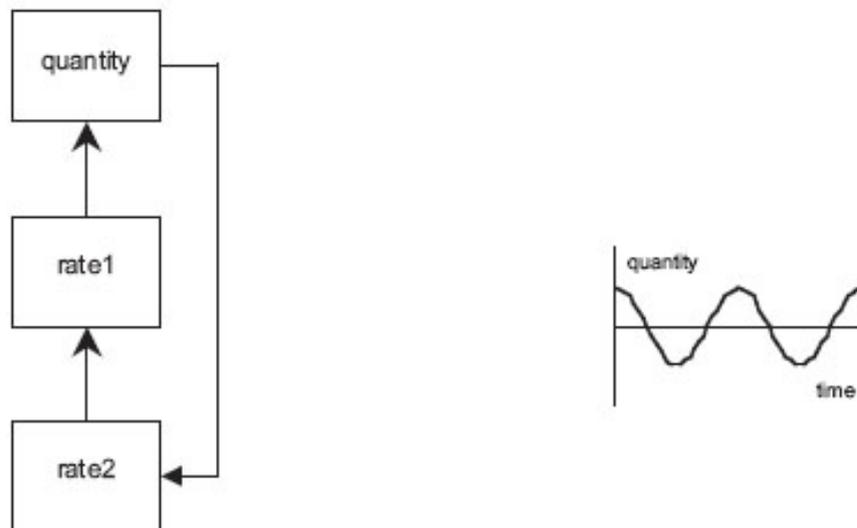
Here we have a quantity (the distance that the ball has dropped) affected by a rate (the velocity) which in turn is affected by another rate (the acceleration). The graph looks a little similar to the graph of exponential change, but although it does get faster and faster, it does not show the steep explosive rise of exponential growth. (Though like an explosion, in practice it would not go on rising forever because there are limits here too – air resistance would mean it would reach a maximum speed.)

What happens when we put feedback into a system like this? An example would be a mass on the end of a spring. Now when the object falls it starts to get pulled back by the spring. The further down it travels the bigger the pull upwards - this is negative feedback. However, even when it is being pulled upwards, it doesn't come back straight away – its momentum carries it down before it comes back. It is negative feedback with a delay and this makes the system oscillate. You met this important idea in dealing with population changes. The change is represented by figure 9.



Figure 8

Figure 9



Again we have a quantity (the distance that the mass has fallen) affected by a rate (the velocity of falling) which in turn is affected by another rate (the acceleration). But here, the acceleration is affected by the distance the mass has fallen.

### Activity 6 Rates that affect rates

- 1 Open 'models5'. This shows the distance travelled by an accelerating car – this is like a falling ball – an object moving with constant acceleration. What happens to the shape of the graph if you change the initial velocity or the acceleration?
- 2 Open 'models6'. This shows an oscillating spring. What do you think will happen if you change the distance of the mass from the centre at the start? Make a prediction and then try it. What do you think will happen if you change the size of the mass? Again make a prediction and then try it.

Feedback can lead to stability, as in a thermostat, but as you have already seen, even in simple systems feedback can cause them to behave in ways that are difficult to predict. Understanding feedback is central to an understanding of complex systems and how they behave. Complex systems like global climate often behave in surprising ways.